STUDIJSKI PROGRAM MEHATRONIKA

K T E T

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

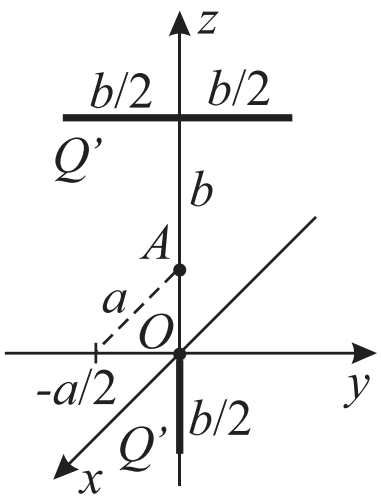
25. 8. 2026.

ZADACI

Zadatak 1. Tanak štap, dužine b, naelektrisan ravnomerno podužnom gustinom naelektrisanja Q', postavljen je u y-z ravan Dekartovog koordinatnog sistema, paralelno sa y osom, na rastojanju b od tačke A, kao što je prikazano na slici 1. Na z osi datog sistema postavljen je i štap dužine $b/2$, naelektrisan ravnomerno istom podužnom gustinom naelektrisanja Q'. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja, koji u tački A stvaraju oba štapa.
b) Izračunati intenzitet datog vektora.
c) Izračunati potencijal tačke A u odnosu na referentnu tačku u beskonačnosti, koji potiče samo od kraćeg štapa.

Brojni podaci su: $a = 1$ cm, $b = 2a\sqrt{3}$, $Q' = 1$ nC/m, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m.

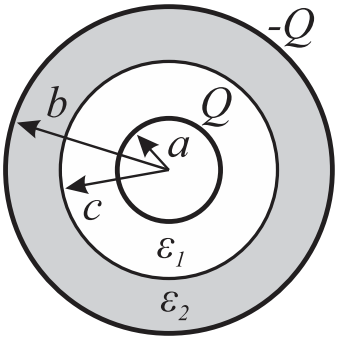


Slika 1.

Zadatak 2. Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\varepsilon_{r1} = 4$ i $\varepsilon_{r2} = 2$. Poluprečnici elektroda kondenzatora su $a = 2$ mm i $b = 6$ mm. Poluprečnik razdvojne površi dva dielektrika je $c = 4$ mm, dok su elektrode kondenzatora naelektrisane naelektrisanjem Q i $-Q$.

a) Razmotriti granične uslove i odrediti kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.
b) Odrediti u opštim brojevima kapacitivnost kondenzatora, a potom i njenu brojnu vrednost.
c) Izračunati količinu vezanog naelektrisanja uz unutrašnju elektrodu kondenzatora, kada se kondenzator priključi na izvor napona $U = 10$ kV.
d) Da li će pri ovom naponu doći do proboja kondenzatora?

Ostali brojni podaci su: $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m, $E_{č1} = 45$ kV/cm, $E_{č2} = 90$ kV/cm.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MEHATRONIKA

K TET

Katedra za teorijsku elektrotehniku

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

25. 8. 2026.

ZADACI

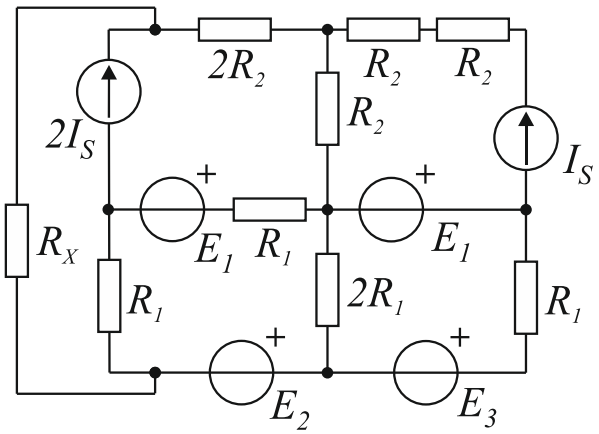
Zadatak 1. U kolu vremenski konstantnih struja, prikazanom na slici 1, poznato je, da se na otporniku R_x razvija maksimalna moguća snaga. Odrediti

a) nepoznatu otpornost R_x, kao i

b) snagu tog otpornika korišćenjem Tevenenove teoreme.

Proračune izvršiti korišćenjem metode konturnih struja.

Brojni podaci su: $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $E_1 = 10$ V, $E_2 = 8$ V, $E_3 = 7$ V, $I_S = 1$ A.



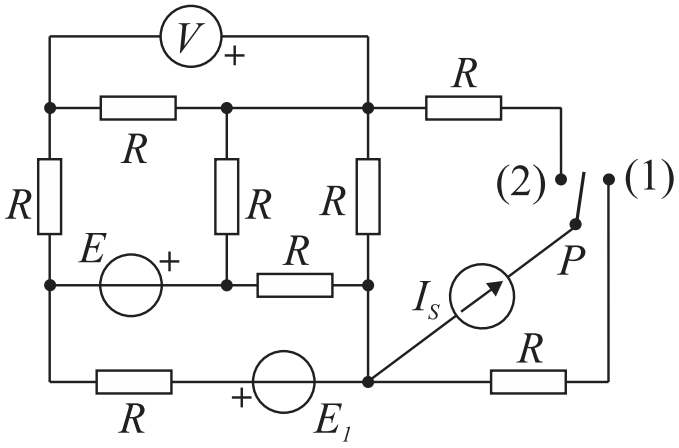
Slika 1.

Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, posle prebacivanja preklopnika P iz položaja (1) u položaj (2), pokazivanje voltmetra se promeni za 1 V.

a) Primenom teoreme superpozicije, odrediti jačinu struje strujnog generatora I_S.

b) Izračunati snagu koju razvijaju naponski generatori, kada je preklopnik u položaju (1).

Brojni podaci su: $E = E_1 = 6{,}5$ V, $R = 6{,}5\ \Omega$.



Slika 2.

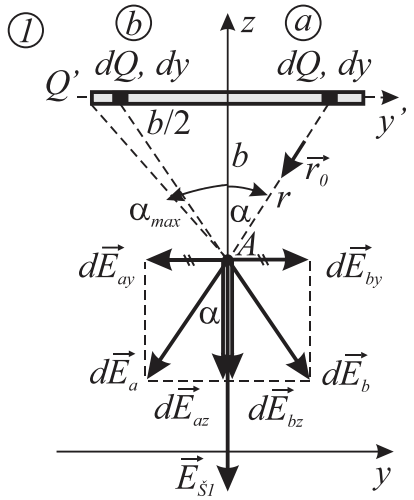
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MH ISPIT, K1, 25. 8. 2026.

K1 Z1

a)



$$d\vec{E}_{ay} + d\vec{E}_{by} = 0 \quad \Rightarrow \quad \boxed{\vec{E}_y = 0}$$

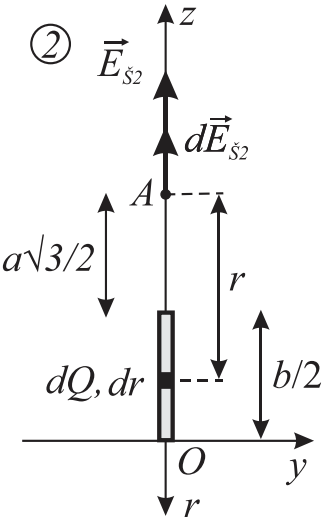
$$dE_{az} = dE_a \cos\alpha = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\alpha = \frac{Q'dy}{4\pi\varepsilon_0 r^2}\cos\alpha \qquad \left(dy = \frac{r\,d\alpha}{\cos\alpha}\right)$$

$$dE_{az} = \frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = \frac{Q'}{4\pi\varepsilon_0\dfrac{b}{\cos\alpha}}d\alpha \qquad \left(r = \frac{b}{\cos\alpha}\right)$$

$$E_{Š1} = 2\int\limits_{\substack{po\ pola\\ štapa}} dE_{az} = 2\frac{Q'}{4\pi\varepsilon_0 b}\int\limits_0^{\alpha_{max}} \cos\alpha\, d\alpha = \frac{Q'}{2\pi\varepsilon_0 b}\left(\sin\alpha_{max} - \sin 0\right)$$

$$E_{Š1} = \frac{Q'}{2\pi\varepsilon_0 b}\cdot\frac{\frac{b}{2}}{\sqrt{b^2 + \left(\frac{b}{2}\right)^2}} = \frac{Q'}{2\pi\varepsilon_0 b}\cdot\frac{\sqrt{5}}{5} = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}$$

$$\boxed{\vec{E}_{Š1} = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}\cdot\left(-\vec{i}_z\right)}$$



$$dE_{Š2} = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'dr}{4\pi\varepsilon_0 r^2}$$

$$E_{Š2} = \frac{Q'}{4\pi\varepsilon_0}\int\limits_{a\sqrt{3}/2}^{a\sqrt{3}/2+b/2}\frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_{a\sqrt{3}/2}^{\left(a\sqrt{3}+b\right)/2}$$

$$E_{Š2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{\frac{a\sqrt{3}}{2}} - \frac{1}{\frac{a\sqrt{3}+b}{2}}\right) = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{2}{a\sqrt{3}} - \frac{2}{a\sqrt{3}+b}\right)$$

$$\vec{E}_{Š2} = \frac{2Q'}{4\pi\varepsilon_0}\left(\frac{1}{a\sqrt{3}} - \frac{1}{a\sqrt{3}+b}\right)\left(-\vec{i}_r\right)$$

$$\boxed{\vec{E}_{Š2} = \frac{Q'}{2\pi\varepsilon_0}\left(\frac{1}{a\sqrt{3}} - \frac{1}{a\sqrt{3}+b}\right)\vec{i}_z}$$

b)

$$\vec{E}_A = \vec{E}_{Š1} + \vec{E}_{Š2} = \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}\cdot\left(-\vec{i}_z\right) + \frac{Q'}{2\pi\varepsilon_0}\left(\frac{1}{a\sqrt{3}} - \frac{1}{a\sqrt{3}+b}\right)\vec{i}_z = \left(\frac{Q'}{2\pi\varepsilon_0 a\sqrt{3}} - \frac{Q'}{2\pi\varepsilon_0(a\sqrt{3}+b)} - \frac{\sqrt{5}\,Q'}{10\pi\varepsilon_0 b}\right)\vec{i}_z$$

$$\vec{E}_A = \frac{Q'}{\pi\varepsilon_0}\left(\frac{1}{2a\sqrt{3}} - \frac{1}{2(a\sqrt{3}+b)} - \frac{\sqrt{5}}{10b}\right)\vec{i}_z$$

$$b = 2a\sqrt{3} \quad \Rightarrow \quad \boxed{\vec{E}_A = \frac{Q'\sqrt{3}}{3\pi\varepsilon_0 a}\left(\frac{2}{3} - \frac{\sqrt{5}}{20}\right)\vec{i}_z} \qquad \boxed{E_A = 3{,}75\,\frac{\mathrm{V}}{\mathrm{m}}}$$

c)

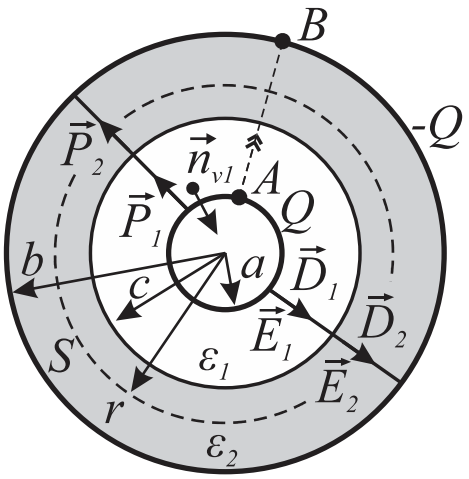
$$dV_A = \frac{dQ}{4\pi\varepsilon_0 r} = \frac{Q'dr}{4\pi\varepsilon_0 r}$$

$$V_A = \int\limits_{po\ štapu} dV_A = \frac{Q'}{4\pi\varepsilon_0}\int_{a\sqrt{3}/2}^{\left(a\sqrt{3}+b\right)/2}\frac{dr}{r} = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{a\sqrt{3}+b}{a\sqrt{3}} = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{b}{a\sqrt{3}} = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{2a\sqrt{3}}{a\sqrt{3}} = \frac{Q'}{4\pi\varepsilon_0}\ln 2$$

$$\boxed{V_A = 6{,}23\,\mathrm{V}}$$

K1 Z2

a)



Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$

$\int_S D\, ds = Q$

$D\, 4r^2\pi = Q \qquad \boxed{D = \frac{Q}{4\pi r^2}}, \qquad a \le r \le b$

$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_1 r^2}}, \qquad a \le r \le c$

$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_2 r^2}}, \qquad c \le r \le b$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr + \int_c^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$U_{AB} = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}\right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\frac{c-a}{ac} + \frac{1}{\varepsilon_{r2}}\frac{b-c}{bc}}} \qquad \boxed{C = 1{,}07\ \text{pF}}$$

c)

$U = 10\ \text{kV} \quad \rightarrow \quad Q = CU = 1{,}07\text{pF} \cdot 10\ \text{kV} = 10{,}7\ \text{nC}$

$$P_1 = D - \varepsilon_0 E_1 = D - \varepsilon_0 \frac{D}{\varepsilon_1} = \left(1 - \frac{1}{\varepsilon_{r1}}\right) D = \left(1 - \frac{1}{4}\right)\frac{Q}{4\pi r^2} = \frac{3}{4}\frac{Q}{4\pi r^2}$$

$$\sigma_{v1} = \vec{P}_1 \cdot \vec{n}_{v1} = -P_1(r = a) = -\frac{3}{4}\frac{Q}{4\pi a^2}$$

$$Q_{v1} = \sigma_{v1}\, 4\pi a^2 = -\frac{3}{4}\frac{Q}{4\pi a^2}\, 4\pi a^2 = -\frac{3}{4}Q = -\frac{3}{4}\cdot 10{,}7\ \text{nC} \qquad \boxed{Q_{v1} = -8{,}03\ \text{nC}}$$

d)

$$E_{1\max} = \frac{Q_{1\max}}{4\pi\varepsilon_1 a^2} \le E_{\check{C}1} \quad \Rightarrow \quad Q_{1\max} = E_{\check{C}1}\, 4\pi\varepsilon_1 a^2 = 8\ \text{nC}$$

$$E_{2\max} = \frac{Q_{2\max}}{4\pi\varepsilon_2 c^2} \le E_{\check{C}2} \quad \Rightarrow \quad Q_{2\max} = E_{\check{C}2}\, 4\pi\varepsilon_2 c^2 = 32\ \text{nC}$$

$Q_{\max} = \min\{Q_{1\max}, Q_{2\max}\} = Q_{1\max} = 8\ \text{nC}$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{8\,\text{nC}}{1{,}07\,\text{pF}} = 7{,}48\,\text{kV}$$

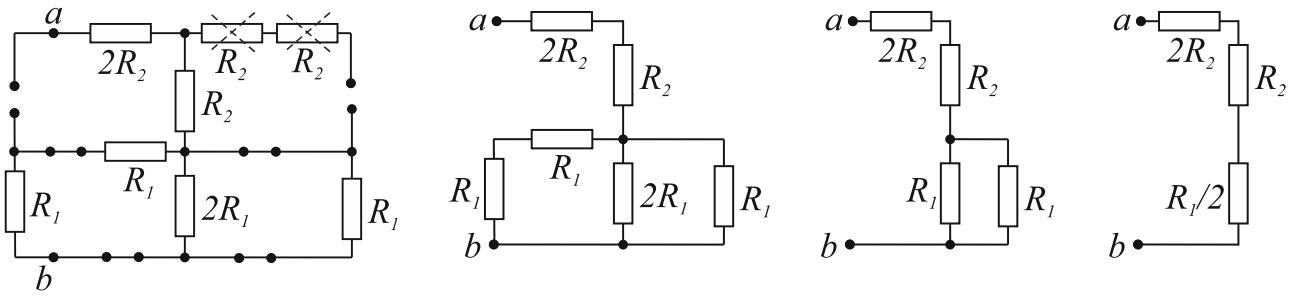
$U = 10\ \text{kV}$

$\boxed{U > U_{\max} \ \Rightarrow}$ doći će do proboja.

MH ISPIT, K2, 28. 8. 2026.

K2 Z1

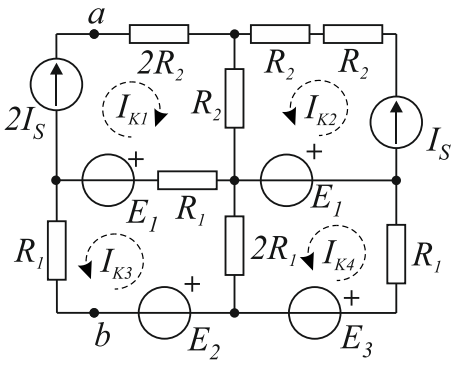
R_T:



$$R_T = R_{ab} = 2R_2 + R_2 + \frac{R_1}{2} = 10 + 5 + 2 \qquad \boxed{R_T = 17\ \Omega}$$

E_T:

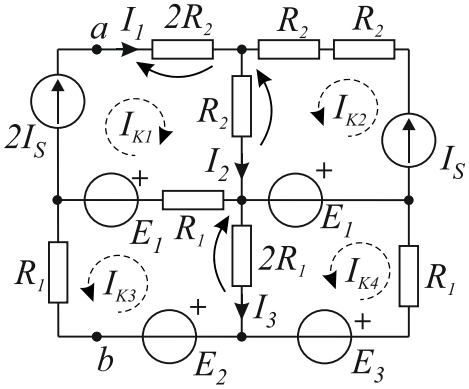
$$MKS:\ n_g - (n_č - 1) - n_s = \underbrace{8 - (5-1)}_{n_k=4} - 2 = 2\ jednačine$$



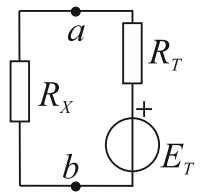
$$I_{K1} = 2I_S = 2\ \text{A}$$
$$I_{K2} = I_S = 1\ \text{A}$$
$$I_{K3}\left(R_1 + R_1 + 2R_1\right) - I_{K4}\,2R_1 + I_{K1}R_1 = E_2 - E_1$$
$$I_{K4}\left(2R_1 + R_1\right) - I_{K3}\,2R_1 = E_3 - E_1$$

$$16\,I_{K3} - 8\,I_{K4} = -10$$
$$-8\,I_{K3} + 24\,I_{K4} = -3$$

$$I_{K4} = -1\ \text{A}$$
$$I_{K3} = -1{,}125\ \text{A}$$



$$I_1 = I_{K1} = 2\ \text{A}$$
$$I_2 = I_{K1} + I_{K2} = 3\ \text{A}$$
$$I_3 = -I_{K3} + I_{K4} = -(-1{,}125) + (-1) = 0{,}125\ \text{A}$$
$$E_T = U_{ab} = +2R_2I_1 + R_2I_2 + 2R_1I_3 + E_2 = 10 \cdot 2 + 5 \cdot 3 + 8 \cdot 0{,}125 + 8$$
$$\boxed{E_T = 44\ \text{V}}$$



$$R_x = R_T = 17\,k\Omega$$
$$P_{Rx} = R_x\left(\frac{E_T}{R_x + R_T}\right)^2 = \frac{E_T{}^2}{4R_T} = 28{,}47\ \text{W}$$

K2 Z2

1. Na osnovu uslova teksta zadatka: $U_V^{(2)} = U_V^{(1)} + \Delta U_V$

2. Na osnovu teoreme superpozicije:

$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_S\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_S\end{matrix}}$$

$$(2) \qquad (1)$$

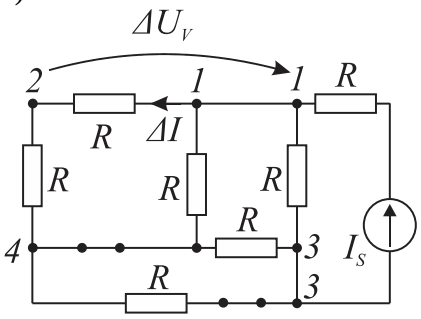
Preklopnik u položaju (2) (deluju svi generatori) $U_V^{(2)}$

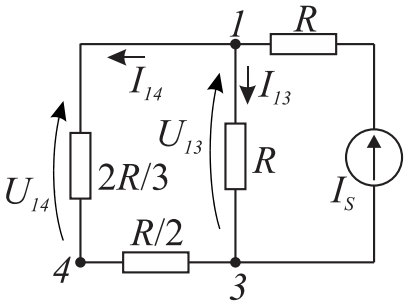
Preklopnik u položaju (1) (svi generatori osim strujnog generotora I_S) $U_V^{(1)}$

Deluje samo strujni generator I_S U_V'

Na osnovu (1) i (2) sledi da je $U_V' = \Delta U_V$

a)





$$\Delta I = \frac{\Delta U_V}{R} = \frac{1}{6,5} = 0,154\ \text{A}$$

$$U_{14} = (R+R)I' = 13 \cdot 0,154 = 2\ \text{V}$$

$$I_{14} = \frac{U_{14}}{2R/3} = \frac{2}{13/3} = 0,462\ \text{A}$$

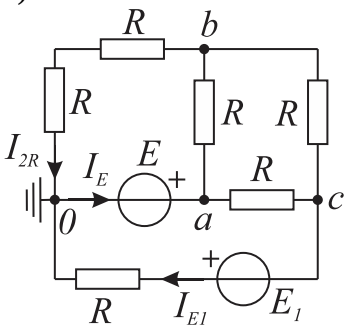
$$U_{13} = (R/2 + 2R/3)I_{14} = 3,503\ \text{V}$$

$$I_{13} = \frac{3,503}{6,5} = \frac{2}{13/3} = 0,538\ \text{A}$$

$$I_S = I_{13} + I_{14} = 0,538 + 0,462$$

$$\boxed{I_S = 1\ \text{A}}$$

b)



$n_č = 4, \quad n_{i.n.g.} = 1$

$MPČ: \quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$

$V_0 = 0\ \text{V}, \qquad V_a = E = 6,5\ \text{V}$

$$V_b\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{2R}\right) - V_a\left(\frac{1}{R}\right) - V_c\left(\frac{1}{R}\right) = 0 \qquad /\cdot 2R$$

$$V_c\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R+0}\right) - V_a\left(\frac{1}{R}\right) - V_b\left(\frac{1}{R}\right) = -\frac{E_1}{R} \qquad /\cdot R$$

$$5V_b - 2V_a - 2V_c = 0$$

$$3V_c - V_a - V_b = -E_1$$

$$\left.\begin{matrix}5V_b - 2V_c = 13\\ 3V_c - V_b = 0\end{matrix}\right\} \Rightarrow \qquad V_b = 3\ \text{V} \qquad V_c = 1\ \text{V}$$

$$I_{E1} = \frac{V_c - V_0 + E_1}{R} = 1,15\ \text{A}$$

$$P_{E1} = +E_1 \cdot I_{E1} = 6,5 \cdot 1,15$$

$$\boxed{P_{E1} = 7,5\ \text{W}}$$

$$I_{2R} = \frac{V_b - V_0}{2R} = 0,23\ \text{A}$$

$$I_E = I_{E1} + I_{2R}$$

$$I_E = 1,15 + 0,23 = 1,38\ \text{A}$$

$$P_E = +E \cdot I_E = 6,5 \cdot 1,38$$

$$\boxed{P_E = 8,97\ \text{W}}$$