

Osnovi elektrotehnike
(I kolokvijum)

K1

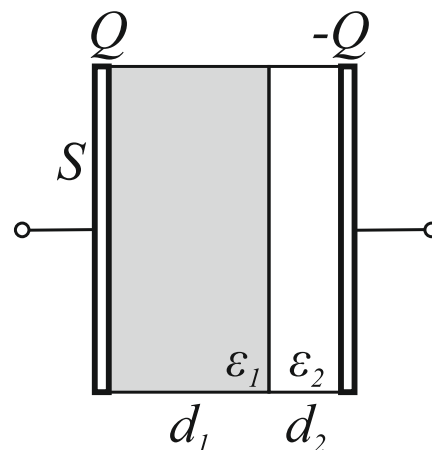
17.09.2026.

ZADACI

Zadatak 1. Na slici 1 je prikazan pločasti kondenzator ispunjen sa dva sloja dielektrika, relativnih permitivnosti $\epsilon_{r1} = 2$ i $\epsilon_{r2} = 4$, debljina d_1 i $d_2 = 0,25 \text{ mm}$ i površine ploča $S = 4 \text{ cm}^2$.

- Izvesti u opštim brojevima izraz za kapacitivnost kondenzatora.
- Odrediti debljinu d_1 , tako da kapacitivnost prvog sloja dielektrika bude šest puta manja od kapacitivnosti drugog sloja.
- Izračunati maksimalni napon na koji sme da se priključi kondenzator.

Brojni podaci su: $E_{C1} = 45 \text{ kV/cm}$ i $E_{C2} = 70 \text{ kV/cm}$.

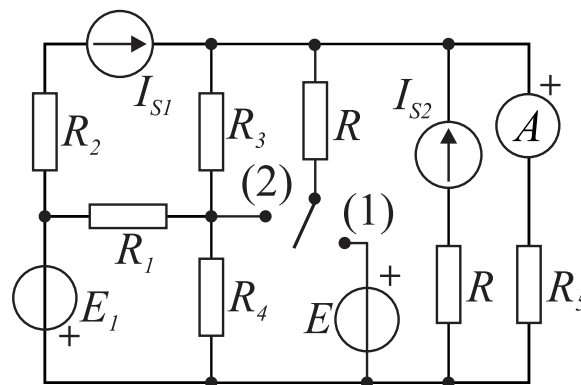


Slika 1.

Zadatak 2. Kada se u mreži sa slike 2, preklopnik prebaci iz položaja (2) u položaj (1), struja koju meri ampermetar se poveća za $\Delta I_A = 90 \text{ mA}$.

- Primenjujući teoremu superpozicije, izračunati vrednost $ems E$.
- Izračunati snagu naponskog generatora $ems E$ kada je prekidač u položaju (1).
- Izračunati snagu strujnog generatora I_{S1} kada je prekidač u položaju (1).

Brojni podaci su: $E_1 = 12 \text{ V}$, $R = R_2 = 10 \Omega$, $R_1 = R_4 = 80 \Omega$, $R_3 = R_5 = 20 \Omega$, $I_{S1} = 100 \text{ mA}$, $I_{S2} = 200 \text{ mA}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum je neophodno sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike
(II kolokvijum)

K2

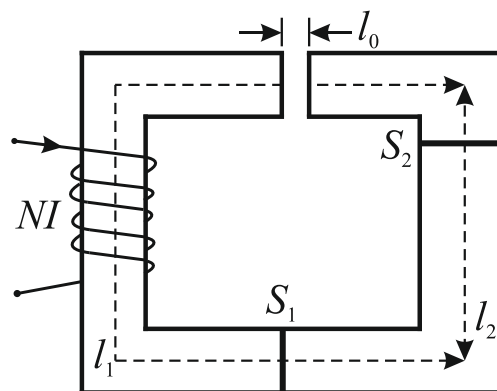
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ZADACI

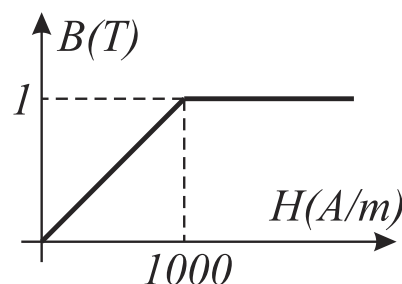
Zadatak 1. Na slici 1a je prikazano tanko magnetsko kolo, načinjeno od feromagnetskog materijala čija je idealizovana magnetska karakteristika data na slici 1b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa $N = 150$ zavoja-
ka u kojima je uspostavljena struja jačine I . Odrediti:

- Jačinu struje, I , tako da intenzitet vektora magnetske indukcije u procepu magnetskog kola ima vrednost $B_0 = 0,81 \text{ T}$. Rasipanja zanemariti.
- Fluks vektora magnetske indukcije kroz poprečni presek S_2 .
- Magnetsku energiju sadržanu u magnetskim granama kola.

Dimenzije jezgra su: $l_1 = 105 \text{ mm}$, $l_2 = 30 \text{ mm}$, $l_0 = 1 \text{ mm}$,
 $S_1 = 1 \text{ cm}^2$, $S_2 = 1,5 \text{ cm}^2$.



Slika 1a.

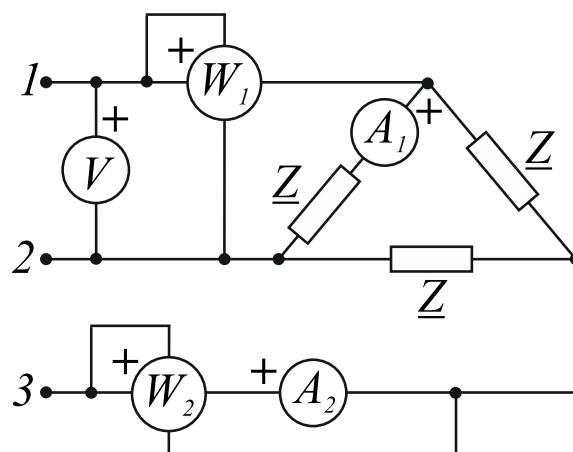


Slika 1b.

Zadatak 2. Simetričan trofazni prijemnik impadanse $\underline{Z}$ vezan u trougao, kao što je prikazano na slici 2, priključen je na mrežu faznog napona $U_f = 230 \text{ V}$, frekvencije f .

- Odrediti pokazivanje idealnih mernih instrumenata.
- Na fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.

Brojni podaci su: $\underline{Z} = (30 + j60) \Omega$, $f = 50 \text{ Hz}$.



Slika 2.

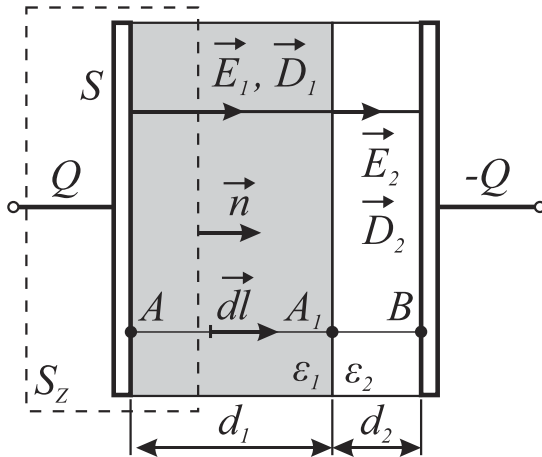
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1

Z1

a)



Granični uslov:

$$D_{n1} = D_{n2}$$

$$D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{slobodno u } S}$$

$$\int_S D dS = Q$$

$$D S = Q$$

$$D = \frac{Q}{S}$$

$$E_1 = \frac{D}{\epsilon_1} = \frac{Q}{\epsilon_1 S}$$

$$E_2 = \frac{D}{\epsilon_2} = \frac{Q}{\epsilon_2 S}$$

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_A^{A_1} \vec{E}_1 \cdot d\vec{l} + \int_{A_1}^B \vec{E}_2 \cdot d\vec{l} = E_1 d_1 + E_2 d_2 = \frac{Q}{S} \left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right)$$

$$C = \frac{Q}{U}$$

$$C = \frac{Q}{\frac{Q}{S} \left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right)} = \frac{1}{\frac{d_1}{S\epsilon_1} + \frac{d_2}{S\epsilon_2}}$$

b)

$$C_1 = \epsilon_1 \frac{S}{d_1}, \quad C_2 = \epsilon_2 \frac{S}{d_2}$$

$$C_1 = \frac{C_2}{6}$$

$$\epsilon_1 \frac{S}{d_1} = \frac{1}{6} \epsilon_2 \frac{S}{d_2}$$

$$d_1 = \frac{6 \cdot \epsilon_1}{\epsilon_2} d_2 = \frac{6 \cdot 2\epsilon_0}{4\epsilon_0} \cdot 0,25 m$$

$$d_1 = 0,75 mm$$

c)

$$E_{1\max} = \frac{Q_{\max 1}}{\epsilon_1 S} = E_{C1} \Rightarrow Q_{\max 1} = E_{C1} \epsilon_1 S = 31,86 nC$$

$$E_{2\max} = \frac{Q_{\max 2}}{\epsilon_2 S} = E_{C2} \Rightarrow Q_{\max 2} = E_{C2} \epsilon_2 S = 99,12 nC$$

$$Q_{\max} = \min\{Q_{\max 1}, Q_{\max 2}\} = Q_{\max 1} = 31,86 nC$$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{31,86 nC}{8,1 pF}$$

$$U_{\max} = 3,93 kV$$

K1

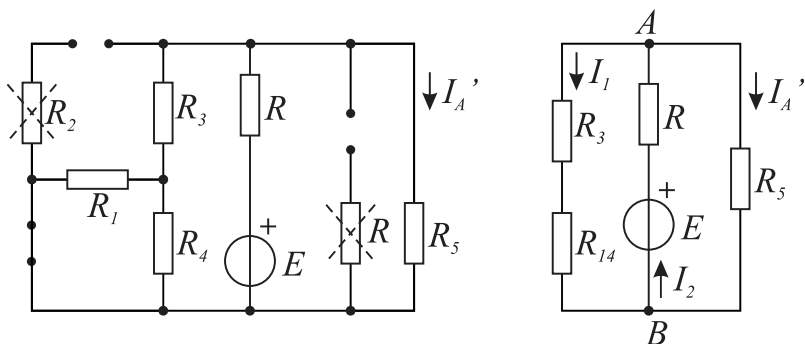
Z2

a)

$$\boxed{\begin{matrix} Svi \\ generatori \end{matrix}} = \boxed{\begin{matrix} Svi \\ sem E \end{matrix}} + \boxed{\begin{matrix} Samo \\ E \end{matrix}}$$

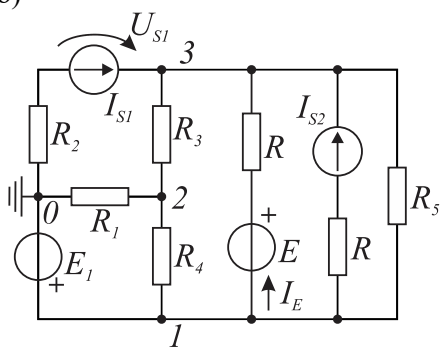
(1) (2)

$$I_A^{(1)} = I_A^{(2)} + I_A' \quad \Rightarrow \quad \Delta I_A = I_A^{(1)} - I_A^{(2)} = I_A' = 90 \text{ mA}$$



$$\begin{aligned} R_{14} &= R_1 \parallel R_4 = 40 \, \Omega \\ U_{AB} &= I_A' R_5 = 90 \text{ m} \cdot 20 = 1,8 \text{ V} \\ I_1 &= \frac{U_{AB}}{R_3 + R_{14}} = \frac{1,8}{20 + 40} = 30 \text{ mA} \\ I_2 &= I_1 + I_A' = 120 \text{ mA} \\ -E + I_2 R + U_{AB} &= 0 \\ E &= I_2 R + U_{AB} = 120 \text{ m} \cdot 10 + 1,8 \\ \boxed{E} &= \boxed{3 \text{ V}} \end{aligned}$$

b)



$$\begin{aligned} V_0 &= 0 \text{ V}, \quad V_1 = E_1 = 12 \text{ V} \\ V_2 \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - V_1 \left(\frac{1}{R_4} \right) - V_3 \left(\frac{1}{R_3} \right) &= 0 \\ V_3 \left(\frac{1}{R_2 + \infty} + \frac{1}{R_3} + \frac{1}{R} + \frac{1}{R + \infty} + \frac{1}{R_5} \right) - V_1 \left(\frac{1}{R} + \frac{1}{R + \infty} + \frac{1}{R_5} \right) - V_2 \left(\frac{1}{R_3} \right) &= \\ &= I_{S1} + \frac{E}{R} + I_{S2} \\ \hline V_2 \left(\frac{1}{80} + \frac{1}{20} + \frac{1}{80} \right) - 12 \cdot \left(\frac{1}{80} \right) - V_3 \left(\frac{1}{20} \right) &= 0 \quad / \cdot 80 \\ V_3 \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{20} \right) - 12 \cdot \left(\frac{1}{10} + \frac{1}{20} \right) - V_2 \left(\frac{1}{20} \right) &= 100 \text{ m} + \frac{3}{10} + 200 \text{ m} \quad / \cdot 20 \\ \hline \left. \begin{aligned} 6V_2 - 4V_3 &= 12 \\ -V_2 + 4V_3 &= 48 \end{aligned} \right\} &\Rightarrow \quad V_2 = 12 \text{ V}, \quad V_3 = 15 \text{ V} \end{aligned}$$

$$-E + I_E R + (V_3 - V_1) = 0$$

$$I_E = \frac{E - V_3 + V_1}{R} = \frac{3 - 15 + 12}{10} = 0 \text{ A}$$

$$P_E = E I_E \quad \boxed{P_E = 0 \text{ W}}$$

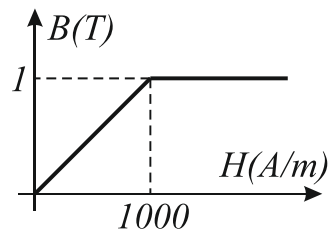
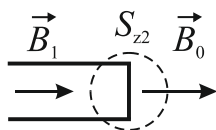
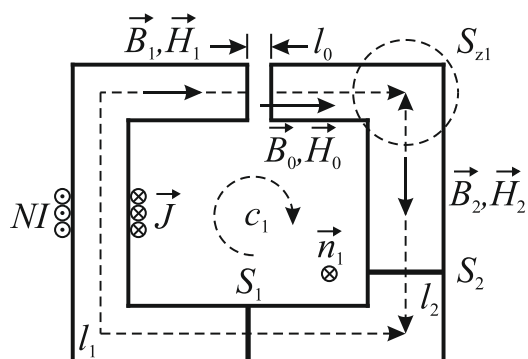
c)

$$U_{S1} = (V_3 - V_0) + I_{S1} R_2 = (15 - 0) + 100 \text{ m} \cdot 10 = 15 + 1 = 16 \text{ V}$$

$$P_{S1} = U_{S1} I_{S1} = 16 \cdot 100 \text{ m} \quad \boxed{P_{S1} = 1,6 \text{ W}}$$

K2

Z1



a)

$$\oint_c \vec{H} \cdot d\vec{l} = NI$$

$$c_1: H_1 l_1 + H_0 l_0 + H_2 l_2 = NI \quad (1)$$

$$\oint_{S_z} \vec{B} \cdot d\vec{s} = 0$$

$$S_{z1}: -B_1 S_1 + B_0 S_0 = 0 \quad (2)$$

$$S_{z2}: -B_1 S_1 + B_2 S_2 = 0 \quad (3)$$

$$B_0 = 0,81 \text{ T} \Rightarrow H_0 = \frac{B_0}{\mu_0} = \frac{0,81}{4\pi \cdot 10^{-7}} = 644,9 \text{ kA/m}$$

$$\text{Iz (2) sledi: } B_1 = B_0 = 0,81 \text{ T} \Rightarrow H_1 = \frac{B_1}{\mu} = \frac{0,81}{\frac{1}{1000}} = 810 \text{ A/m}$$

$$\text{Iz (3) sledi: } B_2 = B_1 \frac{S_1}{S_2} = 0,81 \cdot \frac{1 \cdot 10^{-4}}{1,5 \cdot 10^{-4}} = 0,54 \text{ T} \Rightarrow H_2 = \frac{B_2}{\mu} = \frac{0,54}{\frac{1}{1000}} = 540 \text{ A/m}$$

$$\text{Iz (1) sledi: } I = \frac{H_1 l_1 + H_0 l_0 + H_2 l_2}{N} = \frac{810 \cdot 0,105 + 644,9 \cdot 10^3 \cdot 1 \cdot 10^{-3} + 540 \cdot 0,03}{150}$$

$$I = 4,97 \text{ A}$$

b)

$$\Phi_2 = \int_{S_2} \vec{B}_2 \cdot d\vec{S} = B_2 S_2 = 0,54 \cdot 1,5 \cdot 10^{-4}$$

$$\Phi_2 = 81 \mu\text{Wb}$$

c)

$$W_{m1} = w_{m1} V_1 = w_{m1} l_1 S = \frac{1}{2} B_1 H_1 l_1 S_1 = \frac{1}{2} \cdot 0,81 \cdot 810 \cdot 0,105 \cdot 1 \cdot 10^{-4}$$

$$W_{m1} = 3,44 \text{ mJ}$$

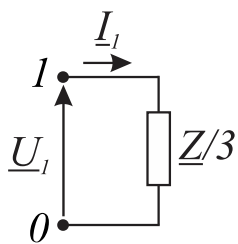
$$W_{m2} = w_{m2} V_2 = w_{m2} l_2 S = \frac{1}{2} B_2 H_2 l_2 S_2 = \frac{1}{2} \cdot 0,54 \cdot 540 \cdot 0,03 \cdot 1,5 \cdot 10^{-4}$$

$$W_{m2} = 0,65 \text{ mJ}$$

K2

Z2

a)



$$\underline{Z} = (30 + j60) \Omega = 67,1 e^{j63,43^\circ} \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}/3} = \frac{230 e^{j0^\circ}}{\frac{1}{3} \cdot 67,1 e^{j63,43^\circ}} = 10,29 e^{-j63,43^\circ} A$$

$$\underline{I}_2 = \underline{I}_1 e^{-j120^\circ} = 10,29 e^{-j183,43^\circ} A$$

$$\underline{I}_3 = \underline{I}_1 e^{-j240^\circ} = 10,29 e^{-j303,43^\circ} A$$

$$I_{A1} = |\underline{I}_{12}| = \frac{|\underline{I}_1|}{\sqrt{3}} = \frac{10,29}{\sqrt{3}} = 5,94 A$$

$$I_{A2} = |\underline{I}_3| = 10,29 A$$

$$U_V = |\underline{U}_{12}| = \sqrt{3} |\underline{U}_1| = \sqrt{3} \cdot 230 = 398,37 V$$

$$P_{W1} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_1^*\} = U_{12} I_1 \cos \angle(\underline{U}_{12}, \underline{I}_1) = \sqrt{3} U_1 I_1 \cos(30^\circ + 63,43^\circ)$$

$$P_{W1} = \sqrt{3} \cdot 230 \cdot 10,286 \cdot \cos 93,43^\circ$$

$$P_{W1} = -245,16 W$$

$$U_{W2} = |\underline{U}_{33}| = 0 V$$

$$P_{W2} = 0 W$$

b)

