

Osnovi elektrotehnike
(I kolokvijum)

K1

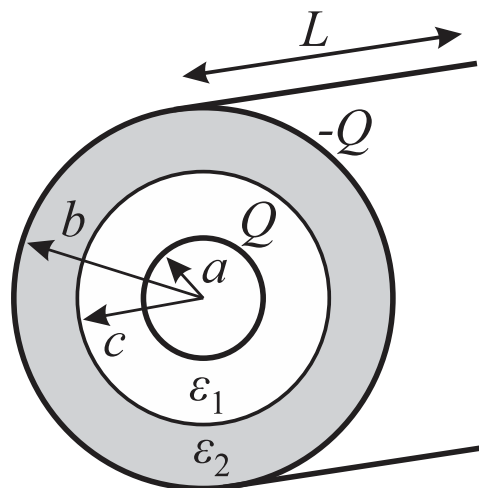
08.09.2026.

ZADACI

Zadatak 1. Na slici 1 je prikazan koaksijalni kabl dužine $L = 1 \text{ m}$, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\varepsilon_{r1} = 6$ i $\varepsilon_{r2} = 4$. Poluprečnici elektroda kabla su $a = 1 \text{ mm}$ i $b = 4 \text{ mm}$. Poluprečnik razdvojne površi dva dielektrika je $c = 2 \text{ mm}$. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i $-Q$.

- Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
- Izračunati maksimalni napon na koji može biti priključen kondenzator.
- Izračunati maksimalnu energiju koja može biti sadržana u kondenzatoru.

Ostali brojni podaci: $E_{\check{c}1} = 70 \text{ kV/cm}$, $E_{\check{c}2} = 80 \text{ kV/cm}$.

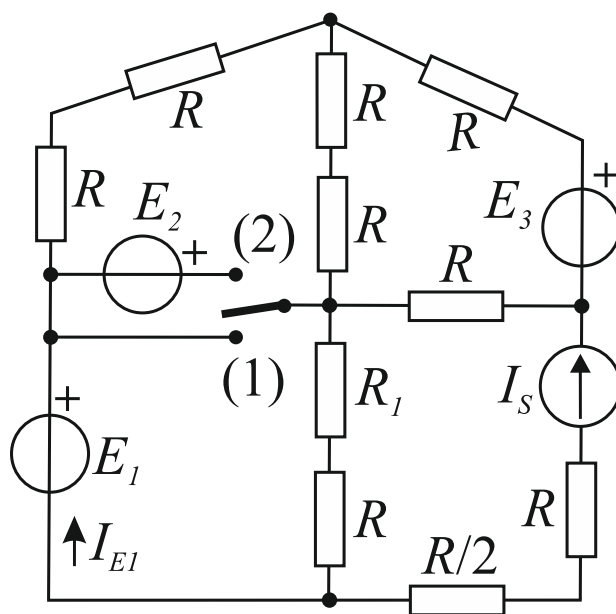


Slika 1.

Zadatak 2. U električnom kolu vremenski konstantnih struja, prikazanom na slici 2, prilikom prebacivanja preklopnika iz položaja (1) u položaj (2) struja kroz naponski generator E_1 se poveća za vrednost $\Delta I_{E1} = 1 \text{ A}$, u naznačenom referentnom smeru.

- Primenjujući teoremu superpozicije, odrediti *ems* naponskog generatora E_2 .
- Odrediti snagu otpornika R_1 , kada je preklopnik u položaju (2).
- Odrediti snagu strujnog generatora I_S , kada je preklopnik u položaju (2).

Brojni podaci su: $R_1 = 10 \Omega$, $R = 10 \Omega$,
 $E_1 = 9 \text{ V}$, $E_3 = 10 \text{ V}$, $I_S = 1 \text{ A}$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum je neophodno sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Osnovi elektrotehnike
(II kolokvijum)

K2

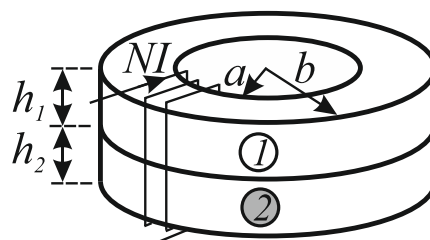
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ZADACI

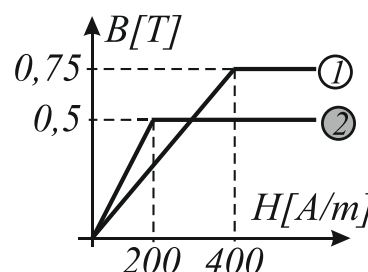
Zadatak 1. Na slici 1a je prikazano debelo torusno jezgro, načinjeno od dva materijala čije su magnetske karakteristike date na slici 1b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa $N = 20$ zavoja u kojima je uspostavljena struja jačine I . Odrediti:

- minimalnu vrednost struje, I , tako da materijal 2 bude ceo u zasićenju,
- režim rada materijala 1, pri struji određenoj pod a).
- fluks kroz površinu poprečnog preseka jezgra, pri struji određenoj pod a).
- magnetsku energiju u delu jezgra načinjenog od materijala 2, pri struji određenoj pod a).

Dimenzije jezgra su: $a = 5 \text{ cm}$, $b = 10 \text{ cm}$,
 $h_1 = h_2 = 5 \text{ cm}$.



Slika 1a.

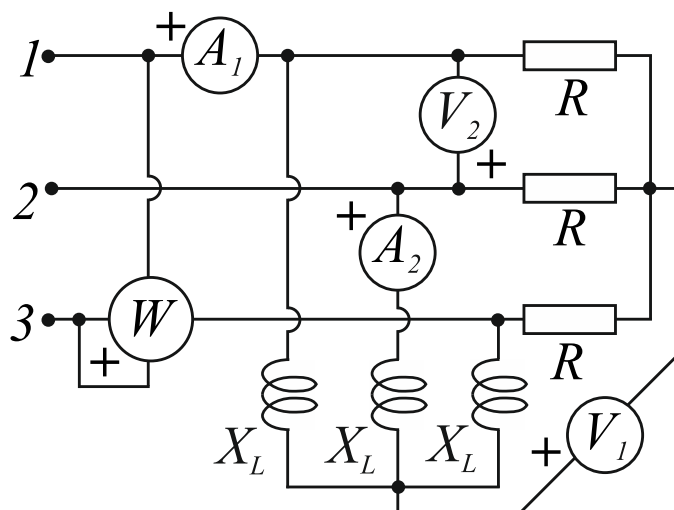


Slika 1b.

Zadatak 2. Simetričan trofazni prijemnik sa slike 2, sastavljen od otpornika otpornosti R i kalemova reaktanse X_L , priključen je na mrežu faznog napona prve faze $U_1 = 230 \text{ V}$.

- Odrediti pokazivanja idealnih mernih instrumenata.
- Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.

Brojni podaci: $R = 10 \Omega$, $X_L = 20 \Omega$.



Slika 2.

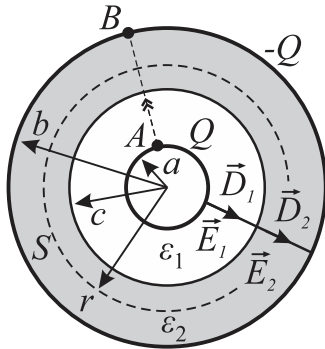
PRAVILA POLAGANJA

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K1

Z1

a)



Granični uslov:

$$D_{n1} = D_{n2} \quad D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{\text{slobodno u } S}$$

$$\int_{S_{OM}} D ds = Q$$

$$D 2\pi r L = Q$$

$$D = \frac{Q}{2\pi r L}$$

$$a \leq r \leq b$$

$$E_1 = \frac{D}{\epsilon_1} = \frac{Q}{2\pi\epsilon_1 r L}$$

$$a \leq r \leq c$$

$$E_2 = \frac{D}{\epsilon_2} = \frac{Q}{2\pi\epsilon_2 r L}$$

$$c \leq r \leq b$$

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_a^b E dr = \int_a^c E_1 dr + \int_c^b E_2 dr = \int_a^c \frac{Q}{2\pi\epsilon_1 r L} dr + \int_c^b \frac{Q}{2\pi\epsilon_2 r L} dr = \frac{Q}{2\pi\epsilon_1 L} \ln \frac{c}{a} + \frac{Q}{2\pi\epsilon_2 L} \ln \frac{b}{c}$$

$$C = \frac{Q}{U_{AB}}$$

$$C = \frac{2\pi\epsilon_0 L}{\frac{1}{\epsilon_{r1}} \ln \frac{c}{a} + \frac{1}{\epsilon_{r2}} \ln \frac{b}{c}}$$

b)

$$E_{1\max} = \frac{Q_{1\max}}{2\pi\epsilon_1 a L} \leq E_{c1} \Rightarrow Q_{1\max} = E_{c1} 2\pi\epsilon_1 a L = 2,33 \mu C$$

$$E_{2\max} = \frac{Q_{2\max}}{2\pi\epsilon_2 c L} \leq E_{c2} \Rightarrow Q_{2\max} = E_{c2} 2\pi\epsilon_2 c L = 3,56 \mu C$$

$$Q_{\max} = \min\{Q_{1\max}, Q_{2\max}\} = Q_{1\max} = 2,33 \mu C$$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{2,33 \mu C}{192,44 pF}$$

$$U_{\max} = 12,11 kV$$

c)

$$W_{e\max} = \frac{1}{2} C U_{\max}^2 = \frac{1}{2} \cdot 192,44 \cdot 10^{-12} \cdot (12,11 \cdot 10^3)^2$$

$$W_{e\max} = 14,11 mJ$$

K1

Z2

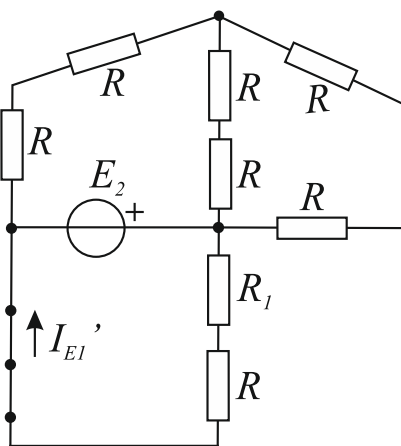
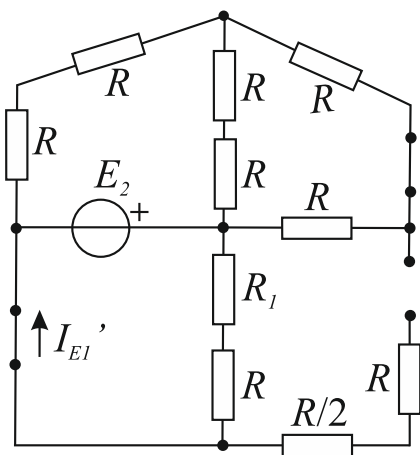
a)

$$\boxed{\begin{matrix} Svi \\ generatori \end{matrix}} = \boxed{\begin{matrix} Svi \\ sem E_2 \end{matrix}} + \boxed{\begin{matrix} Samo \\ E_2 \end{matrix}}$$

(2) (1)

$$I_{E1}^{(2)} = I_{E1}^{(1)} + I_{E1}'$$

$$\Delta I_{E1} = I_{E1}^{(2)} - I_{E1}^{(1)} = I_{E1}' = 1 \text{ A}$$

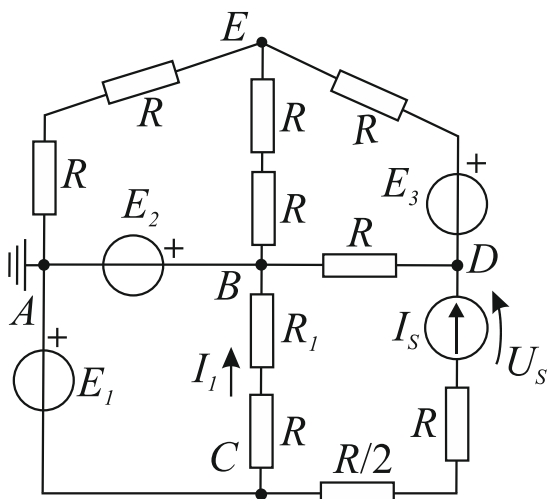


$$E_2 = I_{E1}'(R_1 + R)$$

$$E_2 = 1 \cdot (10 + 10)$$

$$\boxed{E_2 = 20 \text{ V}}$$

b)



$$V_A = 0 \text{ V}, \quad V_B = E_2 = 20 \text{ V}, \quad V_C = -E_1 = -9 \text{ V}$$

$$V_D \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{\infty} \right) - V_B \left(\frac{1}{R} \right) - V_C \left(\frac{1}{\infty} \right) - V_E \left(\frac{1}{R} \right) = I_S - \frac{E_3}{R}$$

$$V_E \left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} \right) - V_B \left(\frac{1}{2R} \right) - V_D \left(\frac{1}{R} \right) = \frac{E_3}{R}$$

$$V_D \left(\frac{1}{10} + \frac{1}{10} \right) - 20 \cdot \frac{1}{10} - V_E \left(\frac{1}{10} \right) = 1 - \frac{10}{10}$$

$$V_E \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20} \right) - 20 \cdot \frac{1}{20} - V_D \left(\frac{1}{10} \right) = \frac{10}{10}$$

$$\left. \begin{array}{l} 2V_D - V_E = 20 \\ -2V_D + 4V_E = 40 \end{array} \right\} \quad V_D = 20 \text{ V}, \quad V_E = 20 \text{ V}$$

$$I_1 = \frac{V_C - V_B}{R_1 + R} = \frac{-9 - 20}{10 + 10} = -1,45 \text{ A}$$

$$P_{R1} = R_1 I_1^2 = 10 \cdot (-1,45)^2$$

$$\boxed{P_{R1} = 21,03 \text{ W}}$$

c)

$$U_S = V_D - V_C + \left(\frac{R}{2} + R \right) I_S = 20 - (-9) + (5 + 10) \cdot 1 = 44 \text{ V}$$

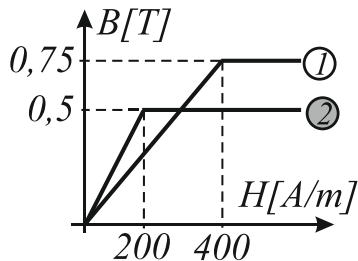
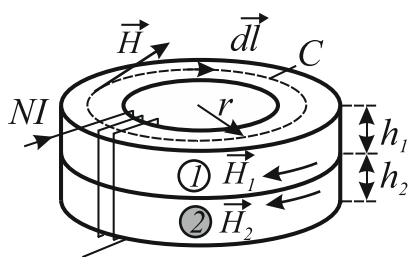
$$P_S = U_S I_S = 44 \cdot 1$$

$$\boxed{P_S = 44 \text{ W}}$$

K2

Z1

a)

Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_C \vec{H} \cdot d\vec{l} = NI$$

$$H 2\pi r = NI$$

$$H = \frac{NI}{2\pi r}$$

$$a \leq r \leq b$$

$$H_{2\min} = \frac{NI}{2\pi b} \geq H_{K2} \Rightarrow I \geq \frac{H_{K2} 2\pi b}{N} = \frac{200 \cdot 2 \cdot 3,14 \cdot 0,1}{20} = 6,28 \text{ A}$$

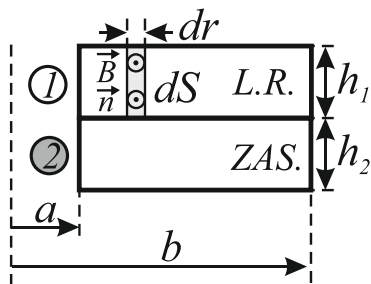
$$I_{\min} = 6,28 \text{ A}$$

b)

$$H_{1\max} = \frac{NI}{2\pi a} = \frac{20 \cdot 6,28}{2 \cdot 3,14 \cdot 0,05} = 400 \text{ A/m} \leq H_{K1}$$

Materijal 1 u linearnom režimu rada

c)



$$\Phi_1 = \int_{S_1} B_1 dS = \int_{S_1} \mu_1 H dr h_1 = \int_a^b \frac{0,75}{400} \frac{NI}{2\pi r} dr h_1 = \frac{0,75}{400} \frac{NI}{2\pi} h_1 \ln \frac{b}{a}$$

$$\Phi_1 = 1299,65 \mu Wb$$

$$\Phi_2 = B_{K2} (b - a) h_2$$

$$\Phi_2 = 1250 \mu Wb$$

$$\Phi_{uk} = \Phi_1 + \Phi_2$$

$$\Phi_{uk} = 2549,65 \mu Wb$$

d)

$$W_m = \int_V w_m dV$$

$$w_{m2} = \frac{1}{2} B_{K2} H_{K2} = \text{const.}$$

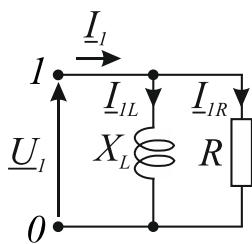
$$W_{m2} = w_{m2} V_2 = \frac{1}{2} B_{K2} H_{K2} (b^2 \pi - a^2 \pi) h_2$$

$$W_{m2} = 58,9 \text{ mJ}$$

K2

Z2

a)



$$\underline{I}_{1R} = \frac{\underline{U}_1}{R} = \frac{230 e^{j0^\circ}}{10} = 23 e^{j0^\circ} \text{ A} = 23 \text{ A}$$

$$\underline{I}_{1L} = \frac{\underline{U}_1}{jX_L} = \frac{230 e^{j0^\circ}}{j20} = \frac{230 e^{j0^\circ}}{20 e^{j90^\circ}} = 11,5 e^{-j90^\circ} \text{ A} = -j11,5 \text{ A}$$

$$\underline{I}_1 = \underline{I}_{1R} + \underline{I}_{1L} = (23 - j11,5) \text{ A} = 25,71 e^{-j26,57^\circ} \text{ A}$$

$$\underline{I}_2 = \underline{I}_1 e^{-j120^\circ} = 25,71 e^{-j146,57^\circ} \text{ A}$$

$$\underline{I}_3 = \underline{I}_1 e^{-j240^\circ} = 25,71 e^{-j266,57^\circ} \text{ A}$$

$$I_{A1} = |\underline{I}_1| = 25,71 \text{ A}$$

$$\underline{I}_{2L} = \underline{I}_{1L} e^{-j120^\circ} = 11,5 e^{-j210^\circ} \text{ A}$$

$$I_{A2} = |\underline{I}_{2L}| = 11,5 \text{ A}$$

$$U_{V1} = |\underline{U}_{00}| = 0 \text{ V}$$

$$U_{V2} = |\underline{U}_{21}| = \sqrt{3} |\underline{U}_1| = \sqrt{3} \cdot 230 = 398,4 \text{ V}$$

$$P_{W1} = \operatorname{Re}\{\underline{U}_{31} \underline{I}_3^*\} = U_{31} I_3 \cos \angle(\underline{U}_{31}, \underline{I}_3)$$

$$P_{W1} = \sqrt{3} U_1 I_1 \cos(30^\circ + 26,57^\circ) = \sqrt{3} \cdot 230 \cdot 25,71 \cdot \cos 56,57^\circ$$

$$P_{W1} = 5642,57 \text{ W}$$

b)

