

Osnovi elektrotehnike 2
(I kolokvijum)

K1

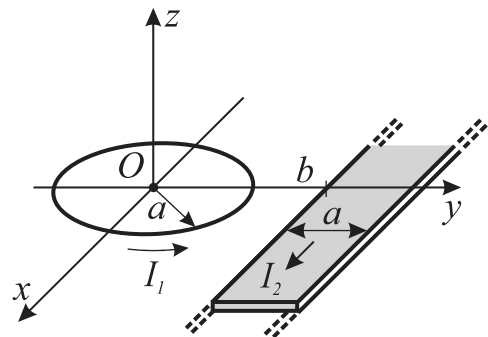
17. 9. 2026.

ZADACI

Zadatak 1. Na slici 1 su prikazani tanka žičana kontura u vidu prstena poluprečnika a , sa vremenski konstantnom strujom jačine I_1 i veoma dugačak, tanak, trakasti provodnik, širine a , sa strujom jačine I_2 . Obe strukture leže u x - y ravni Dekartovog pravouglog koordinatnog sistema. Trakasti provodnik je postavljen paralelno sa x osom na rastojanju b od ose. Sistem se nalazi u vazduhu.

- Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije koji u koordinatnom početku O , stvara prsten.
- Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije koji u koordinatnom početku O , stvara trakasti provodnik.
- Odrediti intenzitet struje I_2 , kroz trakasti provodnik, kako bi rezultatni vektor magnetske indukcije u tački O bio jednak nuli.

Brojni podaci su: $I_1 = 1$ A, $a = 3$ cm, $b = 2a$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



Slika 1.

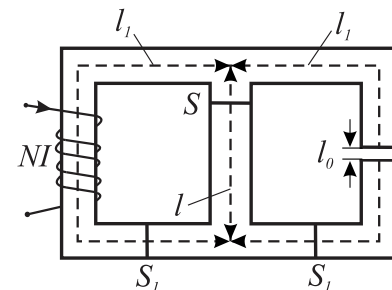
Zadatak 2. Tanko magnetsko kolo, prikazano na slici 2a je načinjeno od feromagnetskog materijala čija je idealizovana kriva prvobitnog magnetećenja data na slici 2b.

- Izračunati broj amperzavojaka, NI , kako bi, nakon uspostavljanja magnetskog polja u jezgru, u vazдушnom procepu postojao magnetski fluks $\Phi_0 = 60 \mu\text{Wb}$. Rasipanja magnetskog fluksa zanemariti.
- Izračunati energiju utrošenu na zagrevanje grane sa procepom, usled histerezisa, u intervalu vremena od 5 minuta, pod pretpostavkom da su namotaji priključeni na generator prostoperiodične struje, amplitude I_m jednake vrednosti izračunatoj pod a), učestanosti $f = 50$ Hz. Histerezisna petlja feromagnetskog materijala od kojeg je načinjeno magnetsko kolo je prikazana na slici 2b.

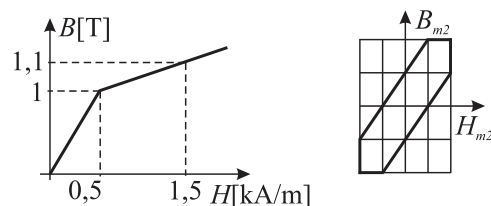
Ostali brojni podaci su:

$$l_0 = 0,5 \text{ mm}, l_1 = 15 \text{ cm}, l = 5 \text{ cm}, S_1 = S = 5 \text{ cm}^2,$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}.$$



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 2
(II kolokvijum)

K2

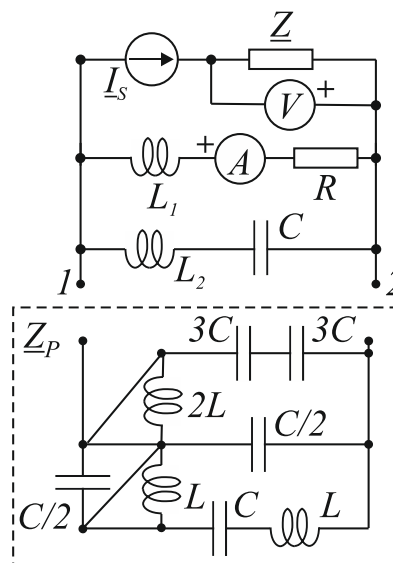
17. 9. 2026.

ZADACI

Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

- Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.
- Odrediti pokazivanja idealnih ampermetra i voltmetra, kada je prijemnik priključen na deo kola između tačaka 1 i 2, pri čemu je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka većoj antirezonantnoj učestanosti prijemnika.

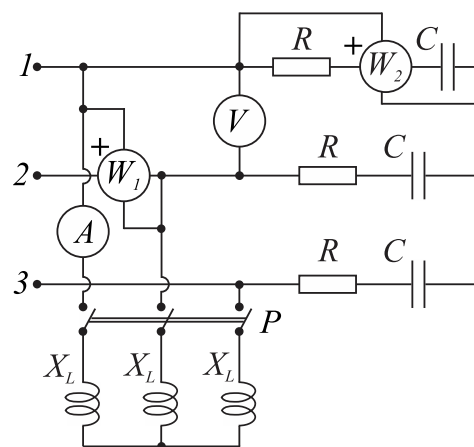
Brojni podaci su: $L = 30 \text{ mH}$, $C = 20 \text{ nF}$, $R = 8 \text{ k}\Omega$, $\underline{Z} = (5 + j3) \text{ k}\Omega$, $\underline{I}_S = (1 + j) \text{ mA}$, $L_1 = 2L$, $L_2 = 4L$.



Slika 1.

Zadatak 2. U simetričnom trofaznom sistemu, prikazanom na slici 2, prijemnici su sačinjeni od otpornika otpornosti $R = 20 \Omega$ i kondenzatora kapacitivnosti $C = 277 \mu\text{F}$. Prijemnici su priključeni na gradsku mrežu ($f = 50 \text{ Hz}$) faznog napona $\underline{U}_1 = 230 \text{ V}$. Pomoću prekidača P u sistem se mogu priključiti kalemovi za popravku faktora snage prijemnika.

- Odrediti pokazivanja idealnih mernih instrumenata pre priključenja kalemova.
- Izračunati reaktanse kalemova X_L kojima se faktor snage grupe povećava na jedinicu.
- Odrediti pokazivanja idealnih mernih instrumenata, posle priključenja kalemova.
- Za svaki vatmetar skicirati fazorski dijagram faznih napona mreže i svih fazora veličina od kojih zavisi njihovo pokazivanje, pre i posle kompenzacije.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1

Z1

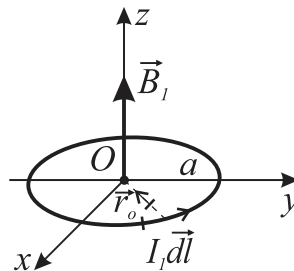
a)

$$dB_1 = \frac{\mu_0}{4\pi} \frac{I_1 d\vec{l} \times \vec{r}_0}{r^2}$$

$$dB_1 = \frac{\mu_0}{4\pi} \frac{I_1 dl}{a^2}$$

$$B_1 = \frac{\mu_0 I_1}{4\pi a^2} \int_0^{2\pi a} dl = \frac{\mu_0 I_1}{4\pi a^2} 2\pi a = \frac{\mu_0 I_1}{2a}$$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2a} \vec{i}_z$$



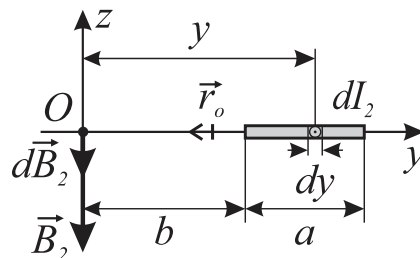
b)

$$\frac{dI_2}{I_2} = \frac{dy}{a}$$

$$dB_2 = \frac{\mu_0 dI_2}{2\pi y} = \frac{\mu_0 I_2}{2\pi y} \frac{dy}{a} = \frac{\mu_0 I_2}{2\pi a} \frac{dy}{y}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \int_b^{b+a} \frac{dy}{y} = \frac{\mu_0 I_2}{2\pi a} \ln \frac{b+a}{b}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi a} \ln \frac{b+a}{b} (-\vec{i}_z)$$



c)

Po uslovu zadatka: $B_1 = B_2$

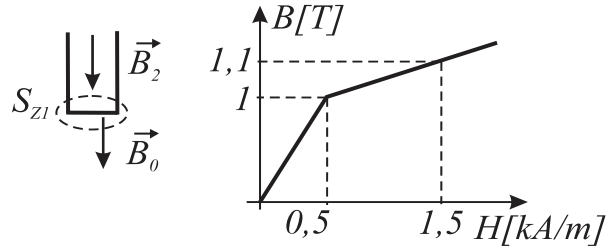
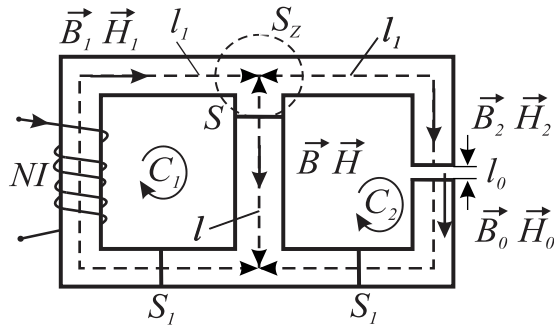
$$\frac{\mu_0 I_1}{2a} = \frac{\mu_0 I_2}{2\pi a} \ln \frac{b+a}{b}$$

$$I_2 = \frac{I_1 \pi}{\ln \frac{b+a}{b}} = \frac{\pi}{\ln \frac{3}{2}} = 7,75 \text{ A}$$

$$I_2 = 7,75 \text{ A}$$

K1

Z2



a)

$$\oint_{S_z} \vec{B} \cdot d\vec{s} = 0$$

$$S_z: -B_1 S_1 + B_2 S_1 + B S = 0 \quad (S_1 = S)$$

$$B_1 = B_2 + B \quad (1)$$

$$S_{z1}: -B_2 S_1 + B_0 S_0 = 0 \quad (S = S_0)$$

$$B_2 = B_0 \quad (2)$$

$$\oint_C \vec{H} \cdot d\vec{l} = NI$$

$$C_1: H_1 l_1 + H l = NI \quad (3)$$

$$C_2: H_2 l_1 + H_0 l_0 - H l = 0 \quad (4)$$

$$H_0 = \frac{B_0}{\mu_0} \quad (5)$$

$$B_0 = \frac{\Phi_0}{S_0} = \frac{60 \cdot 10^{-6}}{5 \cdot 10^{-4}} = 0,12 \text{ T} \quad \Rightarrow \quad H_0 = \frac{B_0}{\mu_0} = \frac{0,12}{4\pi \cdot 10^{-7}} = 95,49 \frac{\text{kA}}{\text{m}} \quad (2) \Rightarrow \quad B_2 = B_0 = 0,12 \text{ T}$$

$$\text{Sa krive magnetisanja sledi } H_2 = \frac{B_2}{\mu} = \frac{0,12}{\frac{1}{500}} = 60 \frac{\text{A}}{\text{m}}$$

$$\text{Iz (4) sledi } H = \frac{H_2 l_1 + H_0 l_0}{l} = \frac{60 \cdot 0,15 + 95,49 \cdot 10^3 \cdot 0,5 \cdot 10^{-3}}{0,05} = 1134,9 \frac{\text{A}}{\text{m}}$$

Jednačina za drugi deo krive magnetisanja:

$$B - 1 = \frac{1,1 - 1}{1500 - 500} (H - 500) \quad \Rightarrow \quad B = \frac{0,1}{1000} (1134,9 - 500) + 1 = 1,06 \text{ T}$$

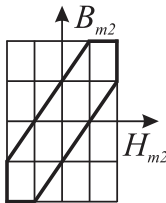
$$\text{Iz (1) sledi } B_1 = B_2 + B = 0,12 + 1,06 = 1,18 \text{ T}$$

$$\text{Sledi da je } H_1 - 500 = \frac{1500 - 500}{1,1 - 1} (B_1 - 1) \quad \Rightarrow \quad H_1 = \frac{1000}{0,1} (1,18 - 1) + 500 = 2334,9 \frac{\text{A}}{\text{m}}$$

$$\text{Iz (3) sledi } NI = H_1 l_1 + H l = 2334,9 \cdot 0,15 + 1134,9 \cdot 0,05$$

$$NI = 406,98 \text{ Azav} \approx 407 \text{ Azav}$$

b)



$$w_{h2} = 7 \frac{B_{m2}}{2} \frac{H_{m2}}{2} = \frac{7}{4} B_{m2} H_{m2} = \frac{7}{4} B_2 H_2$$

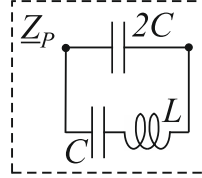
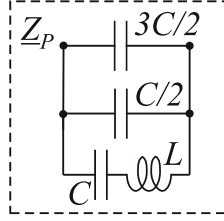
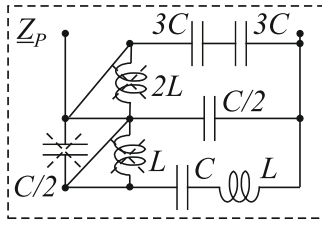
$$W_{h2} = f t W_{h2 \text{ per}} = f t w_{h2} V_2 = f t \frac{7}{4} B_2 H_2 S_2 l_2$$

$$W_{h2} = 50 \cdot 300 \cdot \frac{7}{4} \cdot 0,12 \cdot 60 \cdot 5 \cdot 10^{-4} \cdot 0,15$$

$$W_{h2} = 14,175 \text{ J}$$

K2**Z1**

a)



$$Z_1 = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$Z_P = Z_1 \parallel \frac{1}{j\omega 2C} = \frac{\frac{1 - \omega^2 LC}{j\omega C} \cdot \frac{1}{j\omega 2C}}{\frac{1 - \omega^2 LC}{j\omega C} + \frac{1}{j\omega 2C}} = \frac{\frac{1 - \omega^2 LC}{2(j\omega C)^2}}{\frac{3 - 2\omega^2 LC}{j\omega 2C}} = \frac{1 - \omega^2 LC}{j\omega C(3 - 2\omega^2 LC)}$$

$$Z_P = -j \frac{1 - \omega^2 LC}{\omega C(3 - 2\omega^2 LC)}$$

$$\text{Im}\{Z_P\} = 0 \Rightarrow 1 - \omega^2 LC = 0$$

$$\omega_{r1} = \sqrt{\frac{1}{LC}}$$

$$\omega_{r2} \rightarrow \infty$$

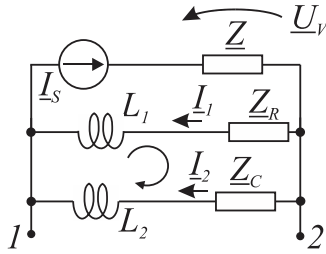
$$\text{Im}\{Y_P\} = 0 \Rightarrow \omega 2C = 0$$

$$\omega_{ar1} = 0$$

$$3 - 2\omega^2 LC = 0$$

$$\omega_{ar2} = \sqrt{\frac{3}{2LC}}$$

b)



$$\omega = \omega_{ar2} = \sqrt{\frac{3}{2LC}} = \sqrt{\frac{3}{2 \cdot 30 \cdot 10^{-3} \cdot 20 \cdot 10^{-9}}} = 5 \cdot 10^4 \frac{\text{rad}}{\text{s}}$$

$$Z_P \rightarrow \infty$$

$$Z_C = \frac{1}{j\omega C} = -j1 \text{ k}\Omega, \quad Z_{L1} = j\omega L_1 = j3 \text{ k}\Omega$$

$$Z_{L2} = j\omega L_2 = j6 \text{ k}\Omega$$

$$I_S = I_1 + I_2 \Rightarrow I_2 = I_S - I_1$$

$$Z_C I_2 + Z_{L2} I_2 - Z_{L1} I_1 - Z_R I_1 = 0$$

$$(-Z_{L1} - Z_R) I_1 + (Z_C + Z_{L2})(I_S - I_1) = 0$$

$$I_1 = \frac{Z_C + Z_{L2}}{Z_C + Z_R + Z_{L1} + Z_{L2}} I_S = \frac{(-j1 + j6)k}{(-j1 + 8 + j3 + j6)k} (1 + j)m = \frac{j5k}{(8 + j8)k} (1 + j)m = \frac{j5}{8} m = j0,625 \text{ mA}$$

$$I_A = |I_1| = \sqrt{0^2 + (0,625)^2} \quad I_A = 0,625 \text{ mA}$$

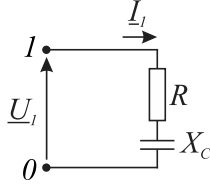
$$U_V = Z I_S = (5 + j3)k \cdot (1 + j)m = (2 + j8) \text{ V}$$

$$U_V = |U_V| = \sqrt{2^2 + 8^2} \quad U_V = 8,25 \text{ V}$$

K2

Z2

a) Pre kompenzacije



$$\underline{Z} = \underline{Z}_R + \underline{Z}_C = R + \left(\frac{-j}{\omega C} \right) = (20 - j11,49) \Omega = 23,1 \cdot e^{-j29,89^\circ} \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1 e^{j0^\circ}}{Z e^{j\varphi}} = \frac{230}{23,1 e^{-j29,89^\circ}} = 9,96 e^{j29,89^\circ} \text{ A}$$

$$P_{W1}^{pre} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_2^*\} = U_{12} I_2 \cos \angle(\underline{U}_{12}, \underline{I}_2) = \sqrt{3} U_1 I_1 \cos \angle(\underline{U}_{12}, \underline{I}_2)$$

$$P_{W1}^{pre} = \sqrt{3} \cdot 230 \cdot 9,96 \cdot \cos(\underbrace{120 + 30 - 29,89^\circ}_{120,11^\circ}) = -1990,48 \text{ W}$$

$$\boxed{P_{W1}^{pre} = -1990,48 \text{ W}}$$

$$P_{W2}^{pre} = \operatorname{Re}\{\underline{U}_1 \underline{I}_1^*\} = U_1 I_1 \cos \angle(\underline{U}_1, \underline{I}_1)$$

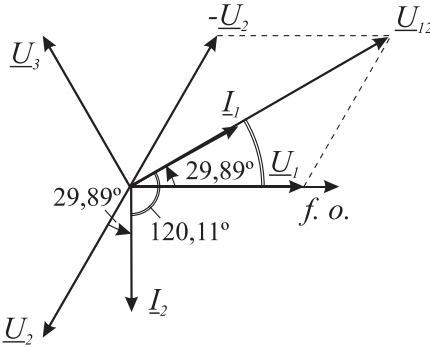
$$P_{W2}^{pre} = U_1 I_1 \cos 29,89^\circ = 230 \cdot 9,96 \cdot \cos 29,89^\circ = 1986,07 \text{ W}$$

$$\boxed{P_{W2}^{pre} = 1986,07 \text{ W}}$$

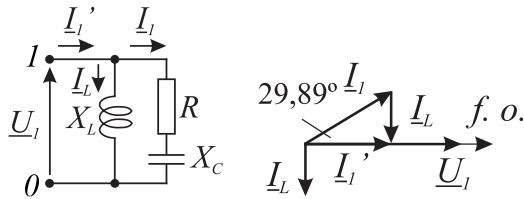
$$U_V^{pre} = |\underline{U}_{12}| = \sqrt{3} \cdot U_1 = \sqrt{3} \cdot 230$$

$$\boxed{U_V^{pre} = 398,37 \text{ V}}$$

$$\boxed{I_A^{pre} = 0 \text{ A}}$$



b) Posle kompenzacije



$$\text{I KZ: } \underline{I}_1' = \underline{I}_1 + \underline{I}_L$$

$$\underline{I}_L = \frac{U_1 e^{j0^\circ}}{jX_L} = \frac{U_1 e^{j0^\circ}}{X_L e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L} e^{-j\frac{\pi}{2}} \text{ A}$$

$$I_L = I_1 \sin 29,89^\circ$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z} \sin 29,89^\circ \rightarrow \frac{1}{X_L} = \frac{\sin 29,89^\circ}{Z} \rightarrow X_L = \frac{23,1}{\sin 29,89^\circ}$$

$$\boxed{X_L = 46,35 \Omega}$$

$$I_1' = I_1 \cos 29,89^\circ = 9,96 \cdot 0,87 = 8,64 \text{ A}$$

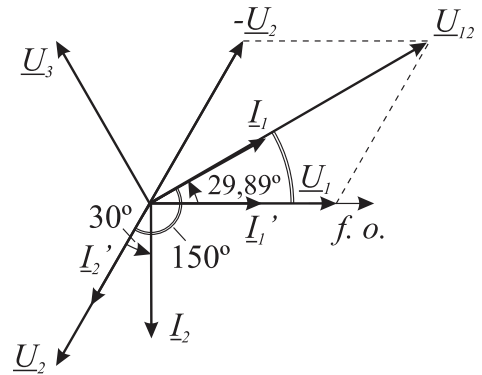
$$P_{W1}^{posle} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_2^*\} = U_{12} I_2' \cos \angle(\underline{U}_{12}, \underline{I}_2') = \sqrt{3} U_1 I_1' \cos \angle(\underline{U}_{12}, \underline{I}_2')$$

$$P_{W1}^{posle} = \sqrt{3} \cdot 230 \cdot 8,64 \cdot \cos(\underbrace{30^\circ + 120^\circ}_{150^\circ}) = -2980,8 \text{ W}$$

$$\boxed{P_{W1}^{posle} = -2980,8 \text{ W}}$$

$$P_{W2}^{posle} = \operatorname{Re}\{\underline{U}_1 \underline{I}_1^*\} = P_{W2}^{pre}$$

$$\boxed{P_{W2}^{posle} = 1986,07 \text{ W}}$$



$$U_V^{posle} = |\underline{U}_{12}| = U_V^{pre}$$

$$\boxed{U_V^{posle} = 398,37 \text{ V}}$$

$$\boxed{I_A^{posle} = I_L = \frac{U_1}{X_L} = \frac{230}{46,15} = 5 \text{ A}}$$