STUDIJSKI PROGRAM MERENJE I REGULACIJA

K T E T

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

2. 9. 2026.

ZADACI

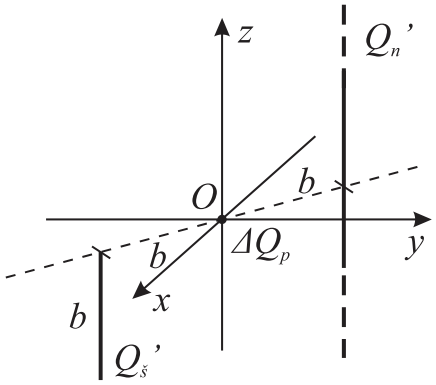
Zadatak 1. Tanak štap od izolacionog materijala, dužine b, naelektrisan je ravnomerno, podužnim naelektrisanjem $Q_š'$ i postavljen je paralelno sa z-osom zadatog koordinatnog sistema. Početak štapa se nalazi na simetrali drugog i četvrtog kvadranta, na rastojanju b od koordinatnog početka. Drugi, beskonačan štap je naelektrisan podužnim naelektrisanjem Q_n', postavljen je u pravcu z ose i preseca simetralu na rastojanju b od koordinatnog početka, kao što je prikazano na slici 1. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u koordinatnom početku, koji potiče od naelektrisanih štapova.

b) Podužno naelektrisanje štapa, tako da horizontalna komponenta vektora jačine električnog polja u tački O bude jednaka nuli,

c) Odrediti silu na probno opterećenje ΔQ_p koje se nalazi u koordinatnom početku.

Brojni podaci su: Q_n' = 1 nC/m, ΔQ_p = 1pC, b = 4 cm,
$\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m



Slika 1.

Zadatak 2. Na slici 2 je prikazan poprečni presek vazdušnog koaksijalnog kabla, poluprečnika elektroda a = 1 mm i b = 7,39 mm, dužine L = 1 m, sa dva sloja dielektrika, permitivnosti $\varepsilon_1 = 6\varepsilon_0$ i $\varepsilon_2 = 3\varepsilon_0$. Poluprečnik granične površine između dielektrika je c = 2,72 mm. Kabl je priključen na izvor napona U. Odrediti:

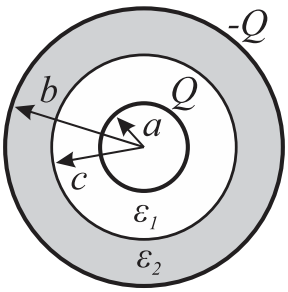
a) Kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja,

b) Kapacitivnost kabla, C.

c) Odrediti ukupno vezno naelektrisanje na graničnoj površini između dva dielektrika.

d) Odrediti maksimalan napon na koji sme da se priključi kabl.

Ostali brojni podaci su:
Q = 1 μC, $E_{č1}$ = 60 kV/cm, $E_{č2}$ = 35 kV/cm, $\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m,



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MERENJE I REGULACIJA

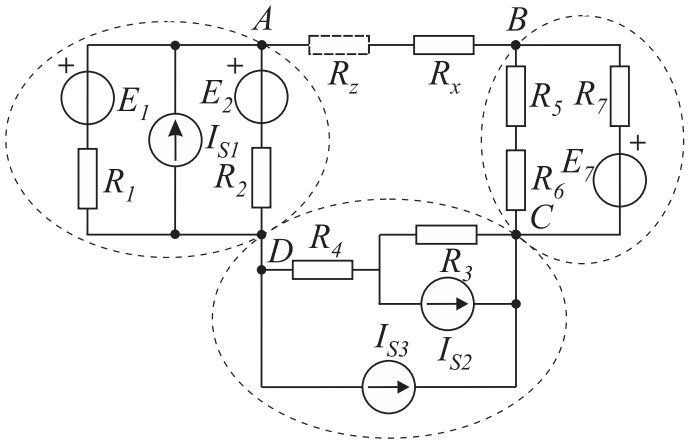
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **2. 9. 2026.**

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

a) Deo mreže između tačaka A i D uprostiti, tako da sadrži samo jedan realan naponski generator.
b) Deo mreže između tačaka C i D uprostiti, tako da sadrži samo jedan realan naponski generator.
c) Deo mreže između tačaka B i C uprostiti, tako da sadrži samo jedan realan naponski generator.
d) Nakon transformacije kola, proveriti da li će otpornik R_x = 2 kΩ, sa maximalnom snagom P_{xmax} = 2 mW, pregoreti?
e) Ako je potrebno, izračunati minimalnu vrednost otpornosti zaštitnog otpornika, R_z, koji treba priključiti na red sa otpornikom otpornosti R_x, da bi ga zaštitio od pregorevanja.

Brojni podaci su:
R_1 = 4 kΩ, R_2 = 2 kΩ, R_3 = R_4 = 2 kΩ,
R_5 = R_6 = 1 kΩ, R_7 = 2 kΩ, E_1 = 4 V, E_2 = 9 V,
E_7 = 2V, I_{S1} = 0,5 mA, I_{S2} = 4 mA, I_{S3} = 3 mA.



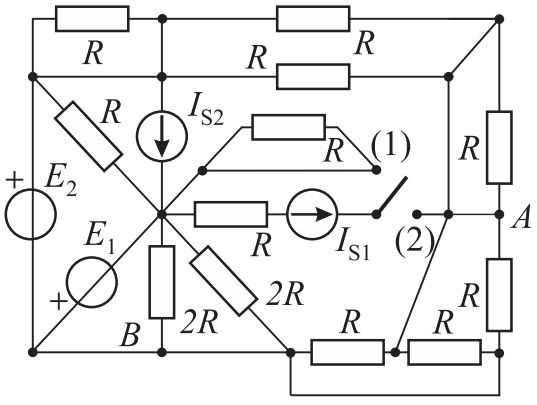
Slika 1.

Zadatak 2. U električnom kolu vremenski konstantnih struja sa slike 2, prilikom prebacivanja prekidača iz položaja (1) u položaj (2), napon između tačaka *A* i *B* se poveća za ΔU_{AB} = 10 V.

a) Izračunati nepoznatu struju strujnog generatora I_{S1} primenom teoreme superpozicije.

b) Izračunati snage generatora I_{S1}, I_{S2} i E_1 kada je prekidač u položaju (2), primenom metode potencijala čvorova.

Brojni podaci su: R = 10 Ω, E_1 = 6V, E_2 = 5V, I_{S2} = 1 A.



Slika 2.

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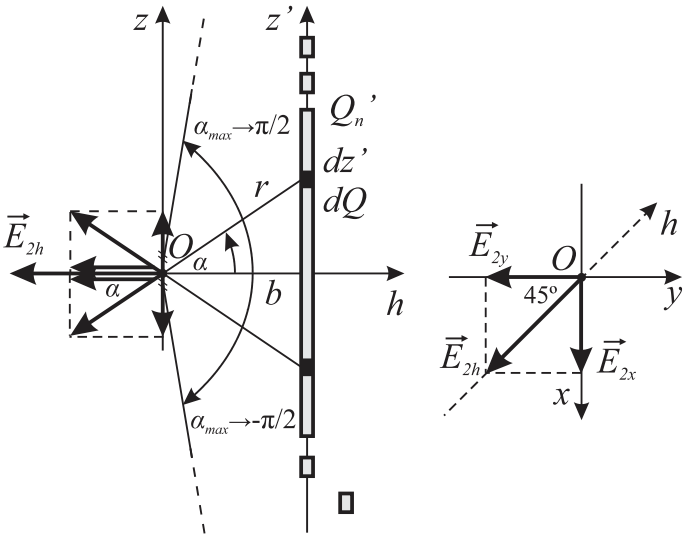
K1 Z1



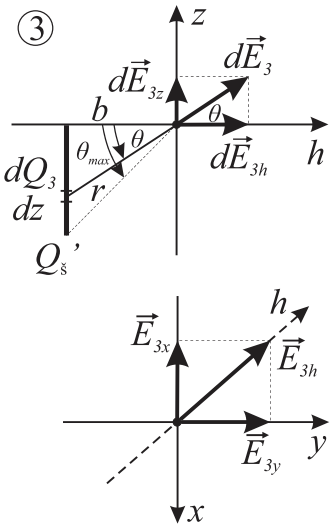
a)

$$E_{\infty niti}=\frac{Q'}{2\pi\varepsilon_0 r}$$

$$E_{nh}=\frac{Q_n'}{2\pi\varepsilon_0 b}$$

$$\boxed{\vec{E}_{nx}=\frac{Q_n'}{2\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\vec{i}_x}$$

$$\boxed{\vec{E}_{ny}=\frac{Q_n'}{2\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\left(-\vec{i}_y\right)}$$



$$dE_{šh}=\frac{dQ_š}{4\pi\varepsilon_0 r^2}\cos\theta=\frac{Q_š' dz}{4\pi\varepsilon_0 r^2}\cos\theta=\frac{Q_š'}{4\pi\varepsilon_0}\frac{r\,d\theta}{\cos\theta}\cos\theta=\frac{Q_š'}{4\pi\varepsilon_0 r}d\theta=\frac{Q_š'}{4\pi\varepsilon_0 b}\cos\theta d\theta$$

$$E_{šh}=\frac{Q_š'}{4\pi\varepsilon_0 b}\int_0^{\theta_{max}}\cos\theta d\theta=\frac{Q_š'}{4\pi\varepsilon_0 b}\sin\theta_{max}$$

$$\vec{E}_{šh}=\frac{Q_š'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\vec{i}_h\ \rightarrow\ \boxed{\vec{E}_{šx}=E_{3h}\frac{\sqrt{2}}{2}\left(-\vec{i}_x\right)}\qquad\boxed{\vec{E}_{šy}=E_{3h}\frac{\sqrt{2}}{2}\vec{i}_y}$$

$$dE_{šz}=\frac{dQ_š}{4\pi\varepsilon_0 r^2}\sin\theta=\frac{Q_š' dz}{4\pi\varepsilon_0 r^2}\sin\theta=\frac{Q_š'}{4\pi\varepsilon_0}\frac{r\,d\theta}{\cos\theta}\sin\theta=\frac{Q_š' d\theta}{4\pi\varepsilon_0 r\cos\theta}\sin\theta$$

$$dE_{šz}=\frac{Q_š' d\theta}{4\pi\varepsilon_0\frac{b}{\cos\theta}\cos\theta}\sin\theta=\frac{Q_š'}{4\pi\varepsilon_0 b}\sin\theta d\theta$$

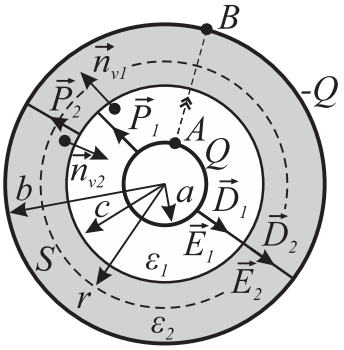
$$E_{šz}=\frac{Q_š'}{4\pi\varepsilon_0 b}\int_0^{\theta_{max}}\sin\theta d\theta=\frac{Q_š'}{4\pi\varepsilon_0 b}\left(-\cos\theta\right)\Big|_0^{\theta_{max}}=\frac{Q_š'}{4\pi\varepsilon_0 b}\left(1-\cos\theta_{max}\right)$$

$$\boxed{\vec{E}_{šz}=\frac{Q_š'}{4\pi\varepsilon_0 b}\left(1-\frac{\sqrt{2}}{2}\right)\vec{i}_z}$$

c) $E_{nh}=E_{šh}\ \rightarrow\ \frac{Q_n'}{2\pi\varepsilon_0 b}=\frac{Q_š'}{4\pi\varepsilon_0 b}\frac{\sqrt{2}}{2}\ \rightarrow\ Q_š'=2\sqrt{2}Q_n'=2{,}83\,\text{nC/m}$

d) $\vec{F}=\Delta Q_p\vec{E}_{šz}=\Delta Q_p\frac{Q_š'}{4\pi\varepsilon_0 b}\left(1-\frac{\sqrt{2}}{2}\right)\vec{i}_z=186{,}33\,\text{pN}\vec{i}_z$

K1 Z2



a)

Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} \neq E_{t2}$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$

$\int_S D\, ds = Q$

$D 2\pi r L = Q \qquad \boxed{D = \frac{Q}{2\pi r L}}, \qquad a \le r \le b$

$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 r L}}, \qquad a \le r \le c$

$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 r L}}, \qquad c \le r \le b$

b)

$C = \frac{Q}{U}$

$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E dr = \int_a^c E_1 dr + \int_c^b E_2 dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 r L} dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 r L} dr = \frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}\right)$

$C = \frac{2\pi L}{\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}} = \frac{2\pi \cdot 1}{\frac{1}{6\varepsilon_0}\ln 2{,}72 + \frac{1}{3\varepsilon_0}\ln 2{,}72}$

$\boxed{C = 110{,}8\ \text{pF}}$

c)

$\sigma_{v1} = \vec{P}_1 \cdot \vec{n}_{v1} = P_1(r=c) = D - \varepsilon_0 E_1 = D - \varepsilon_0 \frac{D}{\varepsilon_1} = \left(1 - \frac{1}{\varepsilon_{r1}}\right) D = \left(1 - \frac{1}{6}\right)\frac{Q}{2\pi c L} = \frac{5}{6}\frac{Q}{2\pi c L}$

$\sigma_{v2} = \vec{P}_2 \cdot \vec{n}_{v2} = -P_2(r=c) = -(D - \varepsilon_0 E_2) = -\left(D - \varepsilon_0 \frac{D}{\varepsilon_2}\right) = -\left(1 - \frac{1}{\varepsilon_{r2}}\right) D = -\left(1 - \frac{1}{3}\right)\frac{Q}{2\pi c L} = -\frac{2}{3}\frac{Q}{2\pi c L}$

$Q_{v(r=c)} = Q_{v1} + Q_{v2} = \sigma_{v1} 2c\pi L + \sigma_{v2} 2c\pi L = \left(\frac{5}{6}\frac{Q}{2\pi c L} - \frac{2}{3}\frac{Q}{2\pi c L}\right) \cdot 2\pi c L = \frac{1}{6} Q$

$\boxed{Q_{v(r=c)} = 166{,}67\ \text{nC}}$

d)

$E_{1\max}(r=a) = \frac{Q_{\max}}{2\pi\varepsilon_1 a L} = \frac{C U_{1\max}}{2\pi\varepsilon_1 a L} \le E_{č1} \quad \rightarrow \quad U_{1\max} = \frac{E_{č1} 2\pi\varepsilon_1 a L}{C} = 180{,}67\ \text{kV}$

$E_{2\max}(r=c) = \frac{Q_{\max}}{2\pi\varepsilon_2 c L} = \frac{C U_{2\max}}{2\pi\varepsilon_2 c L} \le E_{č2} \quad \rightarrow \quad U_{2\max} = \frac{E_{č2} 2\pi\varepsilon_2 c L}{C} = 143{,}33\ \text{kV}$

$U_{\max} = \min\{U_{1\max}, U_{2\max}\} = 143{,}33\ \text{kV}$

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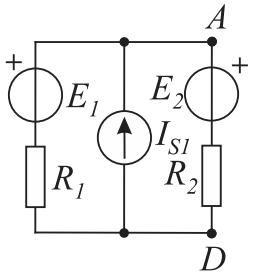
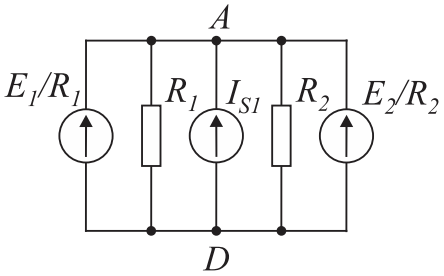
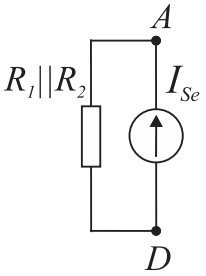
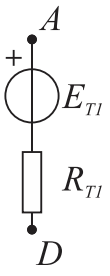
K2 Z1

a)







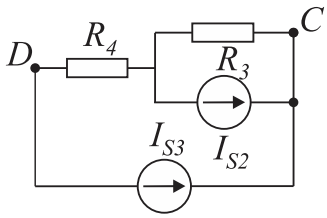


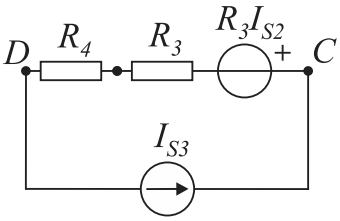
$$I_{Se}=\frac{E_1}{R_1}+I_{S1}+\frac{E_2}{R_2}=\frac{4}{4k}+0.5m+\frac{9}{2k}=6\,\text{mA}$$

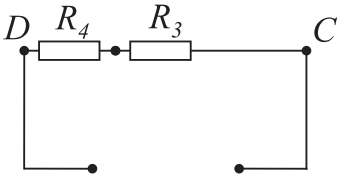
$$\boxed{R_{T1}=R_1\parallel R_2=4k\parallel 2k=\frac{8}{6}\,\text{k}\Omega}$$

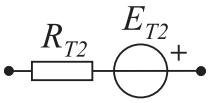
$$\boxed{E_{T1}=R_{T1}\cdot I_{Se}=\frac{8}{6}k\cdot 6m=8\ \text{V}}$$

b)









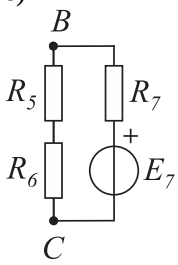
$$E_{T2}=U_{CD}=I_{S2}R_3+\left(R_3+R_4\right)I_{S3}=4m\cdot 2k+4k\cdot 3m$$

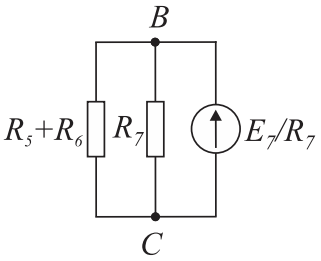
$$\boxed{E_{T2}=20\ \text{V}}$$

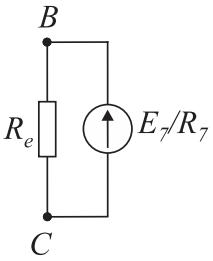
$$R_{T2}=R_3+R_4$$

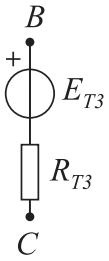
$$\boxed{R_{T2}=4k\Omega}$$

c)







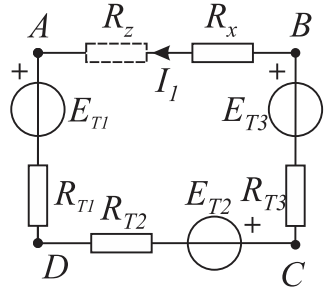


$$\frac{E_7}{R_7}=\frac{2}{2k}=1\text{mA}$$

$$\boxed{R_{T3}=R_e=\left(R_5+R_6\right)\parallel R_7=1\text{k}\Omega}$$

$$\boxed{E_{T3}=\frac{E_7}{R_7}R_e=1m\cdot 1k=1\text{V}}$$

d)



$$I_1=\frac{-E_{T1}+E_{T2}+E_{T3}}{R_{T1}+R_{T2}+R_{T3}+R_x}=1.56\,\text{mA}$$

$$P_{x\max}=R_x I_{\max}{}^2$$

$$I_{\max}=\sqrt{\frac{P_{x\max}}{R_x}}=\sqrt{\frac{2m}{2k}}=1mA$$

$I_1>I_{\max}$ R_x će pregoreti. Neophodno je vezati zaštitni otpornik.

e)

$$I_1=\frac{-E_{T1}+E_{T2}+E_{T3}}{R_{T1}+R_{T2}+R_{T3}+R_x+R_Z}=\frac{E_T}{R_T+R_x+R_Z}\le I_{\max}$$

$$R_T+R_x+R_Z\ge\frac{E_T}{I_{\max}}$$

$$R_Z\ge\frac{E_T}{I_{\max}}-R_T-R_x=\frac{13}{1m}-6,33k-2k$$

$$\boxed{R_{Z\min}=4,67\ k\Omega}$$

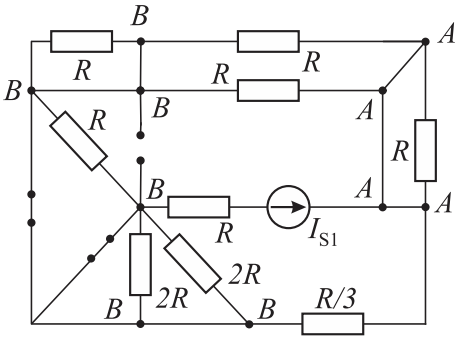
K2 Z2

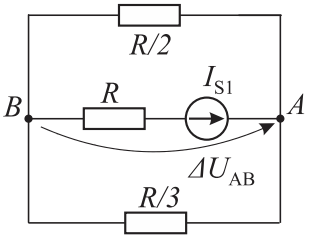
a)

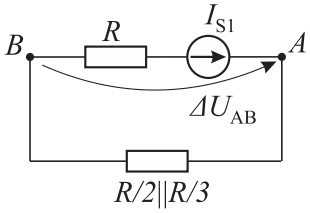
$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_{S1}\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_{S1}\end{matrix}}$$

$$(2) \qquad (1)$$

$$U_{AB}^{(2)} = U_{AB}^{(1)} + \Delta U_{AB} \quad \Rightarrow \quad \Delta U_{AB} = U_{AB}^{(2)} - U_{AB}^{(1)} = 10\,V$$







$$\Delta U_{AB} = I_{S1}\, R_{ek}$$

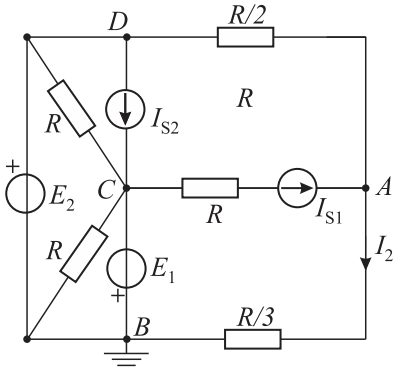
$$R_{ek} = \frac{R}{2} \| \frac{R}{3}$$

$$R_{ek} = 2\Omega$$

$$I_{S1} = \frac{\Delta U_{AB}}{R_{ek}}$$

$$\boxed{I_{S1} = 5A}$$

b)



$$V_B = 0\,V$$

$$V_C = -E_1 = -6\,V$$

$$V_D = E_2 = 5\,V$$

$$V_A\left(\frac{1}{\cancel{R+\infty}^{\,0}} + \frac{1}{R/3} + \frac{1}{R/2}\right) - V_C\left(\frac{1}{\cancel{R+\infty}^{\,0}}\right) - V_D\left(\frac{1}{R/2}\right) = I_{S1}$$

$$0{,}5V_A - 1 = 5$$

$$\boxed{V_A = 12V}$$

$$U_{g1} = R\, I_{S1} + V_A - V_C$$

$$P_{g1} = U_{S1} I_{S1}$$

$$P_{g1} = 68 \cdot 5$$

$$\boxed{P_{g1} = 340W}$$

$$P_{E1} = E_1 I_{E1}$$

$$I_{E1} = I_{S2} - I_{S1} + \frac{V_D - V_C}{R} - \frac{V_C}{R} = 1 - 5 + \frac{5+6}{10} + \frac{6}{10} = -2{,}3\,\mathrm{A}$$

$$\boxed{P_{E1} = -13{,}8W}$$

$$P_{g2} = U_{S2} I_{S2}$$

$$P_{g2} = (V_C - V_D) I_{S2}$$

$$P_{g2} = (-6-5) \cdot 1$$

$$\boxed{P_{g2} = -11W}$$