

Osnovi elektrotehnike 1
(I kolokvijum)

K1

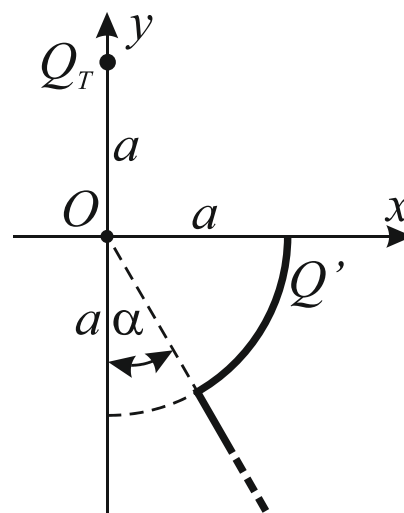
15.09.2026.

ZADACI

Zadatak 1. Veoma dugačak tanak štap, od izolacionog materijala, savijen kao što je prikazano na slici 1, naelektrisan je ravnomerno podužnom gustinom naelektrisanja Q' . Štap se sastoji od dela u obliku luka, poluprečnika a , i jednog pravolinijskog polubeskonačnog segmenta. Struktura se nalazi u x - y ravni zadanog koordinatnog sistema.

- Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u tački O (centar koordinatnog sistema), koji potiče od štapa.
- Odrediti naelektrisanje tačkastog naelektrisanja Q_T , tako da rezultantni vektor jačine električnog polja u tački O ima samo x komponentu.

Brojni podaci su: $a = 1 \text{ cm}$, $Q' = 5 \text{ nC/m}$, $\alpha = 30^\circ$, $\epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$.

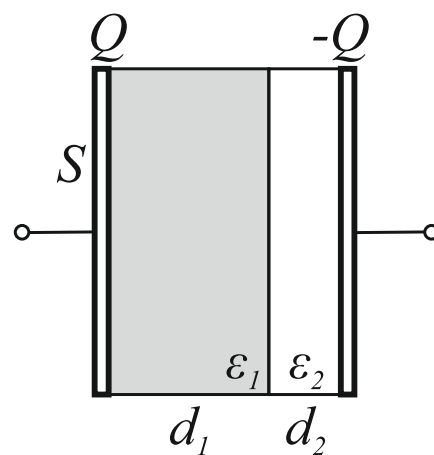


Slika 1.

Zadatak 2. Na slici 2 je prikazan pločasti kondenzator ispunjen sa dva sloja dielektrika, relativnih permitivnosti $\epsilon_{r1} = 2$ i $\epsilon_{r2} = 4$, debljina d_1 i $d_2 = 0,25 \text{ mm}$ i površine ploča $S = 4 \text{ cm}^2$.

- Izvesti u opštim brojevima izraz za kapacitivnost kondenzatora.
- Odrediti debljinu d_1 , tako da kapacitivnost prvog sloja dielektrika bude šest puta manja od kapacitivnosti drugog sloja.
- Izračunati maksimalni napon na koji sme da se priključi kondenzator.

Brojni podaci su: $E_{C1} = 45 \text{ kV/cm}$ i $E_{C2} = 70 \text{ kV/cm}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 1
(II kolokvijum)

K2

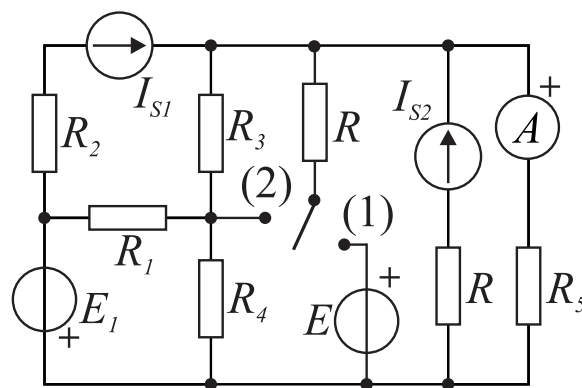
15.09.2026.

ZADACI

Zadatak 1. Kada se u mreži sa slike 1, preklopnik prebaci iz položaja (2) u položaj (1), struja koju meri ampermetar se poveća za $\Delta I_A = 90 \text{ mA}$.

- Primenjujući teoremu superpozicije, izračunati vrednost ems E .
- Izračunati snagu naponskog generatora ems E kada je prekidač u položaju (1).
- Izračunati snagu strujnog generatora I_{S1} kada je prekidač u položaju (1).

Brojni podaci su: $E_1 = 12 \text{ V}$, $R = R_2 = 10 \Omega$, $R_1 = R_4 = 80 \Omega$, $R_3 = R_5 = 20 \Omega$, $I_{S1} = 100 \text{ mA}$, $I_{S2} = 200 \text{ mA}$.

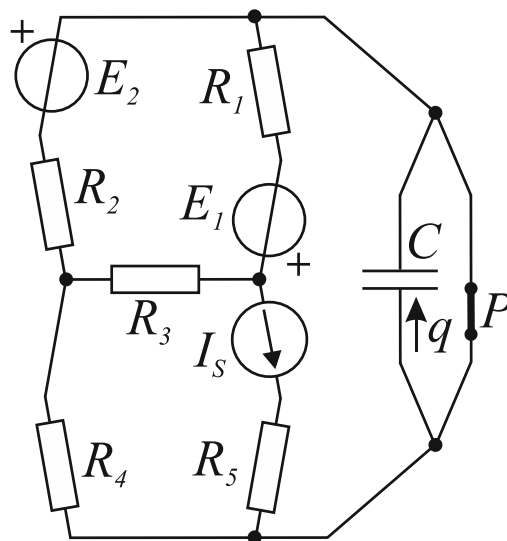


Slika 1.

Zadatak 2. U kolu vremenski konstantne struje sa slike 2 nalazi se kondenzator kapacitivnosti C , kroz koji, pri otvaranju prekidača P , protekne količina naelektrisanja $q = 64 \mu\text{C}$, u naznačenom referentnom smeru.

- Primenjujući Tevenenovu teoremu, odrediti vrednost kapacitivnosti kondenzatora, C .
- Odrediti snagu Tevenenovog generatora E_T , kada je prekidač zatvoren.

Brojni podaci su: $R_1 = R_3 = 10 \Omega$, $R_2 = R_4 = 30 \Omega$, $R_5 = 20 \Omega$, $E_1 = 12 \text{ V}$, $E_2 = 28 \text{ V}$, $I_S = 1 \text{ A}$.



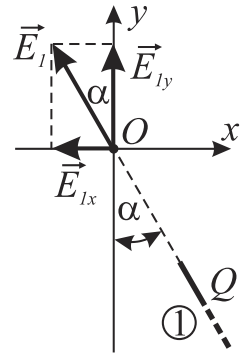
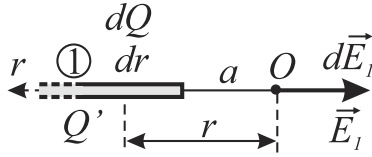
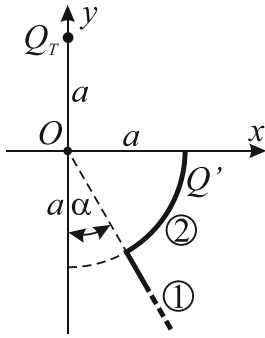
Slika 2.

PRAVILA POLAGANJA

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I-1

a)



$$dE_1 = \frac{dQ}{4\pi\epsilon_0 r^2} = \frac{Q' dr}{4\pi\epsilon_0 r^2}$$

$$E_1 = \int_{\text{po}}^{\infty} dE_1 = \frac{Q'}{4\pi\epsilon_0} \int_a^{\infty} \frac{dr}{r^2} = \frac{Q'}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{\infty} \right) = \frac{Q'}{4\pi\epsilon_0 a}$$

$$E_{1x} = E_1 \sin \alpha = \frac{Q'}{4\pi\epsilon_0 a} \frac{1}{2}$$

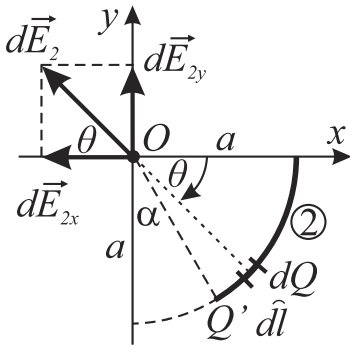
$$E_{1x} = \frac{Q'}{8\pi\epsilon_0 a}$$

$$\vec{E}_{1x} = E_{1x} \cdot (-\vec{i}_x)$$

$$E_{1y} = E_1 \cos \alpha = \frac{Q'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2}$$

$$E_{1y} = \frac{Q'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2}$$

$$\vec{E}_{1y} = E_{1y} \cdot \vec{i}_y$$



$$dE_{2x} = dE_2 \cos \theta = \frac{dQ}{4\pi\epsilon_0 r^2} \cos \theta = \frac{Q' dl}{4\pi\epsilon_0 a^2} \cos \theta = \frac{Q' a d\theta}{4\pi\epsilon_0 a^2} \cos \theta$$

$$E_{2x} = \int_{\text{po}}^{\text{luku}} dE_{2x} = \frac{Q'}{4\pi\epsilon_0 a} \int_0^{90^\circ - \alpha} \cos \theta d\theta = \frac{Q'}{4\pi\epsilon_0 a} (\sin 60^\circ - \sin 0) = \frac{Q'}{4\pi\epsilon_0 a} \left(\frac{\sqrt{3}}{2} - 0 \right)$$

$$E_{2x} = \frac{Q'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2}$$

$$\vec{E}_{2x} = E_{2x} \cdot (-\vec{i}_x)$$

$$dE_{2y} = dE_2 \sin \theta = \frac{dQ}{4\pi\epsilon_0 r^2} \sin \theta = \frac{Q' dl}{4\pi\epsilon_0 a^2} \sin \theta = \frac{Q' a d\theta}{4\pi\epsilon_0 a^2} \sin \theta$$

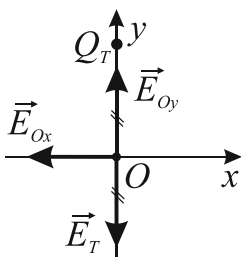
$$E_{2y} = \int_{\text{po}}^{\text{luku}} dE_{2y} = \frac{Q'}{4\pi\epsilon_0 a} \int_0^{90^\circ - \alpha} \sin \theta d\theta = \frac{Q'}{4\pi\epsilon_0 a} (\cos 0 - \cos 60^\circ) = \frac{Q'}{4\pi\epsilon_0 a} \left(1 - \frac{1}{2} \right)$$

$$E_{2y} = \frac{Q'}{8\pi\epsilon_0 a}$$

$$\vec{E}_{2y} = E_{2y} \cdot \vec{i}_y$$

$$\vec{E}_O = \vec{E}_1 + \vec{E}_2 = \left(\frac{Q'}{8\pi\epsilon_0 a} + \frac{Q'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2} \right) \cdot (-\vec{i}_x) + \left(\frac{Q'}{8\pi\epsilon_0 a} + \frac{Q'}{4\pi\epsilon_0 a} \frac{\sqrt{3}}{2} \right) \cdot \vec{i}_y$$

b)



$$E_T = \frac{Q_T}{4\pi\epsilon_0 a^2}$$

$$\vec{E}_T = E_T \cdot (-\vec{i}_y)$$

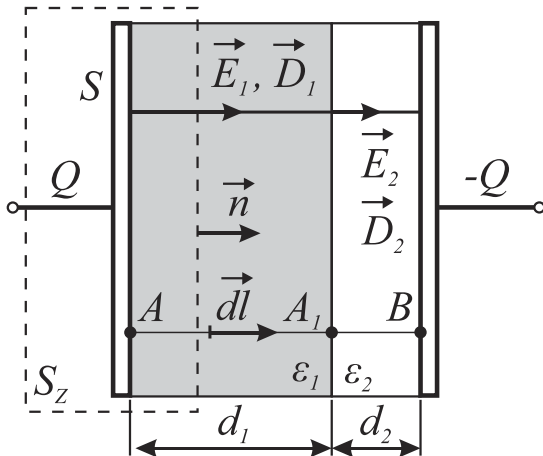
$$\vec{E}_{Oy} = 0 \Rightarrow E_T = E_{Oy}$$

$$\frac{Q_T}{4\pi\epsilon_0 a^2} = \frac{Q'}{8\pi\epsilon_0 a} (1 + \sqrt{3}) \quad Q_T = Q' a \frac{(1 + \sqrt{3})}{2}$$

$$Q_T = 68,3 \text{ pC}$$

I-2

a)



Granični uslov:

$$D_{n1} = D_{n2}$$

$$D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0$$

$$\oint_S \vec{D} \cdot d\vec{S} = Q_{\text{slobodno u } S}$$

$$\int_S D dS = Q$$

$$D S = Q$$

$$D = \frac{Q}{S}$$

$$E_1 = \frac{D}{\epsilon_1} = \frac{Q}{\epsilon_1 S}$$

$$E_2 = \frac{D}{\epsilon_2} = \frac{Q}{\epsilon_2 S}$$

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_A^{A_1} \vec{E}_1 \cdot d\vec{l} + \int_{A_1}^B \vec{E}_2 \cdot d\vec{l} = E_1 d_1 + E_2 d_2 = \frac{Q}{S} \left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right)$$

$$C = \frac{Q}{U}$$

$$C = \frac{Q}{\frac{Q}{S} \left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} \right)} = \frac{1}{\frac{d_1}{S\epsilon_1} + \frac{d_2}{S\epsilon_2}}$$

b)

$$C_1 = \epsilon_1 \frac{S}{d_1}, \quad C_2 = \epsilon_2 \frac{S}{d_2}$$

$$C_1 = \frac{C_2}{6}$$

$$\epsilon_1 \frac{S}{d_1} = \frac{1}{6} \epsilon_2 \frac{S}{d_2}$$

$$d_1 = \frac{6 \cdot \epsilon_1}{\epsilon_2} d_2 = \frac{6 \cdot 2\epsilon_0}{4\epsilon_0} \cdot 0,25 m$$

$$d_1 = 0,75 mm$$

c)

$$E_{1\max} = \frac{Q_{\max 1}}{\epsilon_1 S} = E_{C1} \Rightarrow Q_{\max 1} = E_{C1} \epsilon_1 S = 31,86 nC$$

$$E_{2\max} = \frac{Q_{\max 2}}{\epsilon_2 S} = E_{C2} \Rightarrow Q_{\max 2} = E_{C2} \epsilon_2 S = 99,12 nC$$

$$Q_{\max} = \min\{Q_{\max 1}, Q_{\max 2}\} = Q_{\max 1} = 31,86 nC$$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{31,86 nC}{8,1 pF}$$

$$U_{\max} = 3,93 kV$$

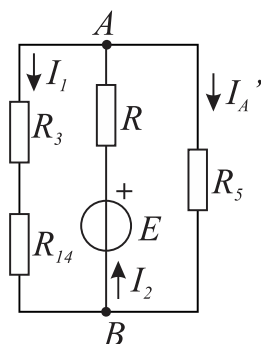
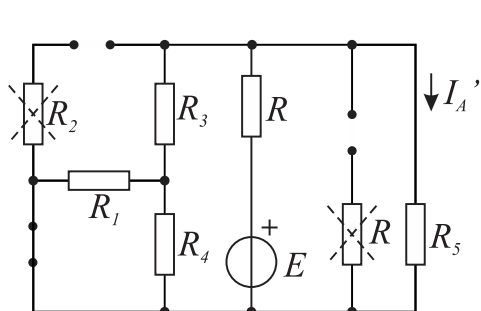
II-1

a)

$$\boxed{\begin{matrix} Svi \\ generatori \end{matrix}} = \boxed{\begin{matrix} Svi \\ sem E \end{matrix}} + \boxed{\begin{matrix} Samo \\ E \end{matrix}}$$

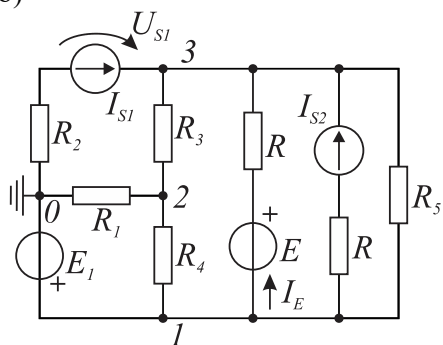
(1) (2)

$$I_A^{(1)} = I_A^{(2)} + I_A' \quad \Rightarrow \quad \Delta I_A = I_A^{(1)} - I_A^{(2)} = I_A' = 90 \text{ mA}$$



$$\begin{aligned} R_{14} &= R_1 \parallel R_4 = 40 \, \Omega \\ U_{AB} &= I_A' R_5 = 90 \text{ mA} \cdot 20 = 1,8 \text{ V} \\ I_1 &= \frac{U_{AB}}{R_3 + R_{14}} = \frac{1,8}{20 + 40} = 30 \text{ mA} \\ I_2 &= I_1 + I_A' = 120 \text{ mA} \\ -E + I_2 R + U_{AB} &= 0 \\ E &= I_2 R + U_{AB} = 120 \text{ mA} \cdot 10 + 1,8 \\ \boxed{E} &= 3 \text{ V} \end{aligned}$$

b)



$$V_0 = 0 \text{ V}, \quad V_1 = E_1 = 12 \text{ V}$$

$$V_2 \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) - V_1 \left(\frac{1}{R_4} \right) - V_3 \left(\frac{1}{R_3} \right) = 0$$

$$V_3 \left(\frac{1}{R_2 + \infty} + \frac{1}{R_3} + \frac{1}{R} + \frac{1}{R + \infty} + \frac{1}{R_5} \right) - V_1 \left(\frac{1}{R} + \frac{1}{R + \infty} + \frac{1}{R_5} \right) - V_2 \left(\frac{1}{R_3} \right) = I_{S1} + \frac{E}{R} + I_{S2}$$

$$V_2 \left(\frac{1}{80} + \frac{1}{20} + \frac{1}{80} \right) - 12 \cdot \left(\frac{1}{80} \right) - V_3 \left(\frac{1}{20} \right) = 0 \quad / \cdot 80$$

$$V_3 \left(\frac{1}{20} + \frac{1}{10} + \frac{1}{20} \right) - 12 \cdot \left(\frac{1}{10} + \frac{1}{20} \right) - V_2 \left(\frac{1}{20} \right) = 100 \text{ mA} + \frac{3}{10} + 200 \text{ mA} \quad / \cdot 20$$

$$\begin{cases} 6V_2 - 4V_3 = 12 \\ -V_2 + 4V_3 = 48 \end{cases} \quad \Rightarrow \quad V_2 = 12 \text{ V}, \quad V_3 = 15 \text{ V}$$

$$-E + I_E R + (V_3 - V_1) = 0$$

$$I_E = \frac{E - V_3 + V_1}{R} = \frac{3 - 15 + 12}{10} = 0 \text{ A}$$

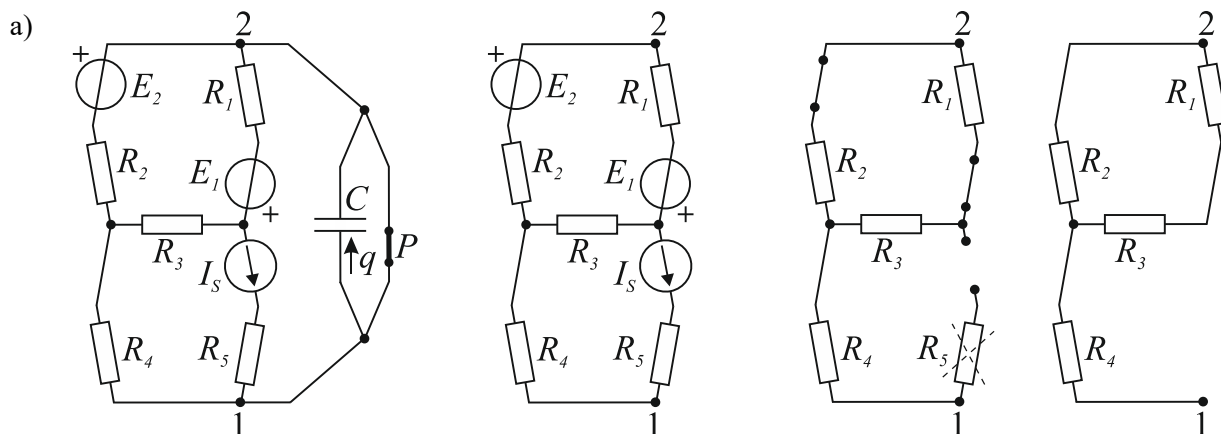
$$P_E = E I_E \quad \boxed{P_E = 0 \text{ W}}$$

c)

$$U_{S1} = (V_3 - V_0) + I_{S1} R_2 = (15 - 0) + 100 \text{ mA} \cdot 10 = 15 + 1 = 16 \text{ V}$$

$$P_{S1} = U_{S1} I_{S1} = 16 \cdot 100 \text{ mA} \quad \boxed{P_{S1} = 1,6 \text{ W}}$$

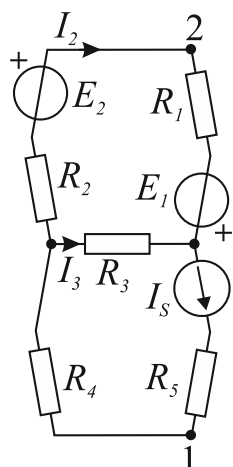
II-2



$$R_T = R_{12} = R_4 + [(R_3 + R_1) \parallel R_2]$$

$$R_T = 30 + [(10 + 10) \parallel 30] = 30 + [20 \parallel 30] = 42$$

$$\boxed{R_T = 42 \Omega}$$



$$R_2 I_2 - E_2 + R_1 I_2 - E_1 - R_3 I_3 = 0$$

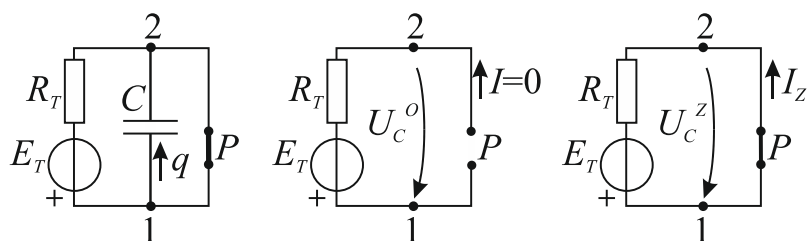
$$I_3 = I_S - I_2$$

$$R_2 I_2 - E_2 + R_1 I_2 - E_1 - R_3 (I_S - I_2) = 0$$

$$I_2 = \frac{E_1 + E_2 + R_3 I_S}{R_2 + R_1 + R_3} = \frac{12 + 28 + 10 \cdot 1}{30 + 10 + 10} = 1 \text{ A}$$

$$E_T = U_{12} = R_4 I_S + R_2 I_2 - E_2 = 30 \cdot 1 + 30 \cdot 1 - 28$$

$$\boxed{E_T = 32 \text{ V}}$$



$$q = C(U_C^{KR} - U_C^{POČ}) = C(U_C^O - U_C^Z)$$

$$C = \frac{q}{U_C^O - U_C^Z}$$

$$U_C^O = E_T = 32 \text{ V}$$

$$U_C^Z = 0 \text{ V}$$

$$C = \frac{q}{U_C^O - U_C^Z} = \frac{64 \cdot 10^{-6}}{32 - 0} = \frac{64 \cdot 10^{-6}}{32} = 2 \cdot 10^{-6} \text{ F}$$

$$\boxed{C = 2 \mu\text{F}}$$

b)

Kada je prekidač zatvoren: $I_Z = \frac{E_T}{R_T}$

$$P_{E_T} = E_T I_Z = E_T \frac{E_T}{R_T} = \frac{E_T^2}{R_T} = \frac{32^2}{42}$$

$\Rightarrow$

$$\boxed{P_{E_T} = 24,4 \text{ W}}$$