

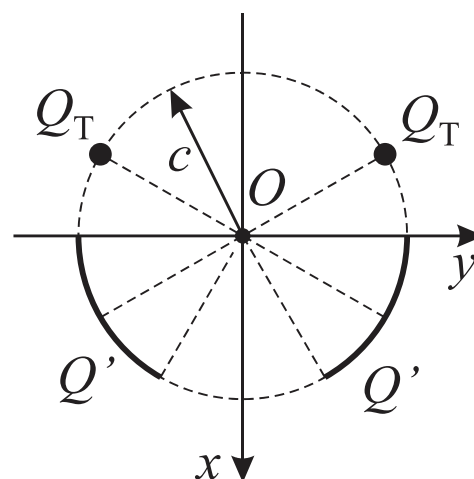
Osnovi elektrotehnike 1
(I kolokvijum)

ZADACI

Zadatak 1. Dva jednaka tanka štapa savijena u obliku luka poluprečnika c , i dva tačkasta naelektrisanja, postavljeni su u Dekartov koordinatni sistem i prikazani su na slici 1. Štapovi su naelektrisani ravnomerno jednakim podužnim naelektrisanjima Q' , dok je količina tačkastih naelektrisanja Q_T .

- Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja koji u tački O stvaraju oba štapa.
- Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u tački O koji stvaraju tačkasta naelektrisanja.
- Odrediti poluprečnik c tako, da ukupan vektor jačine električnog polja u tački O bude jednak nuli.
- Odrediti ukupan potencijal tačke O koji stvaraju tačkasta naelektrisanja, za referentnu tačku u beskonačnosti.

Brojni podaci su: $Q' = (1 + \sqrt{3}) \mu\text{C/m}$, $Q_T = 50 \text{ nC}$,
 $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$.

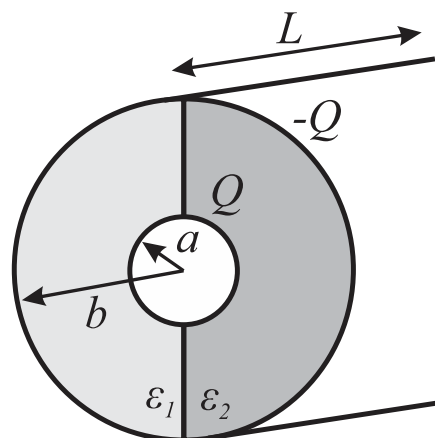


Slika 1.

Zadatak 2. Koaksijalni kabl sa slike 2, poluprečnika elektroda $a = 9 \text{ mm}$ i $b = 15 \text{ mm}$, dužine $L = 2 \text{ m}$, ispunjen je sa dva sloja dielektrika relativnih permitivnosti $\epsilon_1 = 2$ i $\epsilon_2 = 9$. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i $-Q$.

- Razmotriti granične uslove i odrediti kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.
- Izračunati brojnu vrednost kapacitivnosti kabla.
- Izračunati maksimalni napon na koji sme da se priključi kabl.

Ostali brojni podaci: $E_{\check{c}1} = 75 \text{ kV/cm}$, $E_{\check{c}2} = 85 \text{ kV/cm}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 1
(II kolokvijum)

K2

22. 9. 2026.

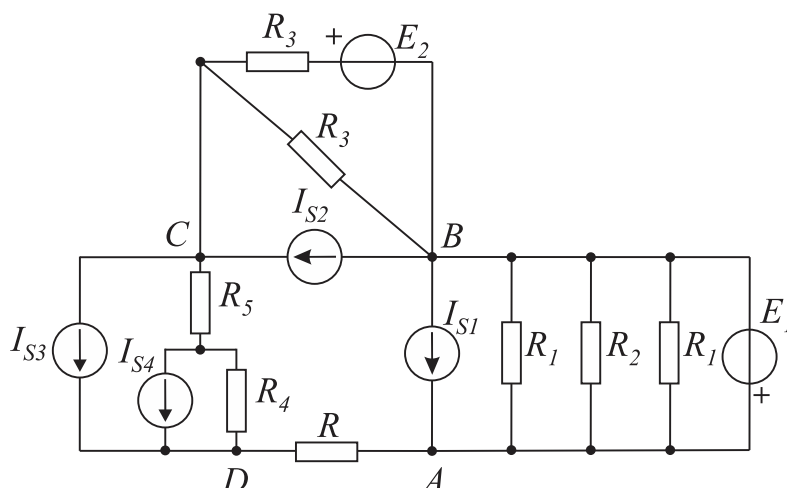
ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

- Deo mreže između tačaka A i B predstaviti ekvivalentnim Tevenenovim generatorom E_{T1} , R_{T1} .
- Deo mreže između tačaka B i C predstaviti ekvivalentnim Tevenenovim generatorom E_{T2} , R_{T2} .
- Deo mreže između tačaka C i D predstaviti ekvivalentnim Tevenenovim generatorom E_{T3} , R_{T3} .
- U uprošćenom kolu, pokazati da je zadovoljena teorema održanja snage.

Brojni podaci su:

$R = 1 \text{ k}\Omega$, $R_1 = R_2 = 3 \text{ k}\Omega$, $R_3 = 4 \text{ k}\Omega$,
 $R_4 = R_5 = 2 \text{ k}\Omega$, $E_1 = 12 \text{ V}$, $E_2 = 6 \text{ V}$,
 $I_{S1} = 1 \text{ mA}$, $I_{S2} = 1,5 \text{ mA}$,
 $I_{S3} = 3 \text{ mA}$, $I_{S4} = 4 \text{ mA}$,
 $C_1 = 6 \text{ }\mu\text{F}$, $C_2 = 4 \text{ }\mu\text{F}$

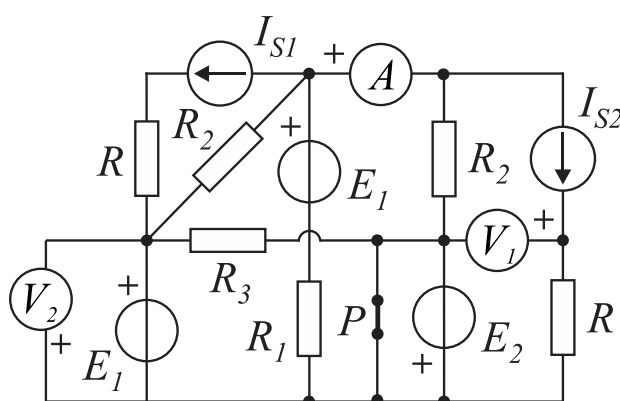


Slika 1.

Zadatak 2. U kolu vremenski konstantne struje sa slike 2 odrediti:

- otpornost nepoznatog otpornika R_1 , tako da se na njemu razvija maksimalna moguća snaga,
- promenu pokazivanja idealnih mernih instrumenata prilikom otvaranja prekidača P i
- snagu strujnog generatora I_{S1} , pri zatvorenom prekidaču.

Brojni podaci su: $E_1 = 3 \text{ V}$, $E_2 = 5 \text{ V}$,
 $R = 50 \text{ }\Omega$, $R_2 = 10 \text{ }\Omega$, $R_3 = 20 \text{ }\Omega$, $I_{S1} = 0,5 \text{ A}$,
 $I_{S2} = 1,5 \text{ A}$.



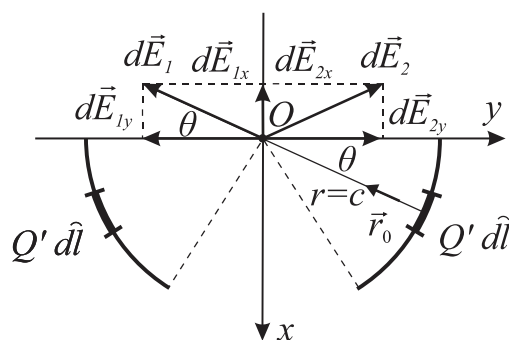
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1

a)



$$d\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \cdot \vec{r}_0, \quad dE_1 = \frac{Q' dl}{4\pi\epsilon_0 c^2} = \frac{Q' dl}{4\pi\epsilon_0 c^2} = \frac{Q' d\theta}{4\pi\epsilon_0 c}$$

$$|d\vec{E}_1| = |d\vec{E}_2|$$

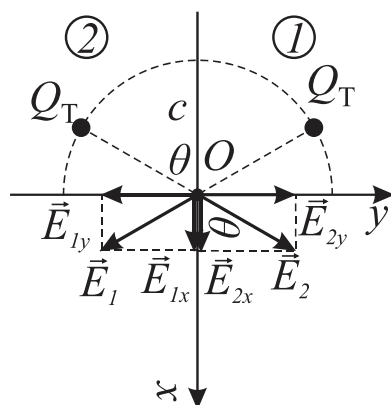
$$d\vec{E}_{1y} = -d\vec{E}_{2y} \Rightarrow E_y = 0$$

$$dE_{1x} = dE_{2x} = dE_1 \sin \theta$$

$$E_l = 2E_{1x} = 2 \int_{\theta=0}^{\theta=\frac{\pi}{3}} \frac{Q'}{4\pi\epsilon_0 c} \cdot \sin \theta \cdot d\theta = 2 \cdot \frac{Q'}{4\pi\epsilon_0 c} \left(\cos 0 - \cos \frac{\pi}{3} \right) = 2 \cdot \frac{Q'}{4\pi\epsilon_0 c} \left(1 - \frac{1}{2} \right)$$

$$E_l = \frac{Q'}{2\pi\epsilon_0 c} \cdot \frac{1}{2} \quad \boxed{\vec{E}_l = \frac{Q'}{4\pi\epsilon_0 c} (-\vec{i}_x)}$$

b)



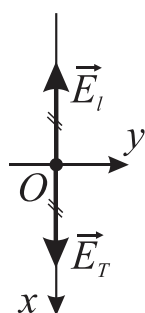
$$E_1 = \frac{Q_T}{4\pi\epsilon_0 c^2}, \quad |\vec{E}_1| = |\vec{E}_2|$$

$$\vec{E}_{1y} = -\vec{E}_{2y} \Rightarrow E_y = 0$$

$$E_{1x} = E_{2x} = E_1 \cos \theta = E_1 \cos \frac{\pi}{3}$$

$$E_{T12} = 2E_{1x} = 2E_1 \cos \frac{\pi}{3} = 2 \cdot \frac{Q_T}{4\pi\epsilon_0 c^2} \cdot \frac{1}{2} = \frac{Q_T}{4\pi\epsilon_0 c^2} \quad \boxed{\vec{E}_{T12} = \frac{Q_T}{4\pi\epsilon_0 c^2} \vec{i}_x}$$

c)



$$\vec{E}_l = \frac{Q'}{4\pi\epsilon_0 c} (-\vec{i}_x)$$

$$\vec{E}_T = \vec{E}_{T12} = \frac{Q_T}{4\pi\epsilon_0 c^2} \vec{i}_x$$

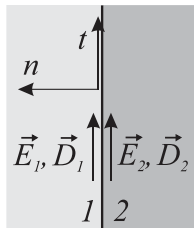
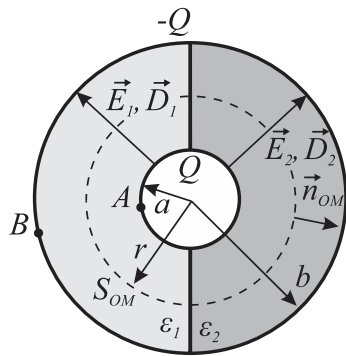
$$\vec{E}_O = 0 \Rightarrow E_l = E_T \Rightarrow \frac{Q'}{4\pi\epsilon_0 c} = \frac{Q_T}{4\pi\epsilon_0 c^2}$$

$$Q' = \frac{Q_T}{c} \Rightarrow c = \frac{Q_T}{Q'} = \frac{50 \cdot 10^{-9}}{(1 + \sqrt{3}) \cdot 10^{-6}} \quad \boxed{c = 1,83 \text{ cm}}$$

d)

$$V_T = V_{T1} + V_{T2} = 2V_{T1} = 2 \frac{Q_T}{4\pi\epsilon_0 c} = \frac{Q_T}{2\pi\epsilon_0 c} \quad \boxed{V_T = 17,99 \text{ kV}}$$

a)



Granični uslov:

$$E_{t1} = E_{t2}$$

$$E_1 = E_2 = E$$

$$D_{n1} = D_{n2} = 0$$

$$D_1 = \varepsilon_1 E$$

$$D_2 = \varepsilon_2 E$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{\text{slobodno u } S}$$

$$\int_{S_{B1}} \vec{D} \cdot d\vec{s} + \int_{S_{B2}} \vec{D} \cdot d\vec{s} + \int_{S_{OM}} \vec{D} \cdot d\vec{s} = Q_{\text{slobodno u } S}$$

$$\angle(\vec{D}, \vec{n}_{B1}) = 90^\circ \quad \angle(\vec{D}, \vec{n}_{B2}) = 90^\circ \quad \angle(\vec{D}, \vec{n}_{OM}) = 0^\circ$$

$$\int_{S_{OM1}} D_1 ds + \int_{S_{OM2}} D_2 ds = Q$$

$$D_1 r \pi L + D_2 r \pi L = Q$$

$$\varepsilon_1 E r \pi L + \varepsilon_2 E r \pi L = Q$$

$$E = \frac{Q}{\varepsilon_1 r \pi L + \varepsilon_2 r \pi L} = \frac{Q}{(\varepsilon_1 + \varepsilon_2) r \pi L}, \quad a < r < b$$

$$D_1 = \varepsilon_1 E = \varepsilon_1 \frac{Q}{(\varepsilon_1 + \varepsilon_2) r \pi L}, \quad a < r < b$$

$$D_2 = \varepsilon_2 E = \varepsilon_2 \frac{Q}{(\varepsilon_1 + \varepsilon_2) r \pi L}, \quad a < r < b$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_a^b E dr = \int_a^b \frac{Q}{(\varepsilon_1 + \varepsilon_2) r \pi L} dr = \frac{Q}{(\varepsilon_1 + \varepsilon_2) \pi L} \ln \frac{b}{a}$$

$$C = \frac{Q}{U_{AB}} \quad C = \frac{(\varepsilon_1 + \varepsilon_2) \pi L}{\ln \frac{b}{a}} \quad C = 1197,4 \text{ pF}$$

c)

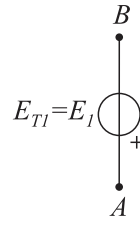
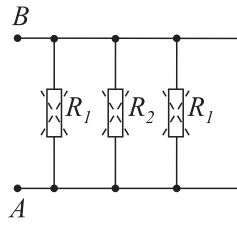
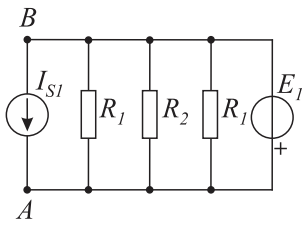
$$E_{\max 1} = E_{\max 2} = E_{\max(r=a)} = \frac{Q_{\max}}{(\varepsilon_1 + \varepsilon_2) a \pi L} = \min\{E_{C1}, E_{C2}\} = E_{C1} = 75 \frac{\text{kV}}{\text{cm}}$$

$$\frac{C U_{\max}}{(\varepsilon_1 + \varepsilon_2) a \pi L} = E_{C1}$$

$$U_{\max} = \frac{E_{C1} (\varepsilon_1 + \varepsilon_2) a \pi L}{C} = \frac{E_{C1} (\varepsilon_1 + \varepsilon_2) a \pi L}{\frac{(\varepsilon_1 + \varepsilon_2) \pi L}{\ln \frac{b}{a}}} = E_{C1} a \ln \frac{b}{a} \quad U_{\max} = 34,5 \text{ kV}$$

K2 Z1

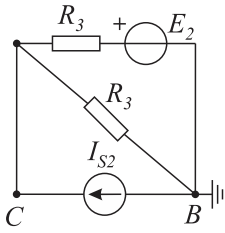
a)



$$R_{T1} = 0$$

$$E_{T1} = E = 12 \text{ V}$$

b)

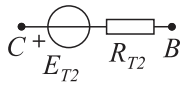


$$R_{T2} = R_3 \parallel R_3 = R_3/2 = 2 \text{ k}\Omega$$

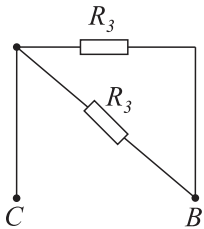
$$\left(\frac{1}{\infty} + \frac{1}{R_3} + \frac{1}{R_3} \right) \cdot V_C = I_{S2} + \frac{E_2}{R_3}$$

$$2V_C = I_{S2}R_3 + E_2$$

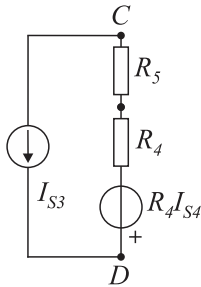
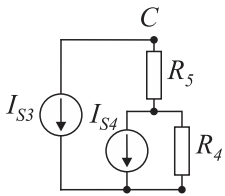
$$V_C = \frac{I_{S2}R_3 + E_2}{2} = \frac{4 \text{ k} \cdot 1,5 \text{ mA} + 6}{2} = 6 \text{ V}$$



$$E_{T2} = U_{CB} = 6 \text{ V}$$



c)

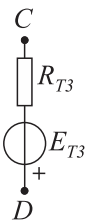
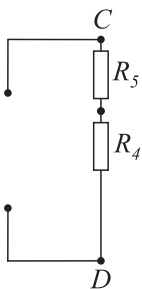


$$E_{T3} = U_{DC} = I_{S4}R_4 + (R_4 + R_5)I_{S3} = 4 \text{ mA} \cdot 2 \text{ k} + 4 \text{ k} \cdot 3 \text{ mA}$$

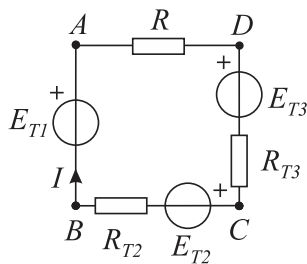
$$E_{T3} = 20 \text{ V}$$

$$R_{T3} = R_4 + R_5$$

$$R_{T3} = 4 \text{ k}\Omega$$



d)



TOS:

$$P_E = P_R$$

$$(-E_{T3} - E_{T2} + E_{T1})I = (R_{T3} + R_{T2} + R)I^2 \Rightarrow$$

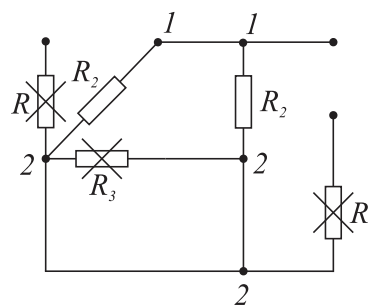
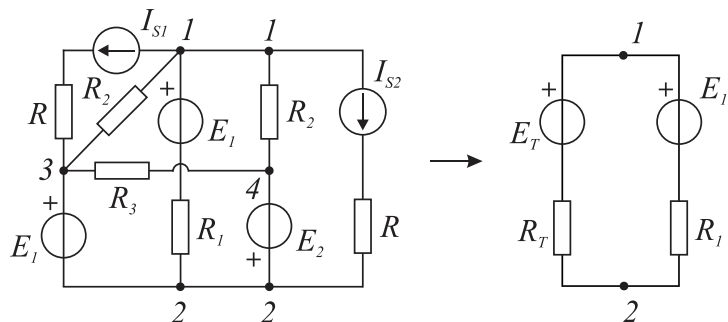
$$-E_{T3} - E_{T2} + E_{T1} = (R_{T3} + R_{T2} + R)I$$

$$20 + 6 - 12 = (2 \text{ k} + 4 \text{ k} + 1 \text{ k}) \cdot 2 \text{ mA}$$

$$14 \text{ W} = 14 \text{ W}$$

K2 Z2

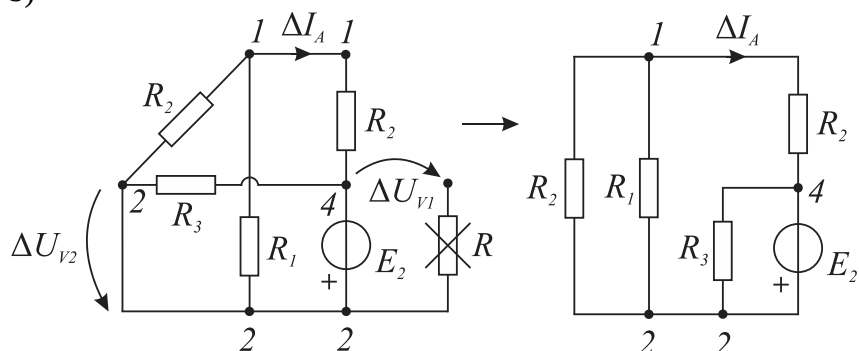
a)



$$R_T = R_{12} = R_2 \parallel R_2 = \frac{R_2}{2} = 5\Omega$$

$$R_1 = R_T = 5\Omega \quad \text{uslov prilagođenja po snazi}$$

b)



$$\Delta U_{v1} = E_2$$

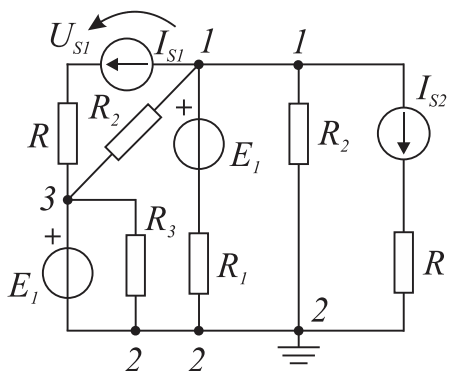
$$\Delta U_{v1} = 5V$$

$$\Delta U_{v2} = 0V$$

$$\Delta I_A = \frac{E_2}{R_1 \parallel R_2 + R_2}$$

$$I_A = 0,375A$$

c)



$$V_2 = 0V$$

$$V_3 = E_1 = 3V$$

$$V_1 \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_2} \right) - V_3 \frac{1}{R_2} = -I_{S1} + \frac{E_1}{R_1} - I_{S2}$$

$$V_1 \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{10} \right) - 3 \frac{1}{10} = -0,5 + \frac{3}{5} - 1,5 / \cdot 50$$

$$20V_1 - 15 = -25 + 30 - 75$$

$$V_1 = -2,75V$$

$$P_{S1} = U_{S1} I_{S1} = (R I_{S1} + V_3 - V_1) I_{S1}$$

$$P_{S1} = 15,375 W$$