

Osnovi elektrotehnike 1
(I kolokvijum)

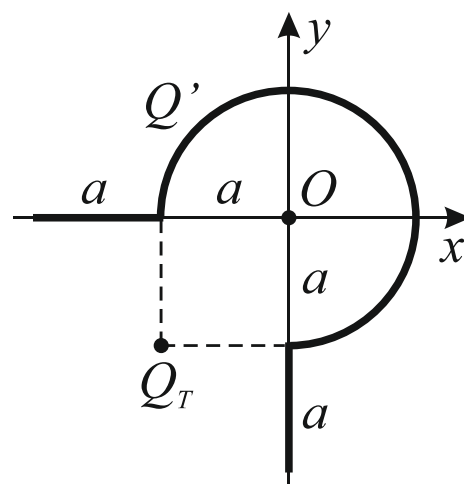
K1

22.09.2026.

ZADACI

Zadatak 1. Tanak štap, od izolacionog materijala, savijen kao što je prikazano na slici 1, naelektrisan je ravnomerno podužnom gustinom naelektrisanja Q' . Štap se sastoji od lučnog dela u obliku 3/4 kruga, poluprečnika a , i dva pravolinijska segmenta, dužine a . Struktura se nalazi u x - y ravni zadatog koordinatnog sistema, u čijem trećem kvadrantu se nalazi tačkasto naelektrisanje, Q_T . Sredina je vazduh. Izvesti, u opštim brojevima, izraz za ukupni vektor jačine električnog polja u tački O (centar koordinatnog sistema).

Brojni podaci su: $a = 5 \text{ cm}$, $Q' = 50 \text{ } \mu\text{C/m}$, $Q_T = 5 \text{ } \mu\text{C}$,
 $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$.

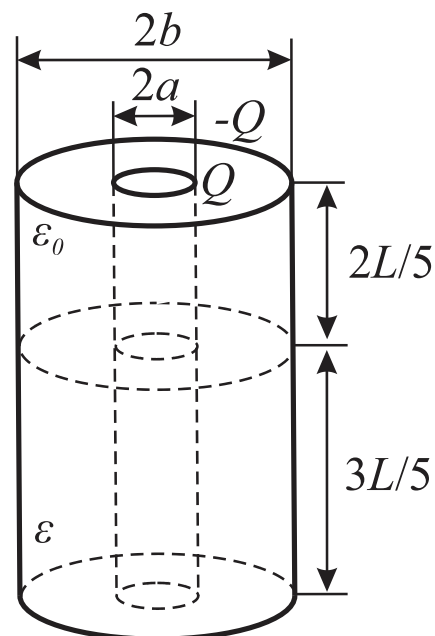


Slika 1.

Zadatak 2. Koaksijalni kabl poluprečnika elektroda a i b , dužine L , ispunjen je do 3/5 svoje zapremine čvrstim dielektrikom relativne permitivnosti $\epsilon_r = 2$, i postavljen u vertikalni položaj, kao što je prikazano na slici 2. Elektrode kondenzatora su naelektrisanje Q i $-Q$.

- Izvesti, u opštim brojevima, izraz za kapacitivnost kondenzatora u vertikalnom položaju.
- Izračunati vrednost kapacitivnosti kondenzatora.
- Za koliko će se promeniti vrednost kapacitivnosti kondenzatora kada se on postavi u horizontalni položaj?
- Odrediti ukupnu količinu vezanog naelektrisanja, koje se formira uz spoljašnju elektrodu.

Brojni podaci su: $a = 1 \text{ mm}$, $b = 2,7 \text{ mm}$, $L = 5 \text{ m}$, $Q = 10 \text{ nC}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 1
(II kolokvijum)

K2

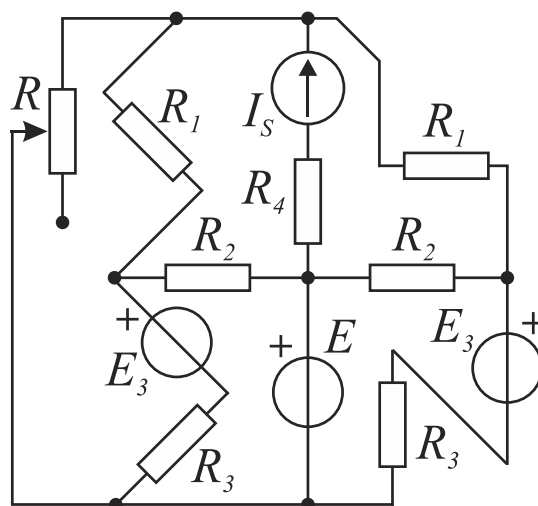
22.09.2026.

ZADACI

Zadatak 1. Klizač promenljivog otpornika postavljen je u položaj u kome je vrednost otpornosti R takva da se na njemu razvija maksimalno moguća snaga, u mreži prikazanoj na slici 1.

- Primenom Tevenenove teoreme, izračunati vrednost otpornosti R i njegove maksimalne snage.
- Izračunati i skicirati grafički kako se menja snaga promenljivog otpornika, ako se vrednost njegove otpornosti menja od vrednosti 0 do vrednosti jednako $5R$.

Brojni podaci su: $R_1 = 4 \Omega$, $R_2 = R_3 = 16 \Omega$, $R_4 = 50 \Omega$, $I_S = 1 \text{ A}$, $E = 4 \text{ V}$, $E_3 = 20 \text{ V}$.

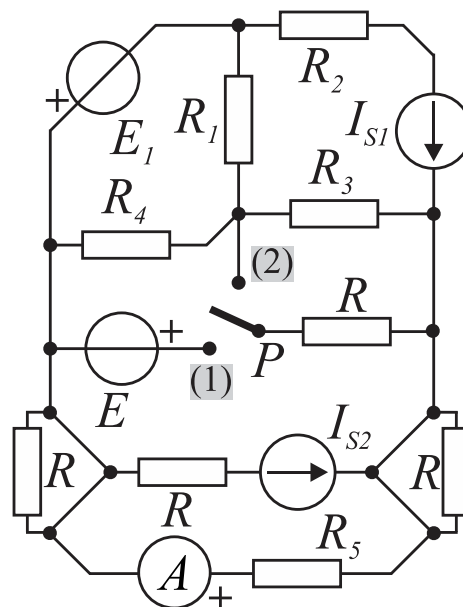


Slika 1.

Zadatak 2. Kada se u mreži sa slike 2, preklopnik prebaci iz položaja (2) u položaj (1), struja koju meri ampermetar se poveća za $\Delta I_A = 90 \text{ mA}$.

- Primenjujući teoremu superpozicije, izračunati vrednost ems naponskog generatora, E .
- Izračunati snagu naponskog generatora ems E , kada je prekidač u položaju (1).
- Izračunati snagu strujnog generatora I_{S1} , kada je prekidač u položaju (1).

Brojni podaci su: $R = R_2 = 10 \Omega$, $R_1 = R_4 = 80 \Omega$, $R_3 = R_5 = 20 \Omega$, $E_1 = 12 \text{ V}$, $I_{S1} = 0,1 \text{ A}$, $I_{S2} = 0,2 \text{ A}$.

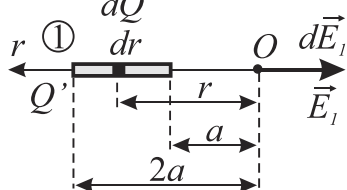
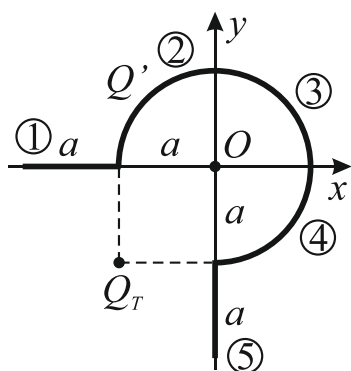


Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

I-1



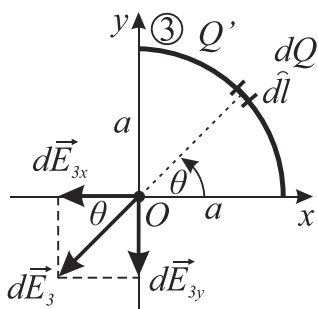
$$dE_1 = \frac{dQ}{4\pi\epsilon_0 r^2} = \frac{Q' dr}{4\pi\epsilon_0 r^2}$$

$$E_1 = \int_{\text{po štapu}} dE_1 = \frac{Q'}{4\pi\epsilon_0} \int_a^{2a} \frac{dr}{r^2} = \frac{Q'}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{2a} \right)$$

$$\vec{E}_1 = \frac{Q'}{8\pi\epsilon_0 a} \cdot \vec{i}_x$$

$$\vec{E}_5 = \frac{Q'}{8\pi\epsilon_0 a} \cdot \vec{i}_y$$

Zbog simetrije je: $\vec{E}_2 + \vec{E}_4 = 0$



$$dE_{3x} = dE_3 \cos \theta = \frac{dQ}{4\pi\epsilon_0 r^2} \cos \theta = \frac{Q' dl}{4\pi\epsilon_0 a^2} \cos \theta = \frac{Q' a d\theta}{4\pi\epsilon_0 a^2} \cos \theta$$

$$E_{3x} = \int_{\text{po luku}} dE_{3x} = \frac{Q'}{4\pi\epsilon_0 a} \int_0^{90^\circ} \cos \theta d\theta = \frac{Q'}{4\pi\epsilon_0 a} (\sin 90^\circ - \sin 0^\circ) = \frac{Q'}{4\pi\epsilon_0 a} (1 - 0)$$

$$E_{3x} = \frac{Q'}{4\pi\epsilon_0 a}$$

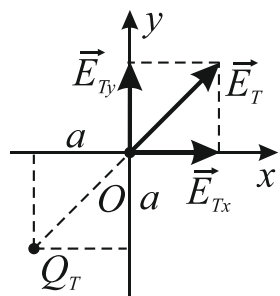
$$\vec{E}_{3x} = E_{3x} \cdot (-\vec{i}_x)$$

$$dE_{3y} = dE_3 \sin \theta = \frac{dQ}{4\pi\epsilon_0 r^2} \sin \theta = \frac{Q' dl}{4\pi\epsilon_0 a^2} \sin \theta = \frac{Q' a d\theta}{4\pi\epsilon_0 a^2} \sin \theta$$

$$E_{3y} = \int_{\text{po luku}} dE_{3y} = \frac{Q'}{4\pi\epsilon_0 a} \int_0^{90^\circ} \sin \theta d\theta = \frac{Q'}{4\pi\epsilon_0 a} (\cos 0^\circ - \cos 90^\circ) = \frac{Q'}{4\pi\epsilon_0 a} (1 - 0)$$

$$E_{3y} = \frac{Q'}{4\pi\epsilon_0 a}$$

$$\vec{E}_{3y} = E_{3y} \cdot (-\vec{i}_y)$$



$$E_T = \frac{Q_T}{4\pi\epsilon_0 (a\sqrt{2})^2} = \frac{Q_T}{8\pi\epsilon_0 a^2}$$

$$E_{Tx} = \frac{Q_T}{8\pi\epsilon_0 a^2} \frac{\sqrt{2}}{2}$$

$$\vec{E}_{Tx} = E_{Tx} \cdot \vec{i}_x$$

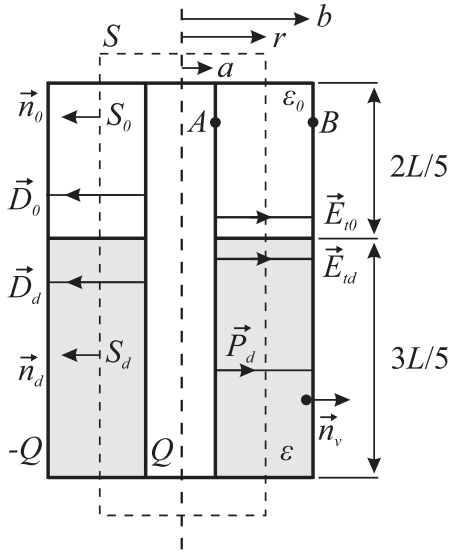
$$E_{Ty} = \frac{Q_T}{8\pi\epsilon_0 a^2} \frac{\sqrt{2}}{2}$$

$$\vec{E}_{Ty} = E_{Ty} \cdot \vec{i}_y$$

$$\vec{E}_O = \vec{E}_1 + \vec{E}_3 + \vec{E}_5 + \vec{E}_T = \left(\frac{Q'}{8\pi\epsilon_0 a} - \frac{Q'}{4\pi\epsilon_0 a} + \frac{Q_T}{8\pi\epsilon_0 a^2} \frac{\sqrt{2}}{2} \right) \cdot \vec{i}_x + \left(\frac{Q'}{8\pi\epsilon_0 a} - \frac{Q'}{4\pi\epsilon_0 a} + \frac{Q_T}{8\pi\epsilon_0 a^2} \frac{\sqrt{2}}{2} \right) \cdot \vec{i}_y$$

I-2

a)



Granični uslov:

$$E_{t0} = E_{td} \quad E_0 = E_d = E$$

$$D_{n0} = D_{nd} = 0$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{uS}$$

$$\int_{S_{OM}} D ds = Q$$

$$\int_{OM_0} D_0 ds + \int_{OM_d} D_d ds = Q$$

$$D_0 2\pi r \frac{2L}{5} + D_d 2\pi r \frac{3L}{5} = Q \quad D_0 = \epsilon_0 E \quad D_d = \epsilon E = 2\epsilon_0 E$$

$$E = \frac{Q}{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi r}, \quad a \leq r \leq b$$

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_a^b E dr = \int_a^b \frac{Q}{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi r} dr = \frac{Q}{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi} \ln \frac{b}{a}$$

$$C = \frac{Q}{U_{AB}} = \frac{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi}{\ln \frac{b}{a}}$$

$$C_V = \frac{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi}{\ln \frac{b}{a}}$$

b)

$$C_V = 444,6 \text{ pF}$$

c)

$$C_V = C_H = \frac{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi}{\ln \frac{b}{a}} = 444,6 \text{ pF}$$

$$\Delta C = C_H - C_V = 0$$

d)

$$P_d = D_d - \epsilon_0 E = (\epsilon - \epsilon_0) E = \epsilon_0 E = \epsilon_0 \frac{Q}{\left(\epsilon_0 \frac{2L}{5} + \epsilon_0 \frac{6L}{5} \right) 2\pi r} = 5 \frac{Q}{16L \pi r}$$

$$\sigma_v = \vec{P}_d \cdot \vec{n}_v = P_d (r=b) = 5 \frac{Q}{16L \pi b}$$

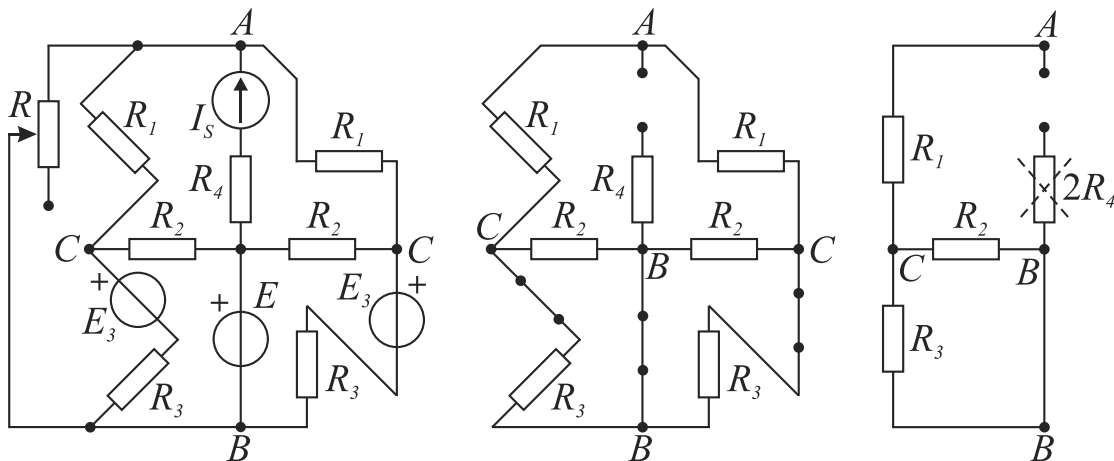
$$Q_v = \sigma_v 2\pi b \frac{3L}{5} = 5 \frac{Q}{16L \pi b} 2\pi b \frac{3L}{5}$$

$$Q_v = \frac{3}{8} Q$$

$$Q_v = 3,75 \text{ nC}$$

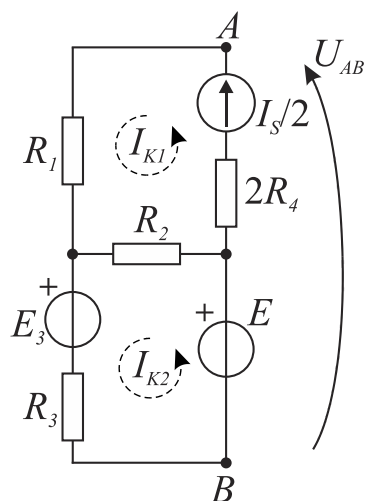
II-1

a)



$$2R_T = R_{AB} = R_1 + (R_2 \parallel R_3) = 4 + (16 \parallel 16) = 4 + 8 = 12 \Omega \quad \Rightarrow \quad R_T = 6 \Omega$$

$$R = R_T = 6 \Omega$$



$$I_{K1} = I_S / 2 = 0,5 \text{ A}$$

$$I_{K2} (R_2 + R_3) - I_{K1} R_2 = E - E_3$$

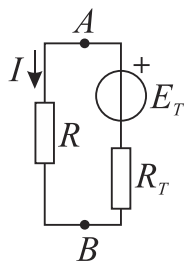
$$I_{K2} (16 + 16) - 0,5 \cdot 16 = 4 - 20$$

$$32 I_{K2} = -8$$

$$I_{K2} = -0,25 \text{ A}$$

$$U_{AB} = \frac{I_S}{2} R_1 + E_3 + I_{K2} R_3 = 0,5 \cdot 4 + 20 + (-0,25) \cdot 16 = 2 + 20 - 4 = 18 \text{ V}$$

$$E_T = U_{AB} = 18 \text{ V}$$



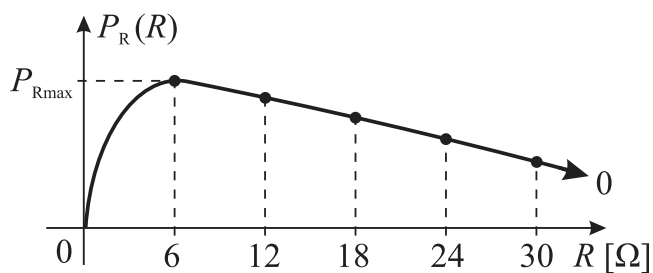
$$I = \frac{E_T}{R_T + R} = \frac{18}{6 + 6} = 1,5 \text{ A}$$

$$P_{R_{\max}} = R I^2 = 6 \cdot 1,5^2$$

$$P_{R_{\max}} = 13,5 \text{ W}$$

b)

$R [\Omega]$	6	12	18	24	30
$P_R [W]$	13,5	12	10,13	8,64	7,5



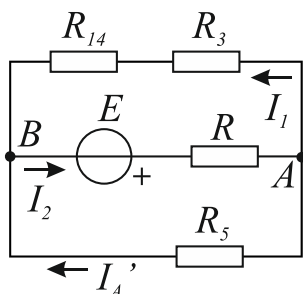
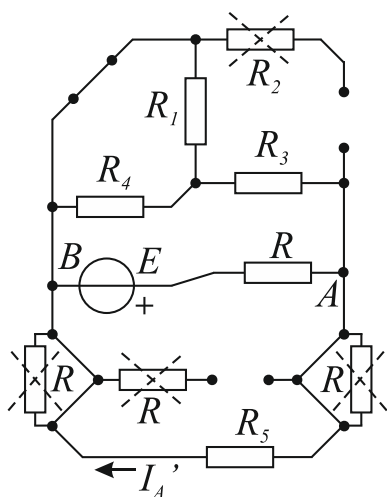
II-2

a)

$$\boxed{\begin{matrix} Svi \\ generatori \end{matrix}} = \boxed{\begin{matrix} Svi \\ sem E \end{matrix}} + \boxed{\begin{matrix} Samo \\ E \end{matrix}}$$

(1) (2)

$$I_A^{(1)} = I_A^{(2)} + I_A' \quad \Rightarrow \quad \Delta I_A = I_A^{(1)} - I_A^{(2)} = I_A' = 90 \text{ mA}$$



$$U_{AB} = I_A' R_5 = 90 \text{ m} \cdot 20 = 1,8 \text{ V}$$

$$R_{14} = R_1 \parallel R_4 = 40 \Omega$$

$$I_1 = \frac{U_{AB}}{R_3 + R_{14}} = \frac{1,8}{20 + 40} = 30 \text{ mA}$$

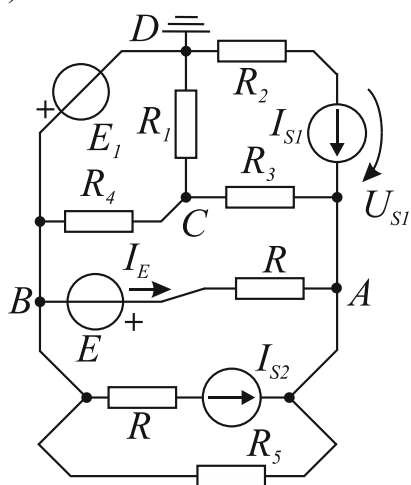
$$I_2 = I_1 + I_A' = 120 \text{ mA}$$

$$-E + I_2 R + U_{AB} = 0$$

$$E = I_2 R + U_{AB} = 120 \text{ m} \cdot 10 + 1,8$$

$$\boxed{E = 3 \text{ V}}$$

b)



$$V_D = 0 \text{ V}, \quad V_B = E_1 = 12 \text{ V}$$

$$V_A \left(\frac{1}{\infty} + \frac{1}{R_3} + \frac{1}{R} + \frac{1}{\infty} + \frac{1}{R_5} \right) - V_B \left(\frac{1}{R} + \frac{1}{\infty} + \frac{1}{R_5} \right) - V_C \left(\frac{1}{R_3} \right) = I_{S1} + \frac{E}{R} + I_{S2}$$

$$V_C \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - V_A \left(\frac{1}{R_3} \right) - V_B \left(\frac{1}{R_4} \right) = 0$$

$$V_A \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{20} \right) - 12 \cdot \left(\frac{1}{10} + \frac{1}{20} \right) - V_C \left(\frac{1}{20} \right) = 0,1 + \frac{3}{10} + 0,2 \quad / \cdot 20$$

$$V_C \left(\frac{1}{80} + \frac{1}{20} + \frac{1}{80} \right) - V_A \left(\frac{1}{20} \right) - 12 \cdot \left(\frac{1}{80} \right) = 0 \quad / \cdot 80$$

$$\left. \begin{array}{l} 4V_A - V_C = 48 \\ -4V_A + 6V_C = 12 \end{array} \right\} \Rightarrow V_A = 15 \text{ V}, \quad V_C = 12 \text{ V}$$

$$-E + I_E R + (V_A - V_B) = 0 \quad \Rightarrow \quad I_E = \frac{E - V_A + V_B}{R} = \frac{3 - 15 + 12}{10} = 0 \text{ A}$$

$$P_E = E I_E \quad \boxed{P_E = 0 \text{ W}}$$

c)

$$U_{S1} = (V_A - V_D) + I_{S1} R_2 = (15 - 0) + 0,1 \cdot 10 = 15 + 1 = 16 \text{ V}$$

$$P_{S1} = U_{S1} I_{S1} = 16 \cdot 0,1 \quad \boxed{P_{S1} = 1,6 \text{ W}}$$