STUDIJSKI PROGRAM MERENJE I REGULACIJA

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (I kolokvijum)	**K1** **29. 8. 2026.**

ZADACI

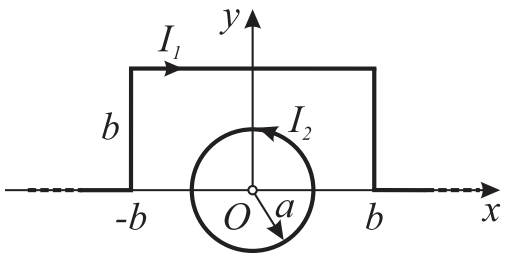
Zadatak 1. Tanak, veoma dugačak, žičani provodnik sa vremenski konstantnom strujom jačine I_1, savijen je u središnjem delu u obliku pravougaonika i postavljen je u x-y ravan zadatog Dekartovog pravouglog koordinatnog sistema, zajedno sa tankom žičanom konturom u vidu kružnog prstena poluprečnika a, sa strujom jačine I_2, kao što je prikazano na slici 1.

a) Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije u koordinatnom početku koji stvara žičani provodnik sa vremenski konstantnom strujom jačine I_1.

b) Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije u koordinatnom početku koji stvara kružni prsten.

c) Odrediti jačinu struje I_2 tako da rezultantni vektor magnetske indukcije u koordinatnom početku bude jednak nuli.

Brojni podaci: $I_1 = 10$ A, $a = 5$ cm, $b = 2a$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



Slika 1.

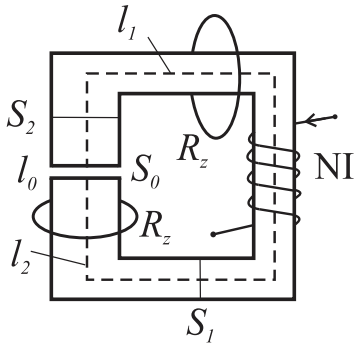
Zadatak 2. Na slici 2a je prikazano tanko magnetsko kolo načinjeno od feromagnetskog materijala čija je idealizovana kriva prvobitnog magnetisanja prikazana na slici 2b. Pre uključivanja struje u namotaju, jezgro je bilo nenamagnetisano. Postepenim povećanjem struje u namotaju, dostgnuta je vrednost magnetskog polja $H_1 = 1{,}5$ kA/m u grani sa namotajem. Odrediti:

a) Broj amperzavojaka, potrebnih da se dostigne data jačina magnetskog polja, H_1.

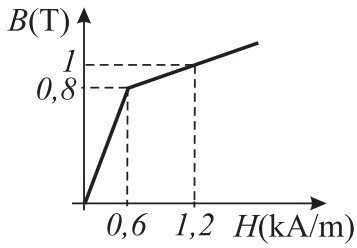
b) Smer i količinu naelektrisanja, q, kroz probne zavojke otpornosti $R_Z = 20\ \Omega$, prilikom uspostavljanja polja

c) Energiju sadržanu u široj grani magnetskog kola.

Brojni podaci su: $l_1 = 14$ cm, $l_2 = 6$ cm, $l_0 = 0{,}5$ mm, $S_1 = 1{,}8$ cm^2, $S_2 = S_0 = 2{,}5$ cm^2.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MERENJE I REGULACIJA

K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (II kolokvijum)	**K2** **29. 8. 2026.**

ZADACI

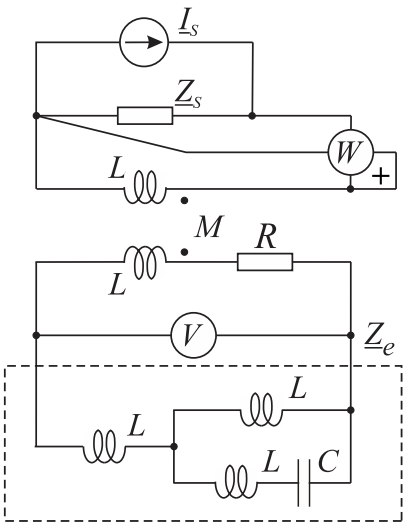
Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_e$.

b) Ako u kolu strujni generator, $\underline{I}_S$, radi na manjoj antirezonantnoj učestanosti, odrediti pokazivanje idealnih mernih instrumenata.

c) Odrediti kompleksnu, aktivnu, reaktivnu i prividnu snagu strujnog generatora $\underline{I}_S$.

Brojni podaci su: $\underline{I}_S = (1+j)$ A, $L = 20$ mH, $C = 250$ nF, $M = 10$ mH, $R = 50\ \Omega$, $\underline{Z}_S = (50-j100)\ \Omega$.

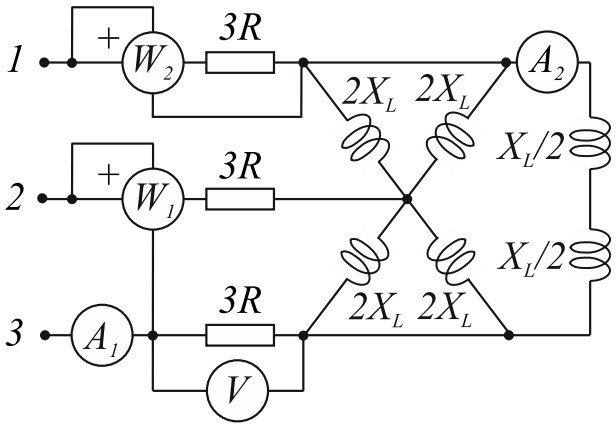


Slika 1.

Zadatak 2. Trofazni prijemnik, reaktanse $X_L = 45\ \Omega$, priključen je preko trofaznog voda otpornosti $R = 5\ \Omega$, na simetričnu trofaznu mrežu faznog napona prve faze $\underline{U}_1 = 230$ V, kao što je prikazano na slici 2.

a) Odrediti pokazivanja idealnih mernih instrumenata.

b) Skicirati fazorski dijagram faznih napona mreže i svih fazora veličina od kojih zavisi pokazivanje instrumenata.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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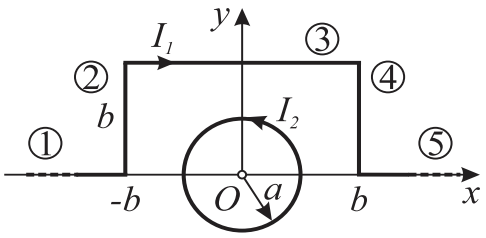
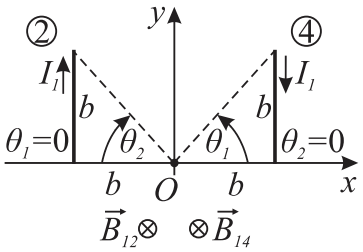
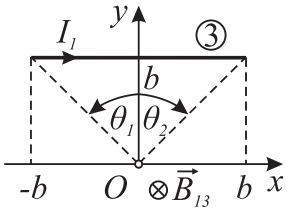
K1 Z1



$\boxed{\vec{B}_{11}=0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right)=0$

$\boxed{\vec{B}_{15}=0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right)=0$





Zbog simetrije: $\vec{B}_{12}=\vec{B}_{14}$

2 $\quad d=b=2a, \quad \theta_1=0, \quad \theta_2=\frac{\pi}{4}$

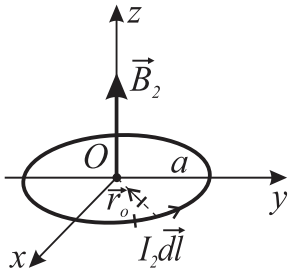
4 $\quad d=b=2a, \quad \theta_1=-\frac{\pi}{4}, \quad \theta_2=0$

3 $\quad d=b=2a, \quad \theta_1=-\frac{\pi}{4}, \quad \theta_2=\frac{\pi}{4}$

$$B_{12}=\frac{\mu_0 I}{4\pi d}\left(\sin\theta_2-\sin\theta_1\right)=\frac{\mu_0 I_1}{4\pi\cdot 2a}\left(\frac{\sqrt{2}}{2}-0\right)=\frac{\mu_0 I_1\sqrt{2}}{16\pi a}=B_{14} \qquad \vec{B}_{12}=\vec{B}_{14}=\frac{\mu_0 I_1\sqrt{2}}{16\pi a}\left(-\vec{i}_z\right)$$

$$B_{13}=\frac{\mu_0 I}{4\pi d}\left(\sin\theta_2-\sin\theta_1\right)=\frac{\mu_0 I_1}{4\pi\cdot 2a}\left[\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{2}}{2}\right)\right]=\frac{\mu_0 I_1\sqrt{2}}{8\pi a} \qquad \vec{B}_{13}=\frac{\mu_0 I_1\sqrt{2}}{8\pi a}\left(-\vec{i}_z\right)$$

$$\vec{B}_1=\vec{B}_{11}+\vec{B}_{12}+\vec{B}_{13}+\vec{B}_{14}+\vec{B}_{15}=\frac{\mu_0 I_1\sqrt{2}}{4\pi a}\left(-\vec{i}_z\right) \qquad \boxed{\vec{B}_1=\frac{\mu_0 I_1\sqrt{2}}{4\pi a}\left(-\vec{i}_z\right)}$$



$$\vec{dB}_2=\frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_2=\frac{\mu_0}{4\pi}\frac{I_2\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right)=\frac{\pi}{2}$$

$$B_2=\int dB_2=\frac{\mu_0 I_2}{4\pi a^2}\int_0^{2a\pi} dl=\frac{\mu_0 I_2}{4\pi a^2}2a\pi=\frac{\mu_0 I_2}{2a}$$

$$\boxed{\vec{B}_2=\frac{\mu_0 I_2}{2a}\vec{i}_z}$$

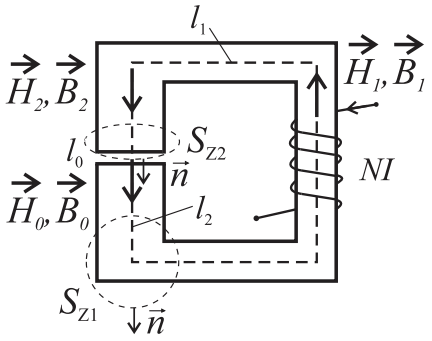
$$\vec{B}_O=\vec{B}_1+\vec{B}_2=0 \quad \Rightarrow \quad B_1=B_2$$

$$\frac{\mu_0 I_1\sqrt{2}}{4\pi a}=\frac{\mu_0 I_2}{2a} \quad \Rightarrow \quad I_2=\frac{I_1\cdot\sqrt{2}}{2}$$

$$\boxed{I_2=7{,}07\ \text{A}}$$

K1 Z2

a)



$$\oint_c \vec{H}\cdot\vec{dl} = NI$$

$$H_1\cdot l_1 + H_2\cdot l_2 + H_0\cdot l_0 = NI \quad (1)$$

$$\oint_{S_{Z1}} \vec{B}\cdot\vec{ds} = 0$$

$$\Phi_2 = \Phi_1$$

$$B_2\cdot S_2 = B_1\cdot S_1 \quad (2)$$

$$\oint_{S_{Z2}} \vec{B}\cdot\vec{ds} = 0$$

$$\Phi_2 = \Phi_0$$

$$B_2\cdot \cancel{S_2} = B_0\cdot \cancel{S_0}, \qquad (S_2 = S_0)$$

$$B_2 = B_0 \quad (3)$$

$H_1 = 1{,}5\dfrac{kA}{m}$ sledi da je B_1 u drugom delu krive

$$B - 0{,}8 = \frac{1-0{,}8}{1200-600}(H-600)$$

$$B = \frac{0{,}2}{600}H + 0{,}6$$

$$B_1 = \frac{0{,}2}{600}\cdot 1500 + 0{,}6 = 1{,}1T$$

Iz (2) sledi $B_2 = \dfrac{S_1}{S_2}B_1 = \dfrac{1{,}8}{2{,}5}\cdot 1{,}1 = 0{,}792T < 0{,}8T$

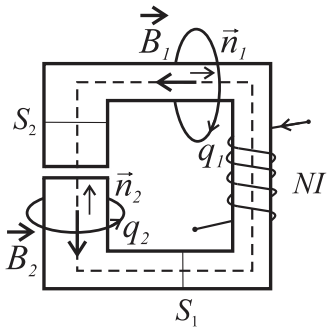
$$H_2 = \frac{B_2}{\mu} = \frac{0{,}792}{\frac{0{,}8}{600}} = 594\frac{A}{m}$$

Iz (3) sledi $B_0 = B_2 = 0{,}792T$

$$H_0 = \frac{B_0}{\mu_0} = 630{,}253\frac{kA}{m}$$

Iz (1) sledi $NI = 1500\cdot 0{,}14 + 594\cdot 0{,}06 + 630{,}253k\cdot 0{,}5m = \boxed{560{,}77Azav}$

b)



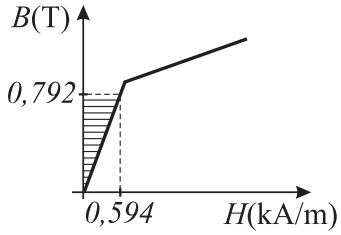
$$q = -\frac{1}{R_Z}\left(\Phi_{kr} - \Phi_{poč}\right)$$

$$\Phi_{poč} = 0$$

$q_1 = q_2$ pošto je $\Phi_1 = \Phi_2$

$$q_1 = q_2 = \frac{1}{R_Z}\cdot B_1S_1 = \frac{1}{R_Z}\cdot B_2S_2 = \boxed{9{,}9\mu C}$$

c)



$$W_m = \int_{V_2} w_m\cdot dV = \frac{1}{2}B_2H_2\cdot l_2S_2 = \frac{1}{2}0{,}792\cdot 594\cdot 0{,}06\cdot 2{,}5\cdot 10^{-4} = \boxed{3{,}53mJ}$$

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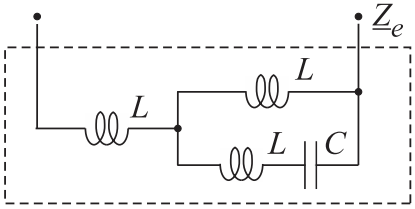
K2 Z1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C}\, j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC-\omega^2 LC}{j\omega C}} = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}$$

$$\underline{Z}_e = \underline{Z}_2 + j\omega L = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC} + j\omega L = \frac{j\omega L\left(1-\omega^2 LC\right) + j\omega L\left(1-2\omega^2 LC\right)}{1-2\omega^2 LC}$$

$$\boxed{\underline{Z}_e = j\frac{\omega L\left(2-3\omega^2 LC\right)}{1-2\omega^2 LC}}$$

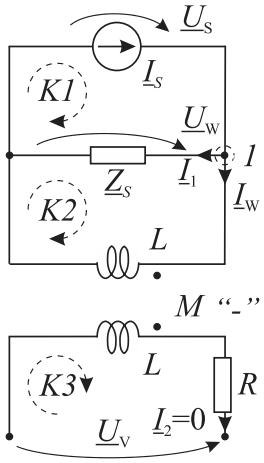
$\mathrm{Im}\{\underline{Z}_P\} = 0 \;\Rightarrow\; \omega L = 0$ $\quad \boxed{\omega_{r1} = 0}$

$2 - 3\omega^2 LC = 0$ $\quad \boxed{\omega_{r2} = \sqrt{\frac{2}{3LC}}}$

$\mathrm{Im}\{\underline{Y}_P\} = 0 \;\Rightarrow\; 1 - 2\omega^2 LC = 0$ $\quad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}}$

$\boxed{\omega_{ar2} \to \infty}$

b)



$$\omega_{ar1} = \frac{1}{\sqrt{2LC}} = \frac{1}{\sqrt{2\cdot 20\cdot 10^{-3}\cdot 25\cdot 10^{-9}}} = 10^4\ \frac{rad}{s} = \omega$$

$\underline{Z}_e = \infty$ $\qquad \underline{Z}_L = j\omega L = j10^4 \cdot 20\cdot 10^{-3} = j200\ \Omega$

$\underline{Z}_M = j\omega M = j10^4 \cdot 10\cdot 10^{-3} = j100\ \Omega$

Č1: $\quad \underline{I}_S = \underline{I}_W + \underline{I}_1$

K1: $\quad -\underline{U}_S + \underline{U}_W = 0$

K2: $\quad -\underline{Z}_S \underline{I}_1 + \underline{Z}_L \underline{I}_W - \underline{Z}_M \cancelto{0}{\underline{I}_2} = 0$

K3: $\quad \cancelto{0}{\underline{Z}_L \underline{I}_2} - \underline{Z}_M \underline{I}_W + \underline{U}_V = 0$

$\underline{I}_W + \underline{I}_1 = 1 + j \quad /\cdot(50 - j100)$

$j200\underline{I}_W - (50 - j100)\underline{I}_1 = 0$

$50\underline{I}_W - j100\underline{I}_W + (50 - j100)\underline{I}_1 = 150 - j50$

$j200\underline{I}_W - (50 - j100)\underline{I}_1 = 0$

$(50 + j100)\underline{I}_W = 150 - j50 \;\Rightarrow\; \underline{I}_W = \frac{150 - j50}{50 + j100} = (0{,}2 - j1{,}4)\ \mathrm{A}$

$\underline{I}_1 = 1 + j - 0{,}2 + j1{,}4 = (0{,}8 + j2{,}4)\ \mathrm{A}$

$\underline{U}_W = \underline{Z}_S \underline{I}_1 = (50 - j100)\cdot(0{,}8 + j2{,}4)$

$\underline{U}_W = (280 + j40)\ \mathrm{V}$

$P_W = \mathrm{Re}\{\underline{U}_W \cdot \underline{I}_W{}^*\}$

$P_W = \mathrm{Re}\{(280 + j40)\cdot(0{,}2 - j1{,}4)^*\}$

$\boxed{P_W = \mathrm{Re}\{j400\} = 0}$ (kalem)

$\underline{U}_V = \underline{Z}_M \underline{I}_W = j100\cdot(0{,}2 - j1{,}4) = (140 + j20)V \quad \Rightarrow \quad \boxed{U_V = |\underline{U}_V| = 141{,}42V}$

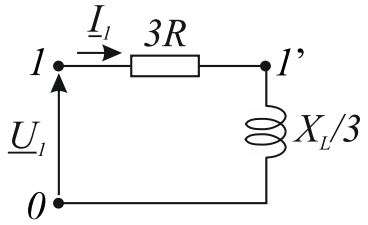
c)

$\underline{S}_{IS} = \underline{U}_S \underline{I}_S{}^* = \underline{U}_W \underline{I}_S{}^* = (280 + j40)\cdot(1 - j) = (320 - j240)\ \mathrm{VA}$ $\quad \boxed{\underline{S}_{IS} = (320 - j240)\ \mathrm{VA}}$

$\boxed{P_{IS} = 320\ \mathrm{W}}$ $\quad \boxed{Q_{IS} = -240\ \mathrm{VAr}}$ $\quad \boxed{S_{IS} = 400\ \mathrm{VA}}$

K2 Z2

a)



$$\underline{I}_1 = \frac{\underline{U}_1}{3R + j\frac{X_L}{3}} = \frac{230\,e^{j0°}}{(15 + j15)} = \frac{230\,e^{j0°}}{15\sqrt{2}e^{j45°}} = 10{,}84\,e^{-j45°}\ A$$

$$I_{A1} = |\underline{I}_3| = I_1$$

$$\boxed{I_{A1} = 10{,}84\ \text{A}}$$

$$U_V = 3R \cdot I_{A1}$$

$$\boxed{U_V = 162{,}6\ \text{V}}$$

$$I_{A2} = \frac{|\underline{I}_1|}{\sqrt{3}} = \frac{10{,}84}{\sqrt{3}}$$

ili $$I_{A2} = \frac{|U'_{13}|}{X_L} = \frac{\sqrt{3}\frac{X_L}{3}|\underline{I}_1|}{X_L}$$

$$\boxed{I_{A2} = 6{,}26\ \text{A}}$$

$$P_{W1} = \text{Re}\left\{\underline{U}_{23}\,\underline{I}_2^{\,*}\right\} = U_{23}\,I_2\cos\sphericalangle(\underline{U}_{23}, \underline{I}_2) = \sqrt{3}U_1\,I_1\cos(30° + 45°) = \sqrt{3}\cdot 230\cdot 10{,}84\cdot\cos 75°$$

$$\boxed{P_{W1} = 1117{,}67\ \text{W}}$$

$$P_{W2} = \text{Re}\left\{\underline{U}_{11'}\,\underline{I}_1^{\,*}\right\} = \text{Re}\left\{3R\underline{I}_1\,\underline{I}_1^{\,*}\right\} = 3RI_1^2$$

$$\boxed{P_{W2} = 1762{,}58\ \text{W}}$$

b)