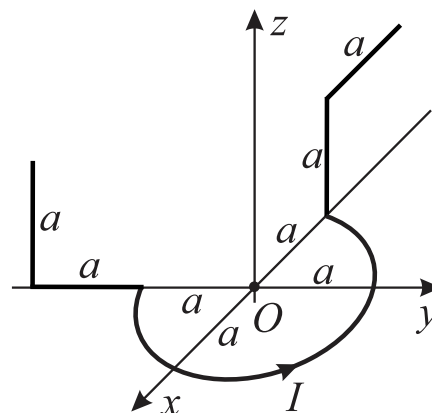


ZADACI

Zadatak 1. Žičani provodnik konačne dužine, sa vremenski konstantnom strujom jačine I , savijen je kao što je prikazano na slici 1. Provodnik se sastoji od četiri pravolinijska i jednog lučnog segmenta, kao što je prikazano na slici. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti su: $I = 3 \text{ A}$, $a = 5 \text{ cm}$.



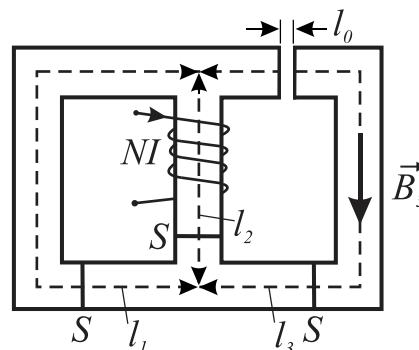
Slika 1.

Zadatak 2. Tanko magnetsko kolo sa slike 2a je načinjeno od materijala čija je idealizovana kriva prvobitnog magnetisanja prikazana na slici 2b. Pre uključivanja struje, jezgro je bilo nenamagnetisano. Kroz pobudni namotaj sa $N = 100$ zavoja propusti se struja intenziteta $I = 2,1 \text{ A}$. Ukoliko je poznato da magnetska indukcija u desnoj grani jezgra iznosi $B_3 = 0,35 \text{ T}$:

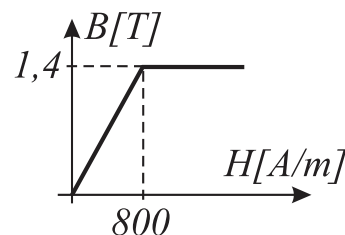
- Odrediti radne tačke u svim granama magnetskog kola i
- (bonus – 5 poena) Izračunati ukupnu energiju potrebnu za uspostavljanje magnetskog polja u kolu.

Ostali brojni podaci su:

$l_1 = 30 \text{ cm}$, $l_2 = 15 \text{ cm}$, $l_3 = 45 \text{ cm}$, $l_0 = 0,11 \text{ mm}$, $S = 1 \text{ cm}^2$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike 2
(II kolokvijum)

K2

16. 7. 2026.

ZADACI

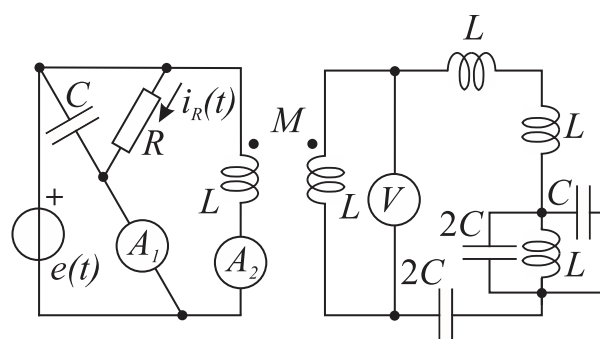
Zadatak 1. U mreži prostoperiodičnih struja, prikazanoj na slici 1, odrediti:

- pokazivanja idealnih mernih instrumenata,
- (**bonus – 5 poena**) kompleksnu, aktivnu, reaktivnu i prividnu snagu koja se razvija na naponskom generatoru $e(t)$.

Brojni podaci su:

$$e(t) = 10\sqrt{2} \cos(1000t + \pi/2) \text{ V},$$

$$L = 10 \text{ mH}, C = 100 \text{ }\mu\text{F}, M = 4 \text{ mH}, R = 10 \text{ }\Omega.$$

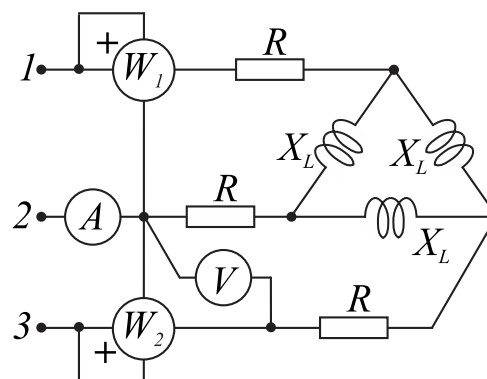


Slika 1.

Zadatak 2. Simetričan trofazni prijemnik priključen je na simetričnu trofaznu mrežu faznog napona prve faze $\underline{U}_1 = 230 \text{ V}$, kao što je prikazano na slici 2.

- Odrediti pokazivanje idealnih mernih instrumenata.
- Skicirati fazorski dijagram svih veličina od kojih zavisi pokazivanje mernih instrumenata.

Ostali brojni podaci su: $R = 30 \text{ }\Omega$, $X_L = 180 \text{ }\Omega$.

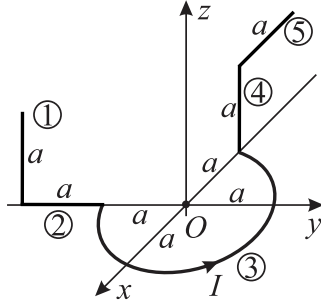


Slika 2.

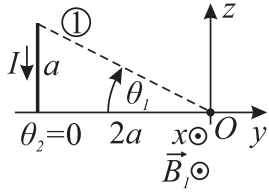
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1



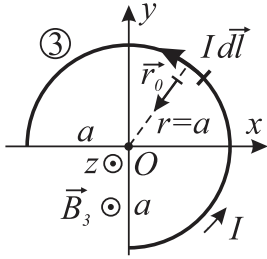
$$\boxed{\vec{B}_2 = 0} \quad \sin \angle(\vec{dl}, \vec{r}_0) = 0$$



$$\boxed{1} \quad d = 2a, \quad \theta_1 < 0, \quad \theta_2 = 0$$

$$B_1 = \frac{\mu_0 I}{4\pi d} (\sin \theta_2 - \sin \theta_1) = \frac{\mu_0 I}{4\pi 2a} \left[0 - \left(-\frac{a}{\sqrt{a^2 + (2a)^2}} \right) \right] = \frac{\mu_0 I}{4\pi 2a} \frac{\sqrt{5}}{5}$$

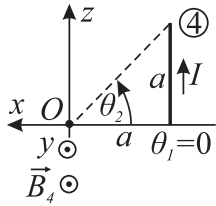
$$\boxed{\vec{B}_1 = \frac{\sqrt{5}\mu_0 I}{40\pi a} \cdot \vec{i}_x}$$



$$\vec{dB}_3 = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}_0}{r^2} \quad dB_3 = \frac{\mu_0}{4\pi} \frac{I dl}{a^2} \quad \angle(\vec{dl}, \vec{r}_0) = \frac{\pi}{2}$$

$$B_3 = \int dB_3 = \frac{\mu_0 I}{4\pi a^2} \int_0^{\frac{3}{4}2\pi} dl = \frac{\mu_0 I}{4\pi a^2} \frac{3}{2} a\pi = \frac{3\mu_0 I}{8a}$$

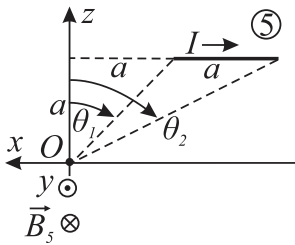
$$\boxed{\vec{B}_3 = \frac{3\mu_0 I}{8a} \cdot \vec{i}_z}$$



$$\boxed{4} \quad d = a, \quad \theta_1 = 0, \quad \theta_2 > 0$$

$$B_4 = \frac{\mu_0 I}{4\pi d} (\sin \theta_2 - \sin \theta_1) = \frac{\mu_0 I}{4\pi a} \left[\frac{a}{\sqrt{a^2 + a^2}} - 0 \right] = \frac{\mu_0 I}{4\pi a} \frac{\sqrt{2}}{2}$$

$$\boxed{\vec{B}_4 = \frac{\sqrt{2}\mu_0 I}{8\pi a} \cdot \vec{i}_y}$$



$$\boxed{5} \quad d = a, \quad \theta_1 > 0, \quad \theta_2 > 0$$

$$B_5 = \frac{\mu_0 I}{4\pi d} (\sin \theta_2 - \sin \theta_1) = \frac{\mu_0 I}{4\pi a} \left(\frac{2a}{\sqrt{(2a)^2 + a^2}} - \frac{a}{\sqrt{a^2 + a^2}} \right) = \frac{\mu_0 I}{4\pi a} \left(\frac{2\sqrt{5}}{5} - \frac{\sqrt{2}}{2} \right)$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{40\pi a} (4\sqrt{5} - 5\sqrt{2}) \cdot (-\vec{i}_y)}$$

$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \frac{\sqrt{5}\mu_0 I}{40\pi a} \cdot \vec{i}_x + \left[\frac{\sqrt{2}\mu_0 I}{8\pi a} - \frac{\mu_0 I}{40\pi a} (4\sqrt{5} - 5\sqrt{2}) \right] \cdot \vec{i}_y + \frac{3\mu_0 I}{8a} \cdot \vec{i}_z}$$

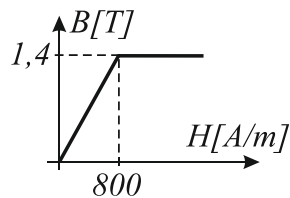
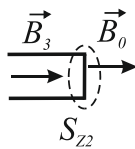
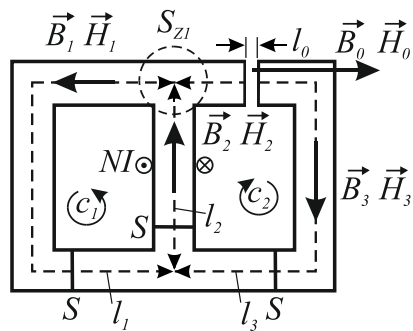
$$\vec{B}_O = 1,34 \mu\text{T} \cdot \vec{i}_x + 3,12 \mu\text{T} \cdot \vec{i}_y + 28,26 \mu\text{T} \cdot \vec{i}_z$$

$$|\vec{B}_O| = \sqrt{(1,34 \mu\text{T})^2 + (3,12 \mu\text{T})^2 + (28,26 \mu\text{T})^2}$$

$$\boxed{|\vec{B}_O| = 28,5 \mu\text{T}}$$

K1 Z2

a)



$$\oint_{S_z} \vec{B} \cdot d\vec{s} = 0$$

$$S_{z1}: B_1 S - B_2 S + B_3 S = 0$$

$$B_2 = B_1 + B_3 \quad (1)$$

$$S_{z2}: -B_3 S + B_0 S_0 = 0$$

$$(S = S_0) \quad B_3 = B_0 \quad (2)$$

$$B_3 = 0,35 \text{ T} \quad \Rightarrow \quad H_3 = \frac{B_3}{\mu} = 200 \text{ A/m}$$

$$\oint_C \vec{H} \cdot d\vec{l} = NI$$

$$c_1: H_2 l_2 + H_1 l_1 = NI \quad (3)$$

$$c_2: H_2 l_2 + H_0 l_0 + H_3 l_3 = NI \quad (4)$$

$$H_0 = \frac{B_0}{\mu_0} \quad (5)$$

$$\mu = \frac{1,4}{800}$$

$$(B_3, H_3) = (0,35 \text{ T}, 200 \text{ A/m})$$

$$\text{Iz (2) sledi } B_0 = B_3 = 0,35 \text{ T} \quad \Rightarrow \quad H_0 = \frac{B_0}{\mu_0} = \frac{0,35}{4\pi \cdot 10^{-7}} = 278,66 \text{ kA/m}$$

$$\text{Iz (4) sledi } H_2 = \frac{NI - H_0 l_0 - H_3 l_3}{l_2} = \frac{100 \cdot 2,1 - 278,66 \cdot 10^3 \cdot 0,11 \cdot 10^{-3} - 200 \cdot 0,45}{0,15} = 595,6 \text{ A/m}$$

$$\Rightarrow B_2 = \mu H_2 = 1,04 \text{ T}$$

$$(B_2, H_2) = (1,04 \text{ T}, 595,6 \text{ A/mA/m})$$

$$\text{Iz (1) sledi } B_1 = B_2 - B_3 = 0,69 \text{ T} \quad \Rightarrow \quad H_1 = \frac{B_1}{\mu} = 394,3 \text{ A/m}$$

$$(B_1, H_1) = (0,69 \text{ T}, 394,3 \text{ A/m})$$

b) (bonus – 5 poena)

$$W_{m1} = w_{m1} V_1 = \frac{1}{2} B_1 H_1 l_1 S_1 = 4,08 \text{ mJ}$$

$$W_{m2} = w_{m2} V_2 = \frac{1}{2} B_2 H_2 l_2 S_2 = 4,64 \text{ mJ}$$

$$W_{m3} = w_{m3} V_3 = \frac{1}{2} B_3 H_3 l_3 S_3 = 1,58 \text{ mJ}$$

$$W_{mUK} = W_{m1} + W_{m2} + W_{m3}$$

$$W_{mUK} = 10,30 \text{ mJ}$$

K2 Z1

$$e(t) = 10\sqrt{2} \cos(1000t + 90^\circ) \text{ V}$$

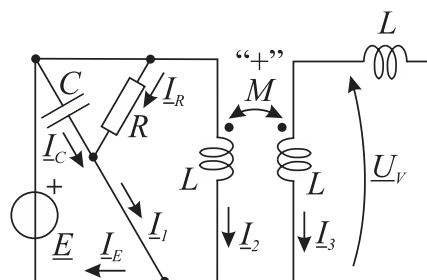
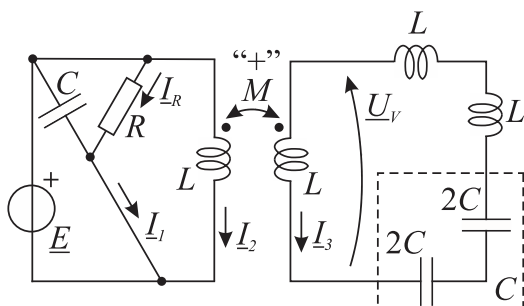
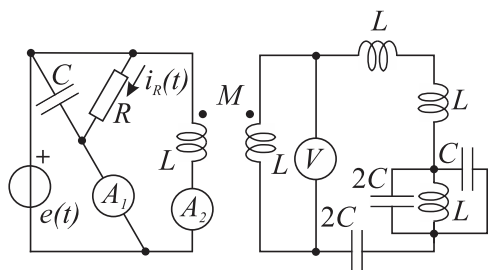
$$\underline{E} = \frac{10\sqrt{2}}{\sqrt{2}} e^{j90^\circ} \text{ V} = 10 e^{j90^\circ} \text{ V} = j10 \text{ V}$$

$$\omega = 1000 \frac{\text{rad}}{\text{s}}$$

$$X_L = j\omega L = j10 \Omega,$$

$$X_C = -j \frac{1}{\omega C} = -j10 \Omega,$$

$$X_M = j\omega M = j4 \Omega$$



a)

$$I_C = \frac{\underline{E}}{-j \frac{1}{\omega C}} = \frac{j10}{-j10} = -1 \text{ A} = 1 e^{j180^\circ} \text{ A}$$

$$I_R = \frac{\underline{E}}{R} = \frac{j10}{10} = j1 \text{ A} = 1 e^{j90^\circ} \text{ A}$$

$$I_1 = I_C + I_R = (-1 + j1) \text{ A} = \sqrt{2} e^{j135^\circ} \text{ A}$$

$$I_{A1} = |I_1| = \sqrt{2} \text{ A}$$

$$(1) \quad \underline{E} = j\omega L I_2 + j\omega M I_3$$

$$(2) \quad j\omega L I_3 + j\omega L I_3 + j\omega M I_2 = 0 \Rightarrow I_3 = -\frac{j\omega M}{2j\omega L} I_2 = -\frac{j4}{2 \cdot j10} I_2 = -\frac{1}{5} I_2$$

$$(1) \quad \underline{E} = j\omega L I_2 + j\omega M \left(-\frac{1}{5} I_2\right)$$

$$I_2 = -\frac{\underline{E}}{j\omega L - \frac{j\omega M}{5}} = \frac{j10}{j10 - j\frac{4}{5}} = \frac{j10}{j9,2} = 1,09 \text{ A} = 1,09 e^{j0^\circ} \text{ A}$$

$$I_{A2} = |I_2| = 1,09 \text{ A}$$

$$\underline{U}_V = -j\omega L I_3 = -j\omega L \left(-\frac{1}{5} I_2\right) = \frac{j\omega L}{5} I_2 = \frac{j10}{5} \cdot 1,09 = j2,18 \text{ V} = 2,18 e^{j90^\circ} \text{ V}$$

$$U_V = |\underline{U}_V| = 2,18 \text{ V}$$

b) (bonus – 5 poena)

$$\underline{I}_E = I_1 + I_2 = (-1 + j1) + 1,09 = (0,09 + j1) \text{ A} = 1,004 e^{j84,86^\circ} \text{ A}$$

$$\underline{S}_E = \underline{E} \underline{I}_E^* = j10 \cdot (0,09 - j1) = (0,9 - j10) \text{ VA}$$

$$\underline{S}_E = (0,9 - j10) \text{ VA}$$

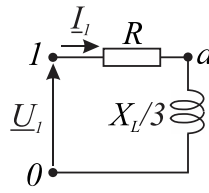
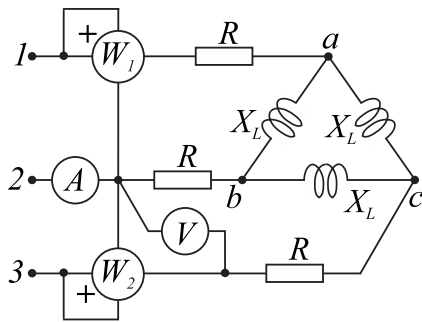
$$S_E = |\underline{S}_E| = 10,04 \text{ VA}$$

$$P_E = \text{Re}\{\underline{S}_E\} \quad P_E = 0,9 \text{ W}$$

$$Q_E = \text{Im}\{\underline{S}_E\} \quad Q_E = -10 \text{ VAr}$$

K2 Z2

a)



$$\underline{Z} = R + jX_L / 3 = (30 + j60) \Omega = 67,1 e^{j63,43^\circ} \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{230 e^{j0^\circ}}{67,1 e^{j63,43^\circ}} = 3,43 e^{-j63,43^\circ} \text{ A}$$

$$\underline{I}_2 = \underline{I}_1 e^{-j120^\circ} = 3,43 e^{-j183,43^\circ} \text{ A}$$

$$\underline{I}_3 = \underline{I}_1 e^{-j240^\circ} = 3,43 e^{-j303,43^\circ} \text{ A}$$

$$I_A = |\underline{I}_2| = 3,43 \text{ A}$$

$$U_V = |U_{32}| = \sqrt{3} |\underline{U}_1| = \sqrt{3} \cdot 230 = 398,37 \text{ V}$$

$$P_{W1} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_1^*\} = U_{12} I_1 \cos \angle(\underline{U}_{12}, \underline{I}_1) = \sqrt{3} U_1 I_1 \cos(30^\circ + 63,43^\circ)$$

$$P_{W1} = \sqrt{3} \cdot 230 \cdot 3,43 \cdot \cos 93,43^\circ$$

$$P_{W1} = -81,75 \text{ W}$$

$$P_{W2} = \operatorname{Re}\{\underline{U}_{32} \underline{I}_3^*\} = U_{32} I_3 \cos \angle(\underline{U}_{32}, \underline{I}_3) = \sqrt{3} U_1 I_1 \cos(63,43^\circ - 30^\circ)$$

$$P_{W2} = \sqrt{3} \cdot 230 \cdot 3,43 \cdot \cos 33,43^\circ$$

$$P_{W2} = 1140,35 \text{ W}$$

b)

