

Osnovi elektrotehnike 1
(I kolokvijum)

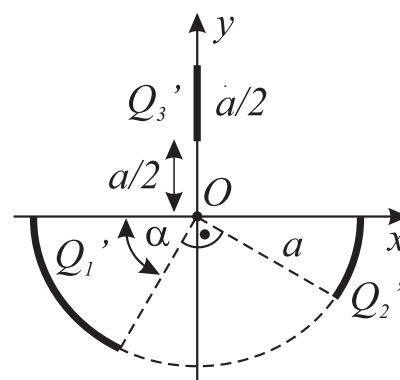
K1

15. 9. 2026.

ZADACI

Zadatak 1. Dva tanka štapa, od izolacionog materijala, savijena kao na slici 1, naelektrisana su ravnomerno podužnim gustinama naelektrisanja $Q_1' > 0$ i $Q_2' > 0$. Štapovi su savijeni u obliku lukova, poluprečnika a , i leže u x - y ravni zadatog koordinatnog sistema. Dodatno, na y osi, na rastojanju $a/2$ od koordinatnog početka, postavljen je štاپ konačne dužine $a/2$, naelektrisan podužnom gustinom $Q_3' > 0$. Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u tački O , koji potiče od naelektrisanih štapova.

Brojni podaci su: $Q_1' = Q_2' = Q_3' = 10 \text{ nC/m}$, $\alpha = \pi/3$,
 $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$.

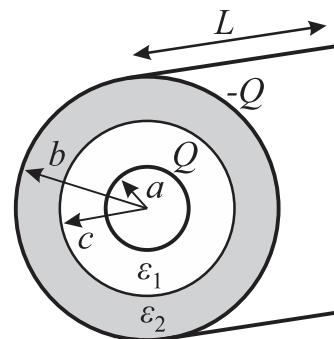


Slika 1.

Zadatak 2. Na slici 2 je prikazan koaksijalni kabl dužine $L = 1 \text{ m}$, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\epsilon_{r1} = 6$ i $\epsilon_{r2} = 4$. Poluprečnici elektroda kabla su $a = 1 \text{ mm}$ i $b = 4 \text{ mm}$. Poluprečnik razdvojne površi dva dielektrika je $c = 2 \text{ mm}$. Elektrode kondenzatora su naelektrisane naelektrisanjima Q i $-Q$.

- Izračunati kapacitivnost kabla.
- Odrediti najveći napon na koji sme da se priključi kabl, ako je maksimalna dozvoljena jačina električnog polja u kondenzatoru $E_{max} = 70 \text{ kV/cm}$.
- Izračunati energiju sadržanu u kabl pri maksimalnom naponu određenom pod b).

Ostali brojni podaci: $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 1
(II kolokvijum)

K2

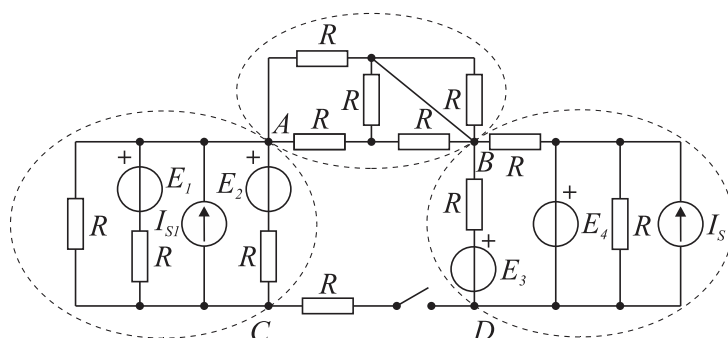
15. 9. 2026.

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

- Transformacijom generatora uprostiti deo mreže između tačaka A i C i predstaviti ga ekvivalentnim realnim naponskim generatorom, E_{T1} , R_{T1} .
- Deo mreže između tačaka B i D predstaviti ekvivalentnim Tevenenovim generatorom, E_{T2} , R_{T2} .
- Deo mreže između tačaka A i B predstaviti ekvivalentnim otpornikom R_e .
- Analizirajući uprošćeno kolo, odrediti za koliko će se promeniti napon između tačaka C i D pri zatvaranju prekidača.

Brojni podaci su: $R = 3 \Omega$, $E_1 = 6 \text{ V}$,
 $E_2 = 3 \text{ V}$, $E_3 = 7 \text{ V}$, $E_4 = 10 \text{ V}$,
 $I_{S1} = 2 \text{ A}$, $I_S = 4 \text{ A}$.

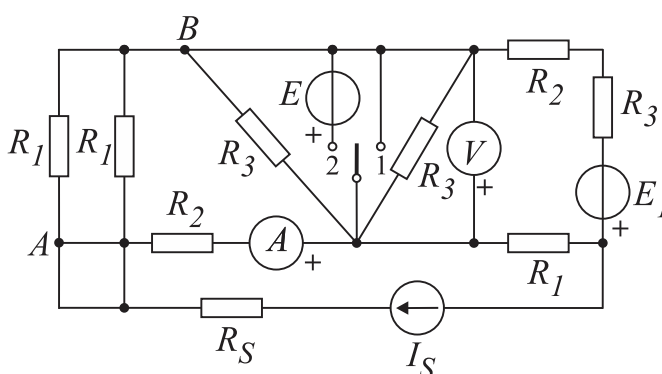


Slika 1.

Zadatak 2. Kada se u mreži sa slike 2 preklopnik prebaci iz položaja (1) u položaj (2), napon između tačaka A i B promeni se za $\Delta U_{AB} = 2 \text{ V}$.

- Primenjujući teoremu superpozicije odrediti elektromotornu silu E .
- Primenom metode potencijala čvorova izračunati snagu strujnog generatora, kada je preklopnik u položaju (2).

Brojni podaci su: $R_1 = 10 \Omega$, $R_2 = 15 \Omega$, $R_3 = 5 \Omega$,
 $R_S = 20 \Omega$, $E_1 = 2 \text{ V}$, $I_S = 0,9 \text{ A}$.



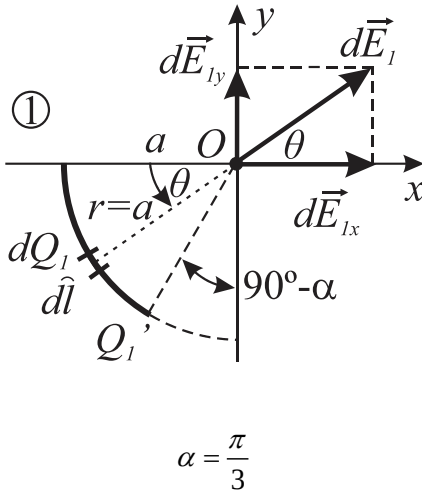
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1

a)



$$dE_{1x} = dE_1 \cos \theta = \frac{dQ_1}{4\pi\epsilon_0 r^2} \cos \theta = \frac{Q_1' dl}{4\pi\epsilon_0 a^2} \cos \theta = \frac{Q_1' a d\theta}{4\pi\epsilon_0 a^2} \cos \theta$$

$$E_{1x} = \int_{\text{po luku 1}} dE_{1x} = \frac{Q_1'}{4\pi\epsilon_0 a} \int_0^{\alpha} \cos \theta d\theta = \frac{Q_1'}{4\pi\epsilon_0 a} \left(\sin \frac{\pi}{3} - \sin 0 \right) = \frac{Q_1'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2}$$

$$E_{1x} = \frac{Q_1'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2}$$

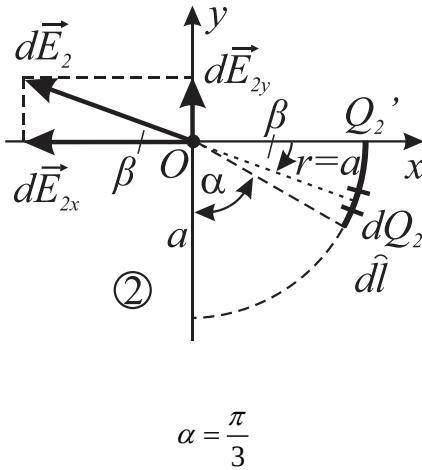
$$\vec{E}_{1x} = E_{1x} \cdot \vec{i}_x$$

$$dE_{1y} = dE_1 \sin \theta = \frac{dQ_1}{4\pi\epsilon_0 r^2} \sin \theta = \frac{Q_1' dl}{4\pi\epsilon_0 a^2} \sin \theta = \frac{Q_1' a d\theta}{4\pi\epsilon_0 a^2} \sin \theta$$

$$E_{1y} = \int_{\text{po luku 1}} dE_{1y} = \frac{Q_1'}{4\pi\epsilon_0 a} \int_0^{\alpha} \sin \theta d\theta = \frac{Q_1'}{4\pi\epsilon_0 a} \left(\cos 0 - \cos \frac{\pi}{3} \right) = \frac{Q_1'}{4\pi\epsilon_0 a} \left(1 - \frac{1}{2} \right)$$

$$E_{1y} = \frac{Q_1'}{8\pi\epsilon_0 a}$$

$$\vec{E}_{1y} = E_{1y} \cdot \vec{i}_y$$



$$dE_{2x} = dE_2 \cos \beta = \frac{dQ_2}{4\pi\epsilon_0 r^2} \cos \beta = \frac{Q_2' dl}{4\pi\epsilon_0 a^2} \cos \beta = \frac{Q_2' a d\beta}{4\pi\epsilon_0 a^2} \cos \beta$$

$$E_{2x} = \int_{\text{po luku 2}} dE_{2x} = \frac{Q_2'}{4\pi\epsilon_0 a} \int_0^{\frac{\pi}{2}-\alpha} \cos \beta d\beta = \frac{Q_2'}{4\pi\epsilon_0 a} \left(\sin \frac{\pi}{6} - \sin 0 \right) = \frac{Q_2'}{4\pi\epsilon_0 a} \frac{1}{2}$$

$$E_{2x} = \frac{Q_2'}{8\pi\epsilon_0 a}$$

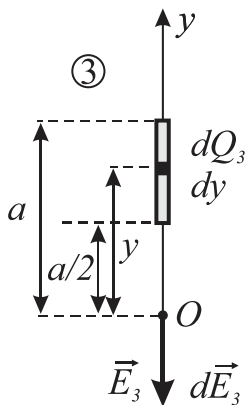
$$\vec{E}_{2x} = E_{2x} \cdot (-\vec{i}_x)$$

$$dE_{2y} = dE_2 \sin \beta = \frac{dQ_2}{4\pi\epsilon_0 r^2} \sin \beta = \frac{Q_2' dl}{4\pi\epsilon_0 a^2} \sin \beta = \frac{Q_2' a d\beta}{4\pi\epsilon_0 a^2} \sin \beta$$

$$E_{2y} = \int_{\text{po luku 2}} dE_{2y} = \frac{Q_2'}{4\pi\epsilon_0 a} \int_0^{\frac{\pi}{2}-\alpha} \sin \beta d\beta = \frac{Q_2'}{4\pi\epsilon_0 a} \left(\cos 0 - \cos \frac{\pi}{6} \right)$$

$$E_{2y} = \frac{Q_2'}{4\pi\epsilon_0 a} \left(1 - \frac{\sqrt{3}}{2} \right)$$

$$\vec{E}_{2y} = E_{2y} \cdot \vec{i}_y$$



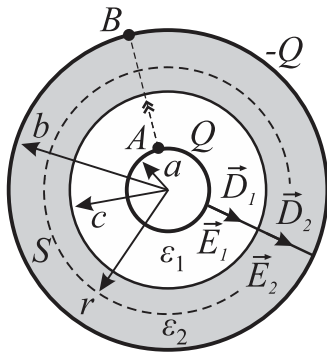
$$dE_3 = \frac{dQ_3}{4\pi\epsilon_0 y^2} = \frac{Q_3' dy}{4\pi\epsilon_0 y^2}$$

$$E_3 = \int_{\text{po štapu}} dE_3 = \frac{Q_3'}{4\pi\epsilon_0} \int_{a/2}^a \frac{dy}{y^2} = \frac{Q_3'}{4\pi\epsilon_0} \left(\frac{2}{a} - \frac{1}{a} \right) = \frac{Q_3'}{4\pi\epsilon_0} \frac{1}{a} = \frac{Q_3'}{4\pi\epsilon_0 a}$$

$$\vec{E}_3 = \frac{Q_3'}{4\pi\epsilon_0 a} \cdot (-\vec{i}_y)$$

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \left(\frac{Q_1'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2} - \frac{Q_2'}{8\pi\epsilon_0 a} \right) \cdot \vec{i}_x + \left(\frac{Q_1'}{8\pi\epsilon_0 a} + \frac{Q_2'}{4\pi\epsilon_0 a} \left(1 - \frac{\sqrt{3}}{2} \right) - \frac{Q_3'}{4\pi\epsilon_0 a} \right) \cdot \vec{i}_y$$

a)



Granični uslov:

$$D_{n1} = D_{n2} \quad D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0 \quad E_1 \neq E_2$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{\text{slobodno u } S}$$

$$\int_{S_{OM}} D ds = Q$$

$$D 2\pi r L = Q$$

$$D = \frac{Q}{2\pi r L}, \quad a \leq r \leq b$$

$$E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi \varepsilon_1 r L}, \quad a \leq r \leq c$$

$$E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi \varepsilon_2 r L}, \quad c \leq r \leq b$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_a^b E dr = \int_a^c E_1 dr + \int_c^b E_2 dr = \int_a^c \frac{Q}{2\pi \varepsilon_1 r L} dr + \int_c^b \frac{Q}{2\pi \varepsilon_2 r L} dr = \frac{Q}{2\pi \varepsilon_1 L} \ln \frac{c}{a} + \frac{Q}{2\pi \varepsilon_2 L} \ln \frac{b}{c}$$

$$U_{AB} = \frac{Q}{2\pi \varepsilon_0 L} \left(\frac{1}{\varepsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{b}{c} \right)$$

$$C = \frac{Q}{U_{AB}} = \frac{2\pi \varepsilon_0 L}{\frac{1}{\varepsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{b}{c}} \quad C = 192,53 \text{ pF}$$

c)

$$U_{\max} = \frac{Q_{\max}}{2\pi \varepsilon_0 L} \left(\frac{1}{\varepsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{b}{c} \right)$$

$$\left. \begin{aligned} E_{1\max} &= \frac{Q_{\max}}{2\pi \varepsilon_1 a L} = \frac{Q_{\max}}{2\pi 6\varepsilon_0 a L} = \frac{Q_{\max}}{12\pi \varepsilon_0 a L} \\ E_{2\max} &= \frac{Q_{\max}}{2\pi \varepsilon_2 c L} = \frac{Q_{\max}}{2\pi 4\varepsilon_0 2a L} = \frac{Q_{\max}}{16\pi \varepsilon_0 a L} \end{aligned} \right\} \Rightarrow E_{1\max} > E_{2\max}, \quad E_{1\max} = E_{\max}$$

$$Q_{\max} = E_{\max} 12\pi \varepsilon_0 a L$$

$$U_{\max} = \frac{E_{\max} 12\pi \varepsilon_0 a L}{2\pi \varepsilon_0 L} \left(\frac{1}{\varepsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{b}{c} \right) = E_{\max} 6a \left(\frac{1}{\varepsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{b}{c} \right)$$

$$U_{\max} = 12,13 \text{ kV}$$

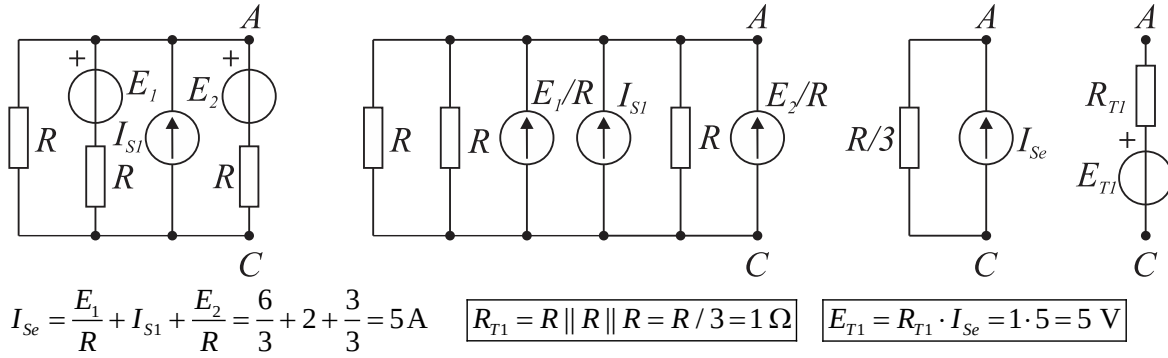
d)

$$W_e = \frac{1}{2} C U_{\max}^2$$

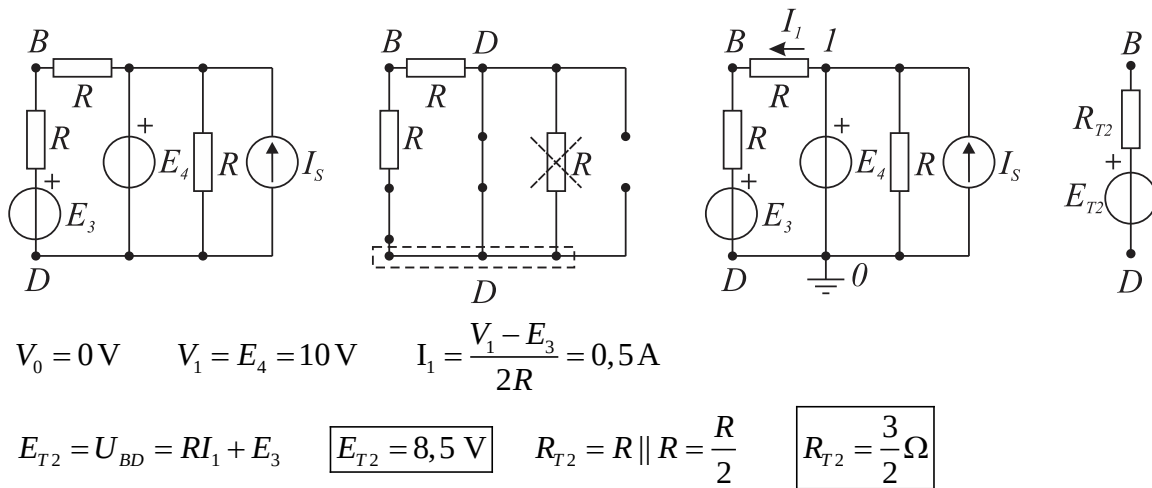
$$W_e = 14,2 \text{ mJ}$$

K2 Z1

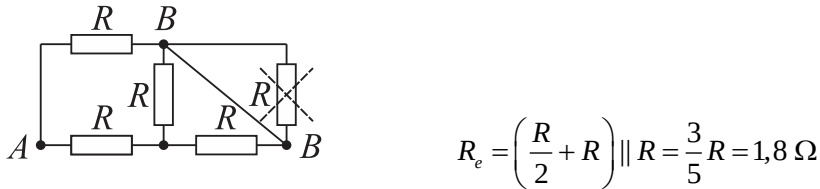
a)



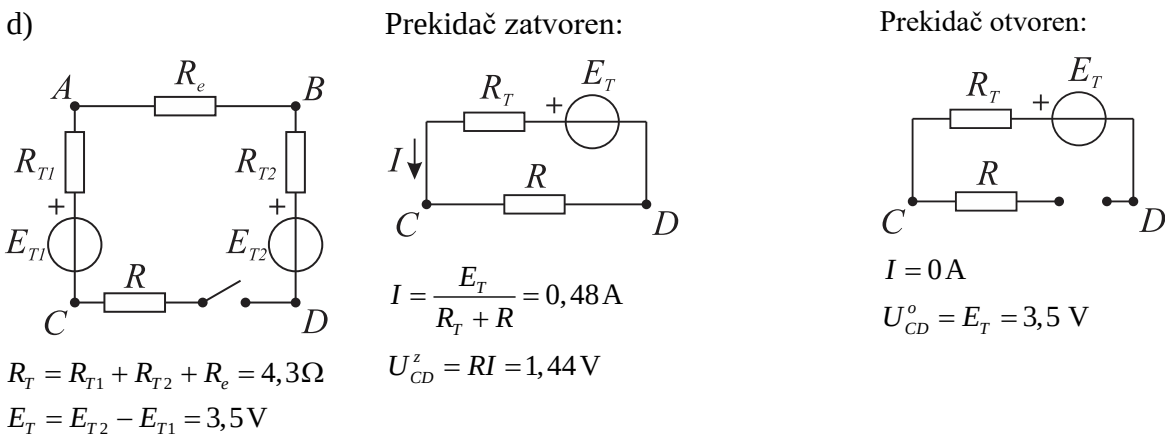
b)



c)



d)



$$\Delta U_{CD} = U_{CD}^z - U_{CD}^o = -2,06 \text{ V}$$

K2 Z2

1. Na osnovu uslova teksta zadatka: $U_{AB}^2 - U_{AB}^1 = \Delta U_{AB} = 2 \text{ V}$
2. Na osnovu teoreme superpozicije:

$$\boxed{\begin{matrix} \text{Svi} \\ \text{generatori} \end{matrix}} = \boxed{\begin{matrix} \text{Svi} \\ \text{sem } E \end{matrix}} + \boxed{\begin{matrix} \text{Samo} \\ E \end{matrix}}$$

(2) (1)

Preklopnik u položaju (2)
(deluju svi generatori)

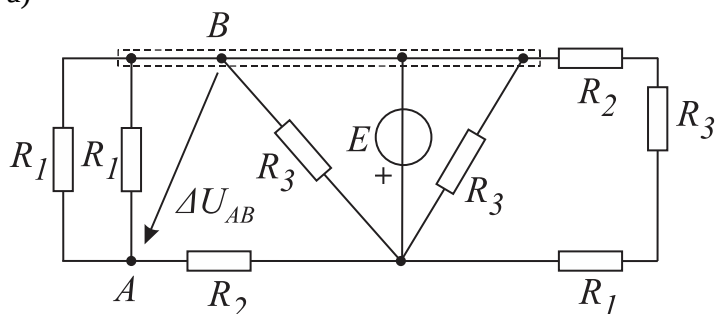
Preklopnik u položaju (1)
(deluju svi generatori osim E)

Deluje samo naponski
generator ems E

$$U_{AB}^{(2)} = U_{AB}^{(1)} + U_{AB}'$$

Na osnovu 1 i 2 sledi da je $U_{AB} = \Delta U_{AB} = 2 \text{ V}$

a)



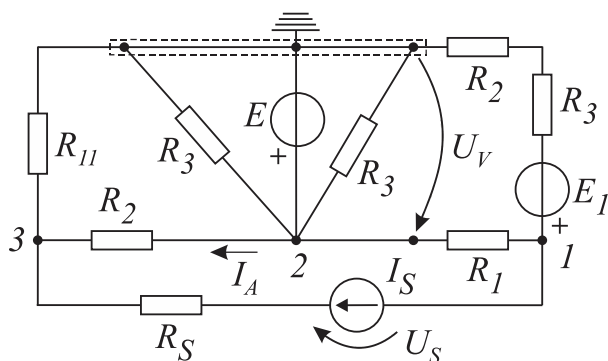
$$R_{11} = R_1 \parallel R_1 = 10 \parallel 10 = 5 \Omega$$

$$\Delta U_{AB} = \frac{R_{11}}{R_{11} + R_2} E$$

$$E = \frac{R_{11} + R_2}{R_{11}} \Delta U_{AB} = \frac{20}{5} \cdot 2$$

$$\boxed{E = 8 \text{ V}}$$

b)



$$\check{C}0: V_0 = 0 \text{ V}$$

$$\check{C}2: V_2 = E = 8 \text{ V}$$

$$\check{C}1: V_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_3 + R_2} \right) - V_2 \cdot \left(\frac{1}{R_1} \right) = -I_s + \frac{E_1}{R_3 + R_2}$$

$$\check{C}3: V_3 \cdot \left(\frac{1}{R_2} + \frac{1}{R_{11}} \right) - V_2 \cdot \left(\frac{1}{R_2} \right) = I_s$$

$$\check{C}1: V_1 \cdot \left(\frac{1}{10} + \frac{1}{20} \right) - V_2 \cdot \left(\frac{1}{10} \right) = -0,9 + \frac{2}{20}$$

$$\check{C}3: V_3 \cdot \left(\frac{1}{15} + \frac{1}{5} \right) - V_2 \cdot \left(\frac{1}{15} \right) = 0,9$$

$$\check{C}1: 3V_1 - 2V_2 = -18 + 2$$

$$\check{C}3: 4V_3 - V_2 = 13,5$$

$$V_1 = 0 \text{ V}, \quad V_3 = 5,375 \text{ V}$$

$$U_s = R_s I_s + V_3 - V_1 = 23,375 \text{ V}$$

$$P_s = U_s I_s = 21,04 \text{ W} \quad \boxed{P_s = 21,04 \text{ W}}$$