STUDIJSKI PROGRAM MEHATRONIKA

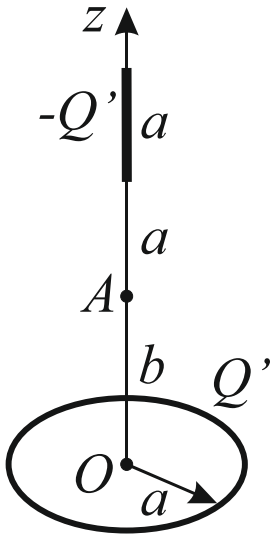
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **2. 9. 2026.**

ZADACI

Zadatak 1. Na slici 1 je prikazan ravnomerno naelektrisan štap i ravnomerno naelektrisana kružna kontura, poluprečnika a. Obe strukture su načinjene od izolacionog materijala i naelektrisane su odgovarajućim podužnim naelektrisanjima. Podužno naelektrisanje štapa je $-Q'$, a kružne konture, Q'. Štap se nalazi na z osi, koja ujedno predstavlja i osu simetrije kružne konture.

a) Odrediti u opštim brojevima rezultantni vektor jačine električnog polja u tački A ($z = b$), koji potiče od štapa i prstena.
b) Izračunati intenzitet vektora jačine električnog polja u tački A.
c) Izračunati napon U_{AO} između tačke A i centra prstena O. Napomena: Napon računati preko razlike potencijala i primenom superpozicije. Smatrati da je referentna tačka u beskonačnosti.

Sistem se nalazi u vazduhu. Brojni podaci su:
$Q' = 10$ nC/m, $a = 3$ cm, $b = 4$ cm, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m.

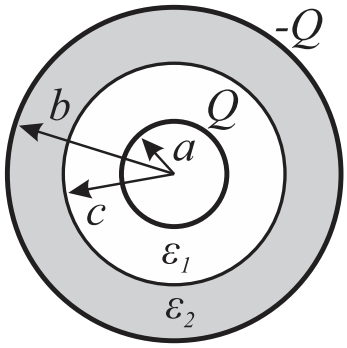


Slika 1.

Zadatak 2. Na slici 2 je prikazan poprečni presek vazdušnog koaksijalnog kabla, poluprečnika elektroda $a = 1{,}2$ cm i $b = 3{,}5$ cm, sa dva sloja dielektrika, relativnih permitivnosti $\varepsilon_{r1} = 10$ i $\varepsilon_{r2} = 3$. Granična površina između dielektrika se nalazi na poluprečniku $c = 2$ cm. Kabl je priključen na izvor napona $U = 100$ kV. Odrediti:

a) Kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.
b) Podužnu kapacitivnost kabla, C'.
c) Kolika mora da bude minimalna električna čvrstina za svaki od dielektrika, da ne dođe do proboja pri zadatom naponu.
d) Površinsku gustinu slobodnog naelektrisanja na unutrašnjoj elektrodi kabla, pri zadatom naponu.

Ostali brojni podaci su: $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MEHATRONIKA

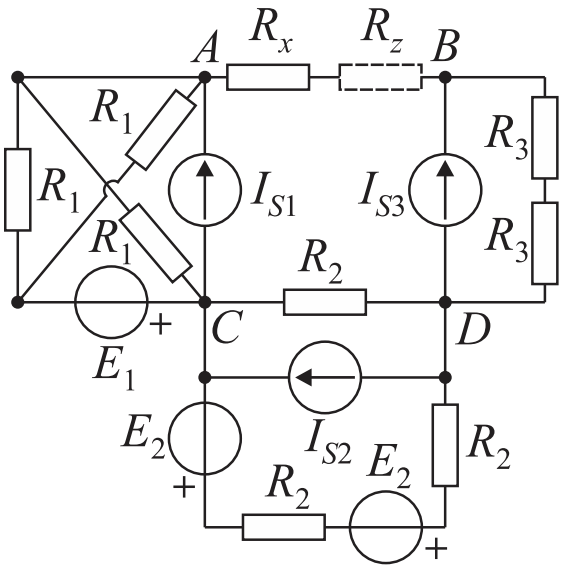
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **2. 9. 2026.**

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

a) Deo mreže između tačaka A i C predstaviti ekvivalentnim Tevenenovim generatorom E_{T1}, R_{T1}.
b) Prostom transformacijom generatora deo mreže između tačaka C i D predstaviti ekvivalentnim naponskim generatorom E_{T2}, R_{T2}.
c) Deo mreže između tačaka B i D predstaviti ekvivalentnim Tevenenovim generatorom E_{T3}, R_{T3}.
d) Nakon transformacije kola, proveriti da li će otpornik R_x = 3 Ω, maksimalne snage P_{xmax} = 3 W, pregoreti?
e) Ako je potrebno, izračunati minimalnu vrednost otpornosti zaštitnog otpornika, R_z, koji treba priključiti na red sa otpornikom otpornosti R_x, da bi ga zaštitio od pregorevanja.

Brojni podaci su:
R_1 = 3 Ω, R_2 = 6 Ω, R_3 = 1 Ω, $E_1 = E_2$ = 6 V, I_{S1} = 5 A, I_{S2} = 2 A, I_{S3} = 10 A.

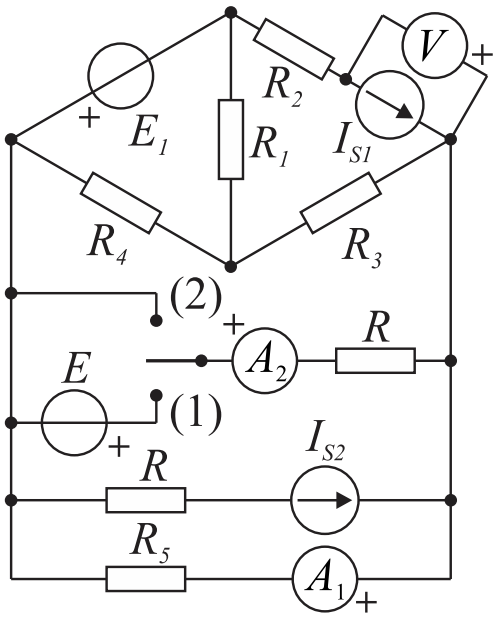


Slika 1.

Zadatak 2. Kada se u mreži sa slike 2, preklopnik prebaci iz položaja (2) u položaj (1), jačina struje koju meri ampermetar A_1 se poveća za ΔI_{A1} = 90 mA.

a) Primenjujući teoremu superpozicije, izračunati vrednost *ems* E.
b) Odrediti pokazivanja idealnih mernih instrumenata kada je prekidač u položaju (1).

Brojni podaci su: $R = R_2$ = 10 Ω, $R_1 = R_4$ = 80 Ω, $R_3 = R_5$ = 20 Ω, E_1 = 12 V, I_{S1} = 100 mA, I_{S2} = 200 mA.



Slika 2.

PRAVILA POLAGANJA

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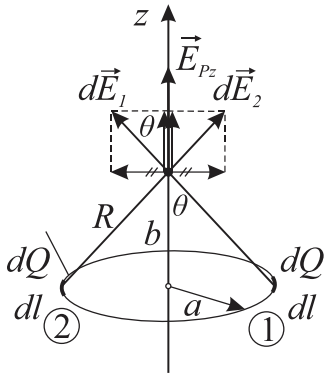
MH ISPIT 2. 9. 2026.

K1 Z1

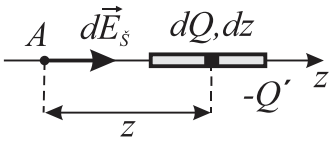


a) $d\vec{E}_{1h} + d\vec{E}_{2h} = 0 \rightarrow \vec{E}_h = 0$

$$dE_z = 2dE_1 \cos\theta = 2\frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = 2\frac{Q'dl}{4\pi\varepsilon_0 R^2}\cos\theta = 2\frac{Q'}{4\pi\varepsilon_0 R^2}\frac{b}{R}dl$$

$$E_z = \int_{po\ prstenu} dE_z = 2\frac{Q'}{4\pi\varepsilon_0 R^2}\frac{b}{R}\int_0^{a\pi} dl$$

$$E_z = \frac{Q'b}{4\pi\varepsilon_0 R^3}2a\pi = \frac{Q'b2a\pi}{4\pi\varepsilon_0\left(b^2+a^2\right)^{\frac{3}{2}}} \qquad \boxed{\vec{E}_P = \frac{Q'ba}{2\varepsilon_0\left(b^2+a^2\right)^{\frac{3}{2}}}\left(\vec{i}_z\right)}$$



$$dE_Š = \frac{dQ}{4\pi\varepsilon_0 z^2} = \frac{Q'dz}{4\pi\varepsilon_0 z^2}$$

$$E_Š = \frac{Q'}{4\pi\varepsilon_0}\int_a^{2a}\frac{dz}{z^2} = \frac{Q'}{4\pi\varepsilon_0}\left(-\frac{1}{z}\right)\Big|_a^{2a} = \frac{Q'}{8\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_Š = \frac{Q'}{8\pi\varepsilon_0 a}\left(\vec{i}_z\right)}$$

$$\boxed{\vec{E}_A = \vec{E}_P + \vec{E}_Š = \left(\frac{Q'ba}{2\varepsilon_0\left(b^2+a^2\right)^{\frac{3}{2}}} + \frac{Q'}{8\pi\varepsilon_0 a}\right)\left(\vec{i}_z\right)}$$

b)

$$E_A = \frac{Q'ba}{2\varepsilon_0\left(b^2+a^2\right)^{\frac{3}{2}}} + \frac{Q'}{8\pi\varepsilon_0 a} \qquad \boxed{E_A = 6{,}92\,\frac{\text{kV}}{\text{m}}}$$

c)

$U_{AO} = V_A - V_O$

$V_A = V_{AP} + V_{AŠ}$

$$dV_{AP} = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{Q'dl}{4\pi\varepsilon_0 R}$$

$$V_{AP} = \int_{po\ prstenu} dV_{AP} = \int_0^{2\pi a}\frac{Q'dl}{4\pi\varepsilon_0 R} = \frac{Q'}{4\pi\varepsilon_0 R}\cdot 2\pi a$$

$$\boxed{V_{AP} = \frac{Q'a}{2\varepsilon_0\left(b^2+a^2\right)^{\frac{1}{2}}}}$$

$$dV_{AŠ} = \frac{dQ}{4\pi\varepsilon_0 z} = \frac{-Q'dz}{4\pi\varepsilon_0 z}$$

$$V_{AŠ} = \int_{po\ štapu} dV_{AŠ} = -\frac{Q'}{4\pi\varepsilon_0}\int_a^{2a}\frac{dz}{z} = \boxed{-\frac{Q'}{4\pi\varepsilon_0}\ln 2}$$

$V_O = V_{OP} + V_{OŠ}$

$$V_{OP} = V_{AP}(b=0) = \boxed{\frac{Q'}{2\varepsilon_0}}$$

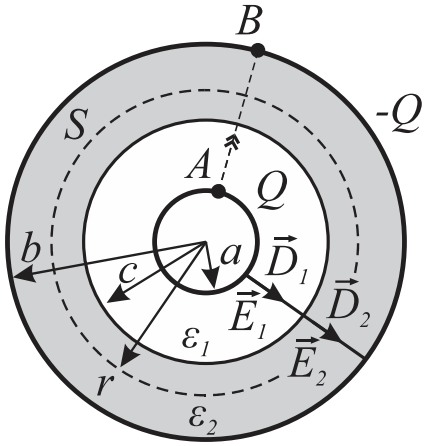
$$V_{OŠ} = -\frac{Q'}{4\pi\varepsilon_0}\int_{b+a}^{b+2a}\frac{dz}{z} = \boxed{-\frac{Q'}{4\pi\varepsilon_0}\ln\frac{b+2a}{b+a}}$$

$U_{AO} = \left(V_{AP} + V_{AŠ}\right) - \left(V_{OP} + V_{OŠ}\right)$

$$U_{AO} = \left(\frac{Q'a}{2\varepsilon_0\left(b^2+a^2\right)^{\frac{1}{2}}} + \left(-\frac{Q'}{4\pi\varepsilon_0}\ln 2\right)\right) - \left(\frac{Q'}{2\varepsilon_0} + \left(-\frac{Q'}{4\pi\varepsilon_0}\ln\frac{b+2a}{b+a}\right)\right)$$

$$\boxed{U_{AO} = -256{,}24\,\text{V}}$$

K1 Z2



a)

Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$

$\int_S D\, ds = Q$

$D 2\pi r L = Q \qquad \boxed{D = \frac{Q}{2\pi r L}}, \qquad a \le r \le b$

$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 r L}}, \qquad a \le r \le c$

$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 r L}}, \qquad c \le r \le b$

b)

$C = \frac{Q}{U}$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E dr = \int_a^c E_1 dr + \int_c^b E_2 dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 r L} dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 r L} dr = \frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}\right)$$

$$C = \frac{2\pi L}{\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}} = \frac{2\pi L}{\frac{1}{4\varepsilon_0}\ln 3 + \frac{1}{2\varepsilon_0}\ln\frac{5}{3}}$$

$$C' = \frac{C}{L} = \frac{2\pi}{\frac{1}{4\varepsilon_0}\ln 3 + \frac{1}{2\varepsilon_0}\ln\frac{5}{3}} = 234\,\text{pF/m} \qquad \boxed{C' = 234\,\frac{\text{pF}}{\text{m}}}$$

c)

$$E_{\check{C}1} = E_{1\max} = \frac{Q}{2\pi\varepsilon_1 a L} = \frac{Q'}{2\pi\varepsilon_1 a} = \frac{U \cdot C'}{2\pi\varepsilon_1 a} = 35\,\frac{\text{kV}}{\text{cm}} \qquad \boxed{E_{\check{C}1} = 35\,\frac{\text{kV}}{\text{cm}}}$$

$$E_{\check{C}2} = E_{2\max} = \frac{Q}{2\pi\varepsilon_2 c L} = \frac{Q'}{2\pi\varepsilon_2 c} = \frac{U \cdot C'}{2\pi\varepsilon_2 c} = 70\,\frac{\text{kV}}{\text{cm}} \qquad \boxed{E_{\check{C}2} = 70\,\frac{\text{kV}}{\text{cm}}}$$

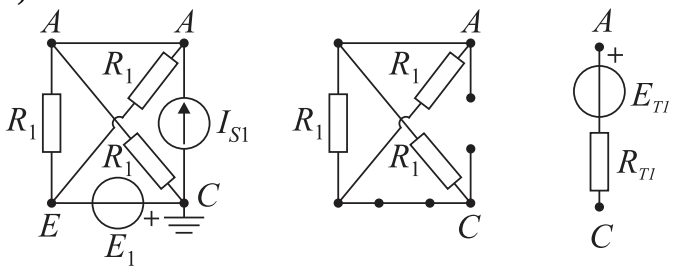
d)

$$\sigma_{sl} = D(r = a) = \frac{Q'}{2\pi a} = 0{,}31\frac{\text{mC}}{\text{m}^2} \qquad \boxed{\sigma_{sl} = 0{,}31\frac{\text{mC}}{\text{m}^2}}$$

MH ISPIT 2. 9. 2016.

K2 Z1

a)



$$V_E = -E_1 = -6\,\text{V}$$

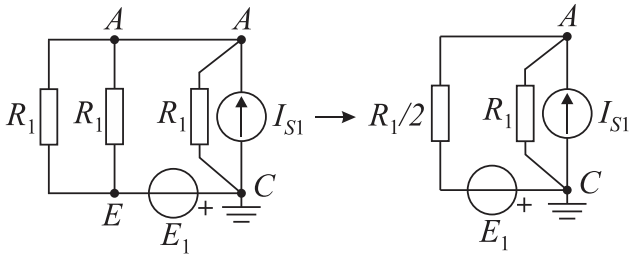
$$V_A\left(\frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_1}\right)-V_E\left(\frac{1}{R_1}+\frac{1}{R_1}\right)=I_{S1}\rightarrow V_A\frac{3}{R_1}-V_E\frac{2}{R_1}=I_{S1}$$

$$V_A = 1\text{V}$$

$$\boxed{E_{T1}=U_{AC}=V_A=1\text{V}}$$

$$R_{T1}=R_{AC}=R_1\,\|\,R_1\,\|\,R_1=\frac{R_1}{3}\rightarrow \boxed{R_{T1}=1\Omega}$$

2. način

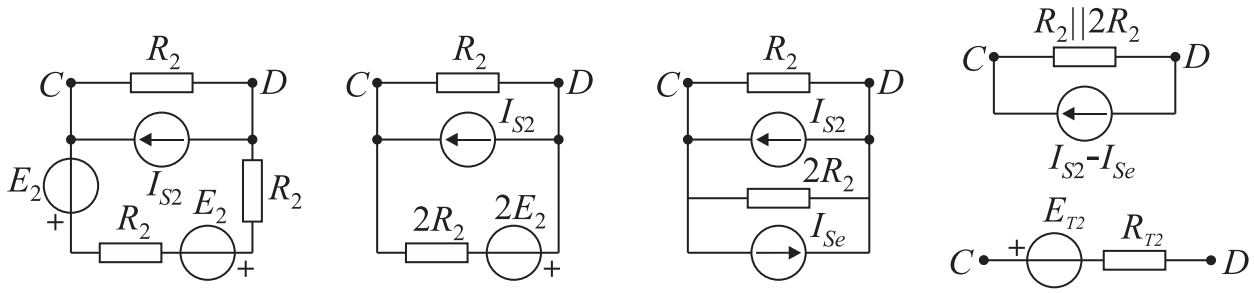


$$V_A\left(\frac{1}{R_1/2}+\frac{1}{R_1}\right)=I_{S1}-\frac{E_1}{R_1/2}\quad /\cdot R_1\rightarrow 3V_A=R_1I_{S1}-2E_1$$

$$V_A = 1\text{V}$$

$$\boxed{E_{T1}=U_{AC}=V_A=1\text{V}}$$

b)



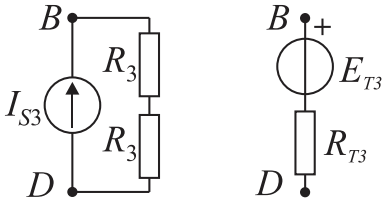
$$I_{Se}=\frac{2E_2}{2R_2}=\frac{E_2}{R_2}=1\text{A}$$

$$E_{T2}=\left(I_{S2}-I_{Se}\right)\cdot\left(R_2\,\|\,2R_2\right)=1\cdot 4\rightarrow \boxed{E_{T2}=4\,\text{V}}$$

$$R_{T2}=R_2\,\|\,2R_2=\frac{2}{3}R_2=\frac{2}{3}\cdot 6$$

$$\boxed{R_{T2}=4\Omega}$$

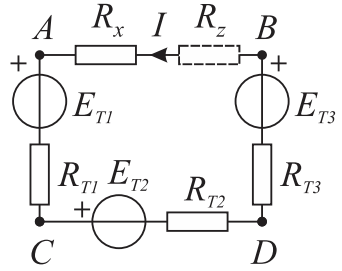
c)



$$E_{T3}=U_{BD}=2R_3\cdot I_{S3}=2\cdot 1\cdot 10\rightarrow \boxed{E_{T3}=20\,\text{V}}$$

$$R_{T3}=2R_3=2\cdot 1\rightarrow \boxed{R_{T3}=2\Omega}$$

d)



$$I_{\max}=\sqrt{\frac{P_{x\max}}{R_x}}=\sqrt{\frac{3}{3}}=1\text{A}$$

$$I=\frac{E_{T3}-E_{T1}-E_{T2}}{R_{T1}+R_{T2}+R_{T3}+R_x}=\frac{20-1-4}{1+4+2+3}=1{,}5\,\text{A}>I_{\max}$$

R_x će pregoreti. Neophodno je vezati zaštitni otpornik.

e)

$$I=\frac{E_{T3}-E_{T1}-E_{T2}}{R_{T1}+R_{T2}+R_{T3}+R_x+R_Z}=\frac{E_T}{R_T+R_x+R_Z}\le I_{\max}$$

$$R_T+R_x+R_Z\ge\frac{E_T}{I_{\max}}$$

$$R_Z\ge\frac{E_T}{I_{\max}}-R_T-R_x=\frac{15}{1}-7-3$$

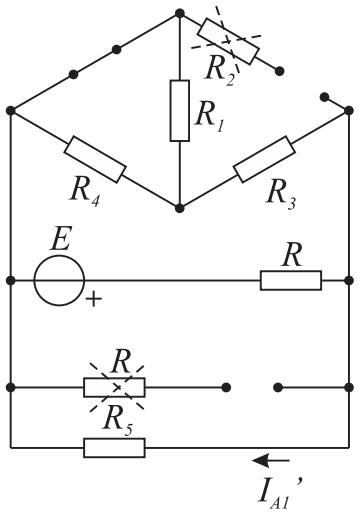
$$\boxed{R_{Z\min}=5\Omega}$$

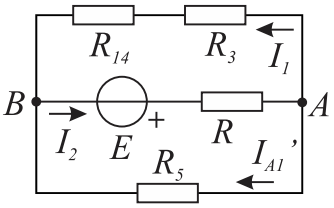
K2 Z2

a)

$$\boxed{\begin{matrix}Svi\\ generatori\\ (1)\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ E\\ (2)\end{matrix}} + \boxed{\begin{matrix}Samo\\ E\end{matrix}}$$

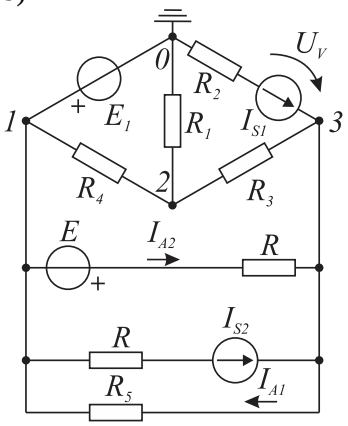
$$I_{A1}^{(1)} = I_{A1}^{(2)} + I_{A1}' \qquad \Rightarrow \qquad \Delta I_{A1} = I_{A1}^{(1)} - I_{A1}^{(2)} = I_{A1}' = 90\ \text{mA}$$





$$R_{14} = R_1 \,||\, R_4 = 40\ \Omega$$
$$U_{AB} = I_{A1}' R_5 = 90\ m \cdot 20 = 1{,}8\ \text{V}$$
$$I_1 = \frac{U_{AB}}{R_3 + R_{14}} = \frac{1{,}8}{20+40} = 30\ \text{mA}$$
$$I_2 = I_1 + I_{A1}' = 120\ \text{mA}$$
$$-E + I_2 R + U_{AB} = 0$$
$$E = I_2 R + U_{AB} = 120\,\text{m} \cdot 10 + 1{,}8$$
$$\boxed{E = 3\ \text{V}}$$

b)



$$V_0 = 0\ \text{V}, \qquad V_1 = E_1 = 12\ \text{V}$$

$$V_2\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}\right) - V_1\left(\frac{1}{R_4}\right) - V_3\left(\frac{1}{R_3}\right) = 0$$

$$V_3\left(\frac{1}{R_2+\infty}+\frac{1}{R_3}+\frac{1}{R}+\frac{1}{R+\infty}+\frac{1}{R_5}\right) - V_1\left(\frac{1}{R}+\frac{1}{R+\infty}+\frac{1}{R_5}\right) - V_2\left(\frac{1}{R_3}\right) = I_{S1} + \frac{E}{R} + I_{S2}$$

$$V_2\left(\frac{1}{80}+\frac{1}{20}+\frac{1}{80}\right) - 12\cdot\left(\frac{1}{80}\right) - V_3\left(\frac{1}{20}\right) = 0 \qquad /\cdot 80$$

$$V_3\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{20}\right) - 12\cdot\left(\frac{1}{10}+\frac{1}{20}\right) - V_2\left(\frac{1}{20}\right) = 100\,\text{m} + \frac{3}{10} + 200\,\text{m} \qquad /\cdot 20$$

$$\left.\begin{aligned} 6V_2 - 4V_3 &= 12 \\ -V_2 + 4V_3 &= 48 \end{aligned}\right\} \qquad \Rightarrow \qquad V_2 = 12\ \text{V}, \qquad V_3 = 15\ \text{V}$$

$$I_{A1} = \frac{V_3 - V_1}{R_5} = \frac{15-12}{20} \qquad \boxed{I_{A1} = 0{,}15\,\text{A}}$$

$$I_{A2} = \frac{V_1 - V_3 + E}{R} = \frac{12-15+3}{10} \qquad \boxed{I_{A2} = 0\,\text{A}}$$

$$U_V = V_3 - V_0 + R_2 I_{S1} = 15 - 0 + 10 \cdot 100\,\text{m} \qquad \boxed{U_V = 16\,\text{V}}$$