STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

29.08.2026.

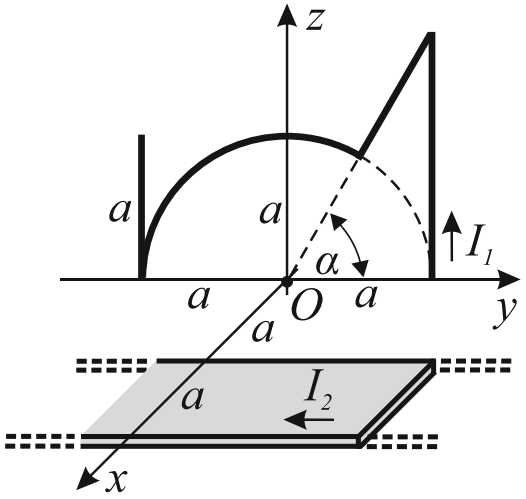
ZADACI

Zadatak 1. Na slici 1 je prikazan žičani provodnik sa vremenski konstantnom strujom jačine I_1, koji leži u y-z ravni Dekartovog pravouglog koordinatnog sistema. Žičani provodnik se sastoji od tri pravolinijska segmenta i lučnog segmenta u obliku dela kruga, poluprečnika a. Veoma dugačak, tanak, trakasti provodnik, širine a, sa strujom jačine I_2, leži u x-y ravni zadatog koordinatnog sistema, paralelno sa y-osom. Sredina je vazduh.

a) Odrediti, u opštim brojevima, vektor magnetske indukcije, B_P, koji u tački O (centar koordinatnog sistema), stvara žičani provodnik.

b) Odrediti, u opštim brojevima, vektor magnetske indukcije, B_T, koji u tački O stvara trakasti provodnik.

Brojne vrednosti: $\alpha = \pi/3$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



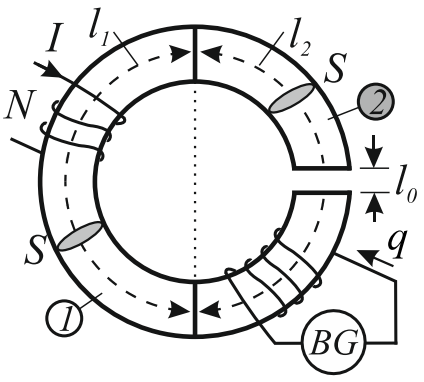
Slika 1.

Zadatak 2. Tanko feromagnetsko torusno jezgro, načinjeno od dva feromagnetska materijala i vazdušnog procepa, prikazano je na slici 2a. Na jezgro je ravnomerno i gusto namotan namotaj sa N zavojaka tanke žice i strujom intenziteta I. U početnom trenutku jezgro je nenamagnetisano i nema struje u namotajima. Krive prvobitnog magnetisanja materijala su prikazane na slici 2b.

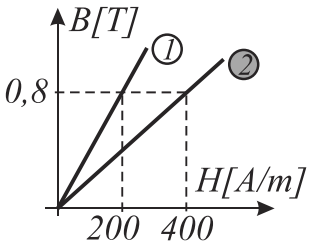
a) Izračunati intenzitet struje I, u namotaju sa N zavojaka, tako da prilikom uspostvljanja polja u jezgru kroz namotaj balističkog galvanometra otpornosti R_{BG}, sa N_{BG} zavojaka, protekne količina naelektrisanja $q = 25\ \mu C$, u naznačenom referentnom smeru. Rasipanja magnetskog fluksa u vazdušnom procepu zanemariti.

b) Izračunati magnetsku energiju u feromagnetskim delovima jezgra, kada struja u namotaju ima intenzitet I određen pod *a)*.

Brojni podaci: $l_1 = 12\ cm$, $l_2 = 11{,}9\ cm$, $l_0 = 1\ mm$, $S = 1\ cm^2$, $N = 100\ zav$, $R_{BG} = 100\ \Omega$, $N_{BG} = 50\ zav$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

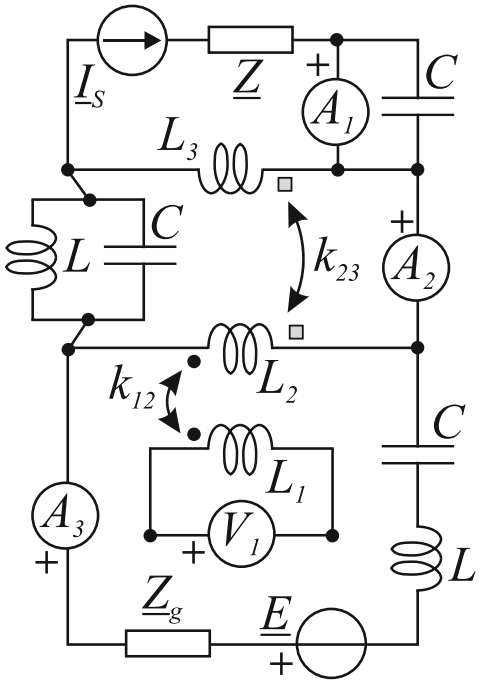
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (II kolokvijum)	**K2** **29.08.2026.**

ZADACI

Zadatak 1. U kolu sa međusobno spregnutim kalemovima sa slike 1:

a) Odrediti pokazivanja idealnih ampermetara.
b) Odrediti pokazivanja idealnih voltmetara.
c) Izračunati aktivnu, reaktivnu i prividnu snagu generatora *ems* $\underline{E}$.

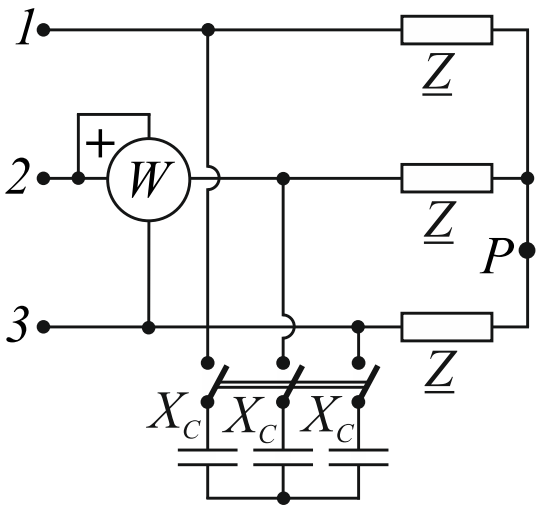
Brojni podaci: $\underline{E} = 96\ V$, $\underline{I}_S = (5 + j10)\ A$,
$\underline{Z}_g = (6 - j7{,}2)\ \Omega$, $\underline{Z} = (8 + j7)\ \Omega$,
$\omega L_1 = 20\ \Omega$, $\omega L_2 = 7{,}2\ \Omega$, $\omega L_3 = 5\ \Omega$,
$k_{12} = 1/3$, $k_{23} = 0{,}5$,
$\omega L = 1/\omega C = 10\ \Omega$.



Slika 1.

Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik impedanse $\underline{Z} = (60 + j150)\ \Omega$, priključen na mrežu faznog napona $\underline{U}_1 = 230\ V$.

a) Odrediti reaktanse kondenzatora koje je potrebno povezati sa prijemnikom, da bi mu se faktor snage popravio na jedinicu.
b) Pokazivanje idealnog vatmetra, kada se priključe kondenzatori.
c) Pokazivanje idealnog vatmetra, ako se nakon priključenja kondenzatora prekine linijski provodnik treće faze u tački P.
d) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



Slika 2.

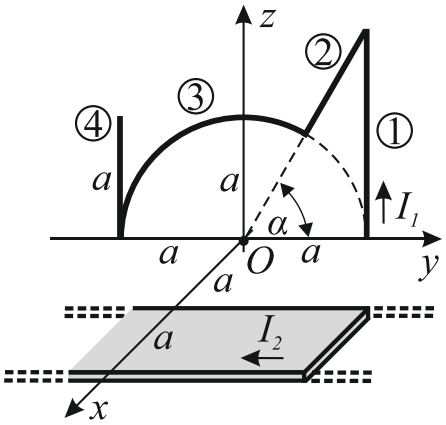
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

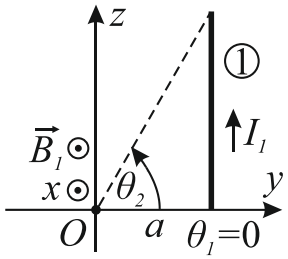
Ispit iz OET2, 29.08.2026.

I-1

a)

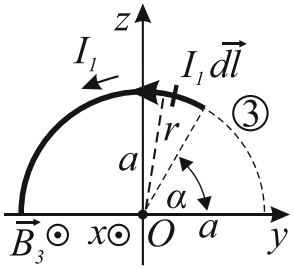


$\boxed{\vec{B}_2 = 0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$



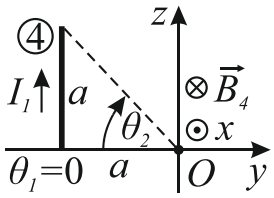
1 $\quad d = a, \qquad \theta_1 = 0, \qquad \theta_2 = \alpha = 60°$

$$B_1 = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi a}\left(\frac{\sqrt{3}}{2} - 0\right) \qquad \boxed{\vec{B}_1 = \frac{\sqrt{3}\,\mu_0 I_1}{8\pi a}\cdot\vec{i}_x}$$



$$\vec{dB}_3 = \frac{\mu_0}{4\pi}\frac{I_1\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_3 = \frac{\mu_0}{4\pi}\frac{I_1\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_3 = \int dB_3 = \frac{\mu_0 I_1}{4\pi a^2}\int_0^{\frac{1}{3}2\pi a} dl = \frac{\mu_0 I_1}{4\pi a^2}\frac{2}{3}\pi a = \frac{\mu_0 I_1}{6a} \qquad \boxed{\vec{B}_3 = \frac{\mu_0 I_1}{6a}\cdot\vec{i}_x}$$

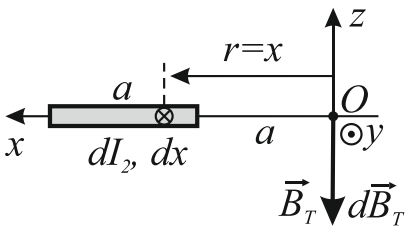


4 $\quad d = a, \qquad \theta_1 = 0, \qquad \theta_2 = 45°$

$$B_4 = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi a}\left(\frac{\sqrt{2}}{2} - 0\right) \qquad \boxed{\vec{B}_4 = \frac{\sqrt{2}\,\mu_0 I_1}{8\pi a}\cdot\left(-\vec{i}_x\right)}$$

$$\boxed{\vec{B}_P = \vec{B}_1 + \vec{B}_3 + \vec{B}_4 = \left(\frac{\sqrt{3}\,\mu_0 I_1}{8\pi a} + \frac{\mu_0 I_1}{6a} - \frac{\sqrt{2}\,\mu_0 I_1}{8\pi a}\right)\cdot\vec{i}_x}$$

b)



$$dB_T = \frac{\mu_0\,dI_2}{2\pi r} = \frac{\mu_0\,I_2\dfrac{dx}{a}}{2\pi x} = \frac{\mu_0 I_2}{2\pi a}\frac{dx}{x} \qquad \frac{dI_2}{I_2} = \frac{dx}{a}$$

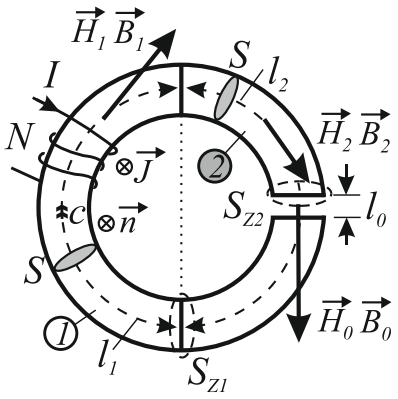
$$B_T = \int dB_T = \frac{\mu_0 I_2}{2\pi a}\int_a^{2a}\frac{dx}{x} = \frac{\mu_0 I_2}{2\pi a}\ln\frac{2a}{a}$$

$$\boxed{\vec{B}_T = \frac{\mu_0 I_2}{2\pi a}\ln 2\cdot\left(-\vec{i}_z\right)}$$

Ispit iz OET2, 29.08.2026.

I-2

a)



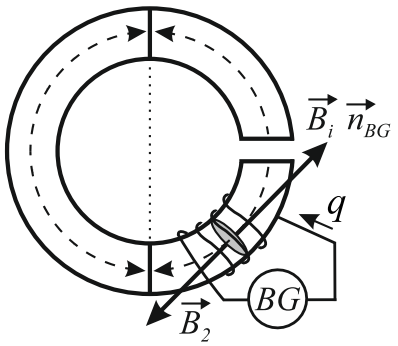
$$\oint_c \vec{H}\cdot\vec{dl} = \int_{Sc} \vec{J}\cdot\vec{dS} \qquad \sphericalangle\left(\vec{H},\vec{dl}\right)=0 \qquad \sphericalangle\left(\vec{J},\vec{n}\right)=0$$

$$H_1\,l_1 + H_2\,l_2 + H_0\,l_0 = NI \qquad (1)$$

$$\oint_{S_z} \vec{B}\cdot\vec{dS} = 0$$

$$S_{Z1}: \qquad B_1\,S - B_2\,S = 0 \qquad B_1 = B_2 \qquad (2)$$

$$S_{Z2}: \qquad -B_2\,S + B_0\,S_0 = 0 \qquad B_0 = B_2 \qquad (3)$$



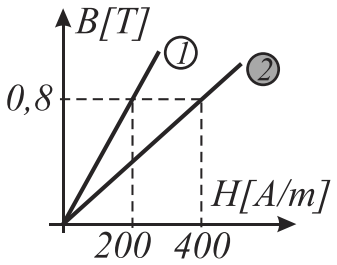
$$\Phi_{POČ} = 0\,Wb$$

$$\Phi_{KR} = \int_S \vec{B}_2\cdot\vec{dS} = -B_2\,S \qquad \leq\left(\vec{B}_2,\vec{n}_{BG}\right)=180°$$

$$q = -\frac{N_{BG}}{R_{BG}}\left(\Phi_{KR}-\Phi_{POČ}\right) = -\frac{N_{BG}}{R_{BG}}\left(-B_2\,S\right) = \frac{N_{BG}}{R_{BG}}B_2\,S$$

$$B_2 = \frac{q\,R_{BG}}{N_{BG}\,S} = \frac{25\cdot10^{-6}\cdot100}{50\cdot1\cdot10^{-4}} = 50\cdot10^{-2} = 0{,}5\,T$$

$$H_2 = \frac{B_2}{\mu_2} = \frac{0{,}5}{\frac{0{,}8}{400}} = 250\,\frac{A}{m}$$



$$(2) \quad \Rightarrow \quad B_1 = B_2 = 0{,}5\,T$$

$$H_1 = \frac{B_1}{\mu_1} = \frac{0{,}5}{\frac{0{,}8}{200}} = 125\,\frac{A}{m}$$

$$(3) \quad \Rightarrow \quad B_0 = B_2 = 0{,}5\,T$$

$$H_0 = \frac{B_0}{\mu_0} = \frac{0{,}5}{4\pi\cdot10^{-7}} = 398{,}1\,\frac{kA}{m}$$

$$(1) \quad \Rightarrow \quad I = \frac{H_1\,l_1 + H_2\,l_2 + H_0\,l_0}{N} \qquad \boxed{I = 4{,}43\ A}$$

b)

$$W_{m1} = w_{m1}\,V_1 = w_{m1}\,l_1\,S = \frac{1}{2}B_1\,H_1\,l_1\,S$$

$$W_{m1} = \frac{1}{2}\cdot0{,}5\cdot125\cdot0{,}12\cdot1\cdot10^{-4} \qquad \boxed{W_{m1} = 0{,}375\ mJ}$$

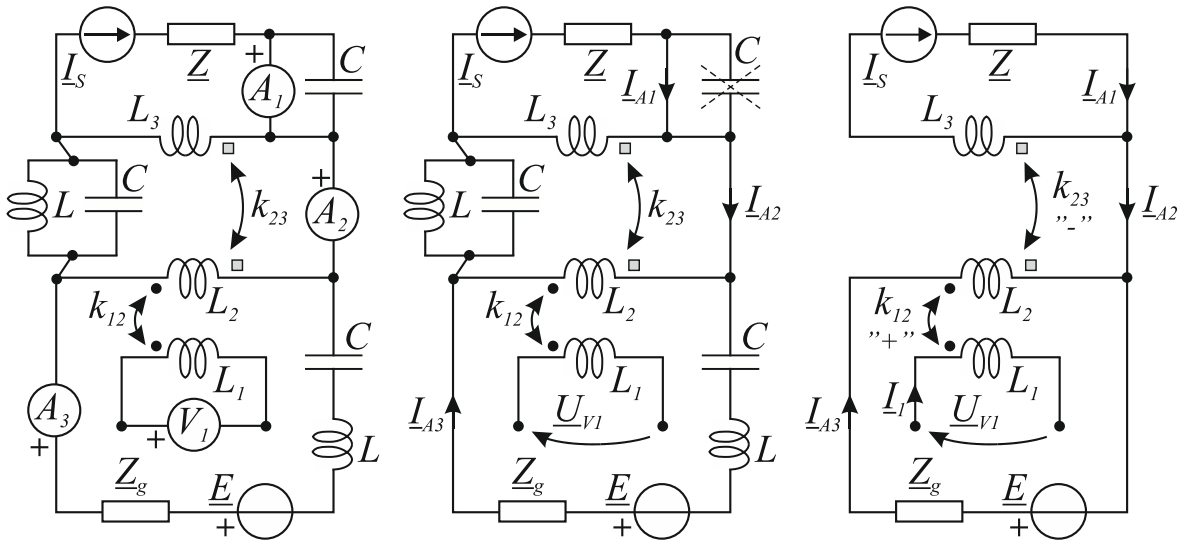
$$W_{m2} = w_{m2}\,V_2 = w_{m2}\,l_2\,S = \frac{1}{2}B_2\,H_2\,l_2\,S$$

$$W_{m2} = \frac{1}{2}\cdot0{,}5\cdot250\cdot0{,}119\cdot1\cdot10^{-4} \qquad \boxed{W_{m2} = 0{,}744\ mJ}$$

Ispit iz OET2, 29.08.2026.

II-1

a)



$\omega L_1 = 20\,\Omega, \qquad \omega L_2 = 7,2\,\Omega, \qquad \omega L_3 = 5\,\Omega$

$\omega L_{12} = k_{12}\sqrt{\omega L_1\ \omega L_2} = 4\,\Omega, \qquad \omega L_{23} = k_{23}\sqrt{\omega L_2\ \omega L_3} = 3\,\Omega$

$$-\underline{E} + \underline{Z}_g\,\underline{I}_{A3} + j\omega L_2\,\underline{I}_{A3} + j\omega L_{12}\,\underline{I}_1^{\,0} - j\omega L_{23}\,\underline{I}_S = 0$$

$$\underline{I}_{A3} = \frac{j\omega L_{23}\,\underline{I}_S + \underline{E}}{\underline{Z}_g + j\omega L_2} = \frac{j3\cdot(5+j10)+96}{(6-j7,2)+j7,2} = \frac{66+j15}{6} = (11+j2,5)\,A = 11,28\,e^{j12,8^\circ}\,A$$

$$I_{A3} = |\underline{I}_{A3}| \qquad \boxed{I_{A3} = 11,28\,A}$$

$$\boxed{I_{A2} = 0\,A}$$

$$I_{A1} = |\underline{I}_S| = \sqrt{5^2+10^2} \qquad \boxed{I_{A1} = 11,18\,A}$$

b)

$$\underline{U}_{V1} = j\omega L_1\,\underline{I}_1^{\,0} + j\omega L_{12}\,\underline{I}_{A3} = j4\cdot(11+j2,5) = (-10+j44)\,V$$

$$\underline{U}_{V1} = 45,12\,e^{j102,8^\circ}\,V$$

$$\boxed{U_{V1} = |\underline{U}_{V1}| = 45,12\,V}$$

c)

$$\underline{S}_E = \underline{E}\,\underline{I}_{A3}^{\,*} = 96\cdot(11-j2,5) = (1056-j240)\,VA = 1082,93\,e^{-j12,8^\circ}\,VA$$

$$\boxed{P_E = \mathrm{Re}\{\underline{S}_E\} = 1056\,W}$$

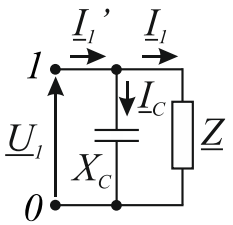
$$\boxed{Q_E = \mathrm{Im}\{\underline{S}_E\} = -240\,VAr}$$

$$\boxed{S_E = |\underline{S}_E| = 1082,93\,VA}$$

Ispit iz OET2, 29.08.2026.

II-2

a)

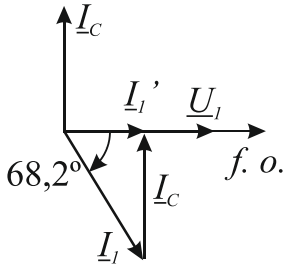


$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{230\,e^{j0°}}{60 + j150} = \frac{230\,e^{j0°}}{161{,}55\,e^{j68{,}2°}} = 1{,}42\,e^{-j68{,}2°}\ A$$

$$I_C = I_1 \sin 68{,}2°$$

$$\frac{U_1}{X_C} = \frac{U_1}{Z}\sin 68{,}2° \quad \Rightarrow \quad X_C = \frac{Z}{\sin 68{,}2°}$$

$$\boxed{X_C = 174\,\Omega}$$



$$I_1' = I_1 \cos 68{,}2° = 1{,}42 \cdot \cos 68{,}2° = 0{,}53\ A$$

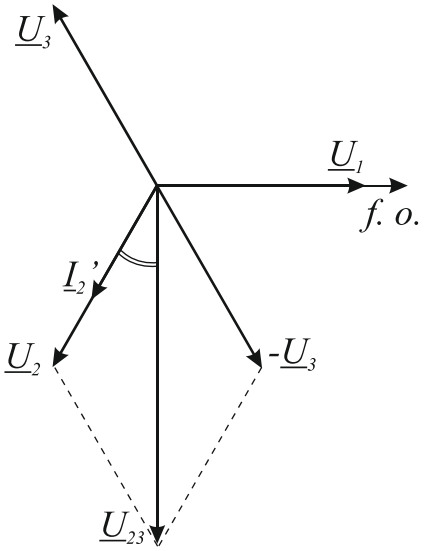
$$\frac{U_1}{R_e} = \frac{U_1}{Z}\cos 68{,}2° \quad \Rightarrow \quad R_e = \frac{Z}{\cos 68{,}2°} \qquad R_e = 435\,\Omega$$

b)

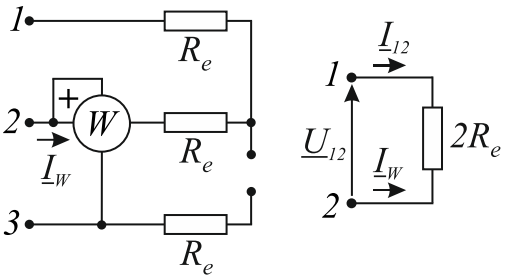
$$P_W = \mathrm{Re}\left\{\underline{U}_{23}\,\underline{I}_2'^{*}\right\} = \sqrt{3}\,U_1\,I_1'\cos\sphericalangle\left(\underline{U}_{23}, \underline{I}_2'\right)$$

$$P_W = \sqrt{3}\cdot 230 \cdot 0{,}53 \cdot \cos 30°$$

$$\boxed{P_W = 182{,}85\,W}$$



c)



$$\underline{I}_W = -\underline{I}_{12} = -\frac{\underline{U}_{12}}{2R_e} = -\frac{\sqrt{3}\,U_1\,e^{j30°}}{870} = -0{,}46\,e^{j30°}\ A$$

$$\underline{I}_W = 0{,}46\,e^{-j150°}\ A$$

$$P_W' = \mathrm{Re}\left\{\underline{U}_{23}\,\underline{I}_W^{*}\right\} = \sqrt{3}\,U_1\,I_W\cos\sphericalangle\left(\underline{U}_{23}, \underline{I}_W\right)$$

$$P_W' = \sqrt{3}\cdot 230 \cdot 0{,}46 \cdot \cos 60°$$

$$\boxed{P_W' = 91{,}63\,W}$$