STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

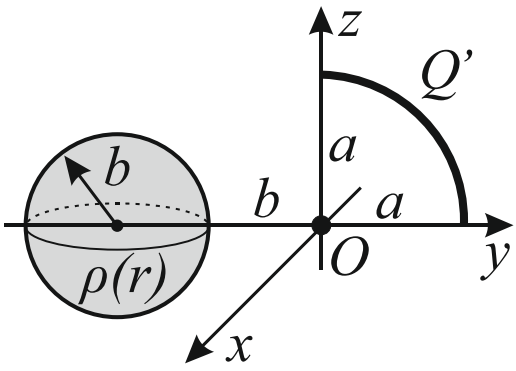
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **25.08.2026.**

ZADACI

Zadatak 1. Zapreminska gustina sferno simetrične raspodele naelektrisanja, poluprečnika b, nije konstantna nego se menja u funkciji rastojanja od centra opterećenja po zakonu $\rho(r)$. Centar sfernog naelektrisanja se nalazi na y osi zadatog Dekartovog koordinatnog sistema, na udaljenosti $2b$ od centra sistema, kao što je prikazano na slici 1. Tanak štap, naelektrisan podužnom gustinom naelektrisanja Q', savijen je u obliku 1/4 kruga, poluprečnika a, i leži u y-z ravni. Sistem se nalazi u vazduhu.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja, koji u tački O (centar koordinatnog sistema) stvara zapreminski raspoređeno naelektrisanje.
b) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja, koji u tački O stvara štap.
c) Izračunati konstantu, ρ_0, tako da rezultantni vektor jačine električnog polja u tački O ima samo z komponentu.

Brojni podaci su: $\rho(r)=\frac{\rho_0}{2b^2}r^2$, $a = 1\ cm$, $b = 0{,}5\ cm$, $Q' = 1\ nC/m$, $\varepsilon_0 = 8{,}85\cdot10^{-12}\ F/m$.

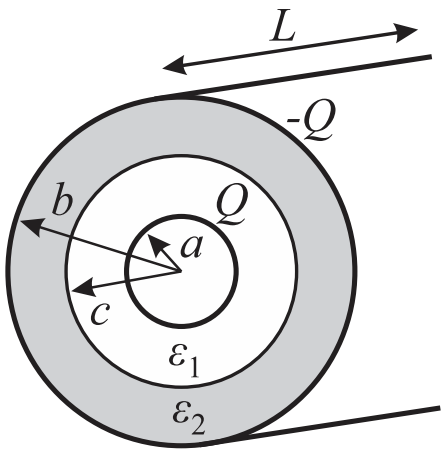


Slika 1.

Zadatak 2. Na slici 2 je prikazan koaksijalni kabl dužine $L = 1\ m$, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\varepsilon_{r1} = 6$ i $\varepsilon_{r2} = 4$. Poluprečnici elektroda kabla su $a = 1\ mm$ i $b = 4\ mm$. Poluprečnik razdvojne površi dva dielektrika je $c = 2\ mm$. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i $-Q$.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) Izračunati kapacitivnost kabla.
c) Da li kabl sme da se priključi na napon $U = 15\ kV$?
d) Izračunati maksimalnu energiju koja može biti sadržana u kablu.

Ostali brojni podaci: $E_{Č1} = 70\ kV/cm$, $E_{Č2} = 80\ kV/cm$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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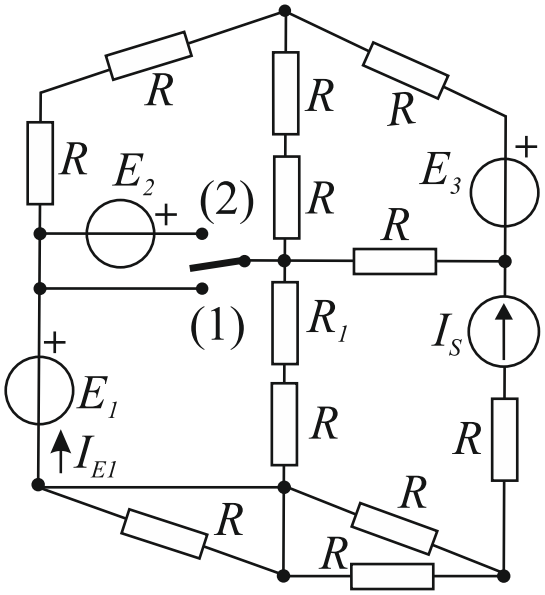
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **25.08.2026.**

ZADACI

Zadatak 1. U električnom kolu vremenski konstantnih struja, prikazanom na slici 1, prilikom prebacivanja preklopnika iz položaja (1) u položaj (2) struja kroz naponski generator E_1 se poveća za vrednost $\Delta I_{E1} = 1\ A$, u naznačenom referentnom smeru.

a) Primenjujući teoremu superpozicije, odrediti *ems* naponskog generatora E_2.
b) Odrediti snage naponskog generatora E_1 i strujnog generatora I_S, kada je preklopnik u položaju (2), primenom metode potencijala čvorova.

Brojni podaci su: $R_1 = 10\ \Omega$, $R = 10\ \Omega$, $E_1 = 9\ V$, $E_3 = 10\ V$, $I_S = 1\ A$.

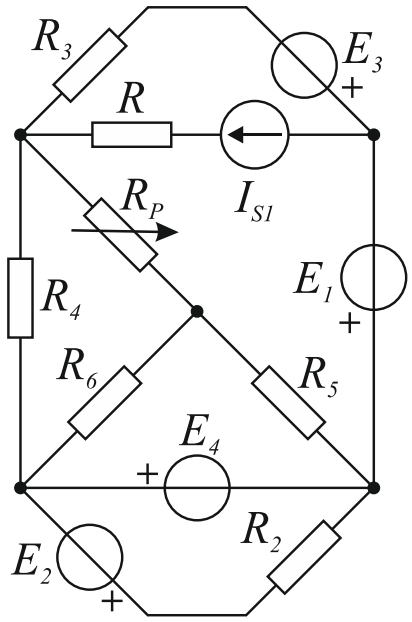


Slika 1.

Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, klizač promenljivog otpornika R_P se nalazi u položaju u kome se na ovom otporniku razvija maksimalno moguća snaga.

a) Izračunati vrednost otpornosti otpornika R_P.
b) Izračunati snagu koja se razvija na otporniku R_P.
c) Skicirati grafički kako se menja snaga promenljivog otpornika R_P, ako se vrednost njegove otpornosti menja od vrednosti 0 do vrednosti $5R_P$.

Brojni podaci su: $R = R_2 = 100\ \Omega$, $R_3 = R_4 = 40\ \Omega$, $R_5 = R_6 = 60\ \Omega$, $E_1 = E_2 = 12\ V$, $E_3 = 8\ V$, $E_4 = 3\ V$, $I_{S1} = 0{,}2\ A$.



Slika 2.

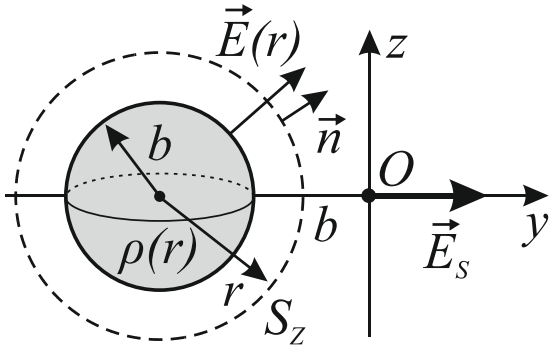
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 25.08.2026.

I-1

a)



Pretpostavka $\rho_0 > 0$

$$\oint_{S_Z} \vec{E}\cdot\overrightarrow{dS} = \frac{Q_{u\,S_Z}}{\varepsilon_0} \qquad \sphericalangle\left(\vec{E},\vec{n}\right)=0$$

$$\oint_{S_Z} E(r)\,dS = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

$$E(r)4r^2\pi = \frac{1}{\varepsilon_0}\int_0^b \frac{\rho_0}{2b^2} r^2\, 4r^2\pi\, dr = \frac{1}{\varepsilon_0}\frac{\rho_0}{2b^2}4\pi\int_0^b r^4 dr$$

$$= \frac{1}{\varepsilon_0}\frac{\rho_0}{2b^2}4\pi\frac{b^5}{5} = \frac{2}{5}\frac{\rho_0}{\varepsilon_0}\pi b^3$$

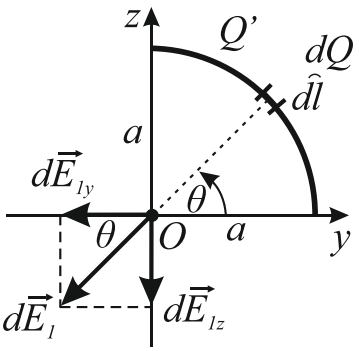
$$\boxed{E(r) = \frac{\rho_0 b^3}{10\varepsilon_0 r^2}}, \qquad r > b$$

$r = 2b$

$$E_S(r=2b) = \frac{\rho_0 b^3}{10\varepsilon_0 (2b)^2} = \frac{\rho_0 b^3}{40\varepsilon_0 b^2} = \frac{\rho_0 b}{40\varepsilon_0}$$

$$\boxed{\vec{E}_S = \frac{\rho_0 b}{40\varepsilon_0}\cdot\vec{i}_y}$$

b)



$$dE_{1y} = dE_1\cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\cos\theta = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta$$

$$E_{1y} = \int_{\substack{po\\luku}} dE_{1y} = \frac{Q'}{4\pi\varepsilon_0 a}\int_{0°}^{90°}\cos\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\sin 90° - \sin 0°\right) = \frac{Q'}{4\pi\varepsilon_0 a}(1-0)$$

$$\boxed{E_{1y} = \frac{Q'}{4\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_{1y} = E_{1y}\cdot\left(-\vec{i}_y\right)}$$

$$dE_{1z} = dE_1\sin\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\sin\theta = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\sin\theta$$

$$E_{1z} = \int_{\substack{po\\luku}} dE_{1z} = \frac{Q'}{4\pi\varepsilon_0 a}\int_{0°}^{90°}\sin\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\cos 0° - \cos 90°\right) = \frac{Q'}{4\pi\varepsilon_0 a}(1-0)$$

$$\boxed{E_{1z} = \frac{Q'}{4\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_{1z} = E_{1z}\cdot\left(-\vec{i}_z\right)}$$

c)

$$\vec{E}_O = \vec{E}_S + \vec{E}_{1y} + \vec{E}_{1z} = \left[\frac{\rho_0 b}{40\varepsilon_0} - \frac{Q'}{4\pi\varepsilon_0 a}\right]\cdot\vec{i}_y + \frac{Q'}{4\pi\varepsilon_0 a}\cdot\left(-\vec{i}_z\right)$$

$$\vec{E}_{Oy} = 0 \quad \Rightarrow \quad \frac{\rho_0 b}{40\varepsilon_0} - \frac{Q'}{4\pi\varepsilon_0 a} = 0$$

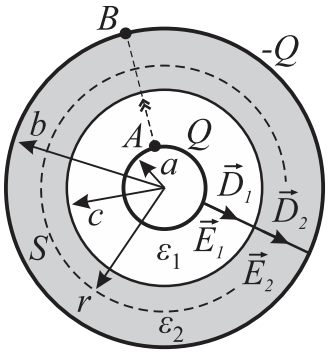
$$\rho_0 = \frac{10\,Q'}{\pi a b}$$

$$\boxed{\rho_0 = 63{,}7\ \mu C/m^3}$$

Ispit iz OET1, 25.08.2026.

I-2

a)



Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{OM}} D\, ds = Q$$

$$D\, 2r\pi\, L = Q \qquad \boxed{D = \frac{Q}{2\pi r\, L}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 r\, L}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 r\, L}}, \qquad c \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 r\, L}\, dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 r\, L}\, dr = \frac{Q}{2\pi\varepsilon_1\, L}\ln\frac{c}{a} + \frac{Q}{2\pi\varepsilon_2\, L}\ln\frac{b}{c}$$

$$C = \frac{Q}{U_{AB}} \qquad \boxed{C = \frac{2\pi\varepsilon_0\, L}{\frac{1}{\varepsilon_{r1}}\ln\frac{c}{a} + \frac{1}{\varepsilon_{r2}}\ln\frac{b}{c}}}$$

b)

$$\boxed{C = 192{,}44\ pF}$$

c)

$$E_{1\max} = \frac{Q_{1\max}}{2\pi\varepsilon_1 a\, L} \le E_{\check{C}1} \qquad \Rightarrow \qquad Q_{1\max} = E_{\check{C}1}\, 2\pi\varepsilon_1 a\, L = 2{,}33\ \mu C$$

$$E_{2\max} = \frac{Q_{2\max}}{2\pi\varepsilon_2 c\, L} \le E_{\check{C}2} \qquad \Rightarrow \qquad Q_{2\max} = E_{\check{C}2}\, 2\pi\varepsilon_2 c\, L = 3{,}56\ \mu C$$

$$Q_{\max} = \min\{Q_{1\max}, Q_{2\max}\} = Q_{1\max} = 2{,}33\ \mu C$$

$$U_{\max} = \frac{Q_{\max}}{C} = \frac{2{,}33\ \mu C}{192{,}44\ pF}$$

$$\boxed{U_{\max} = 12{,}11\ kV}$$

$$\boxed{U = 15\ kV, \qquad U > U_{\max} \qquad \textit{kabl ne sme da se priključi na napon od } 15\ kV}$$

d)

$$W_{e\max} = \frac{1}{2} C\, U_{\max}{}^2 = \frac{1}{2}\cdot 192{,}44\cdot 10^{-12}\cdot\left(12{,}11\cdot 10^3\right)^2$$

$$\boxed{W_{e\max} = 14{,}11\ mJ}$$

Ispit iz OET1, 25.08.2026.

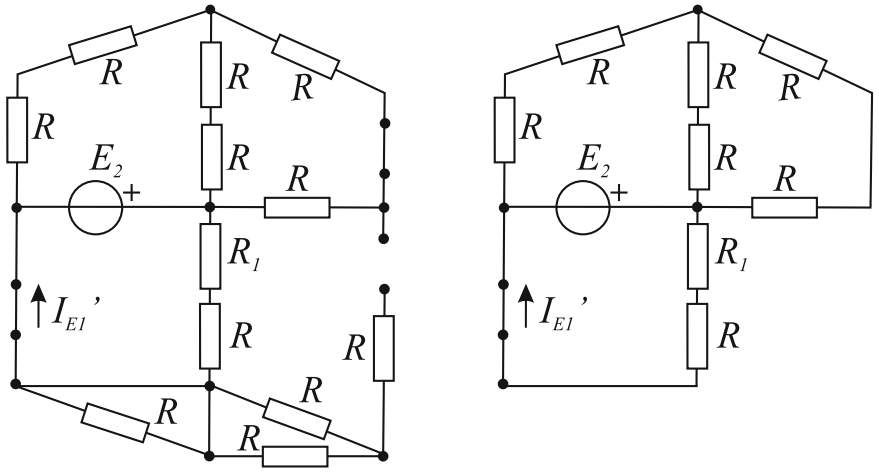
II-1

a)

$$\boxed{\begin{matrix}\text{Svi}\\ \text{generatori}\\ (2)\end{matrix}} = \boxed{\begin{matrix}\text{Svi}\\ \text{sem } E_2\\ (1)\end{matrix}} + \boxed{\begin{matrix}\text{Samo}\\ E_2\\ \end{matrix}}$$

$$I_{E1}^{(2)} = I_{E1}^{(1)} + I_{E1}'$$

$$\Delta I_{E1} = I_{E1}^{(2)} - I_{E1}^{(1)} = I_{E1}' = 1\,A$$

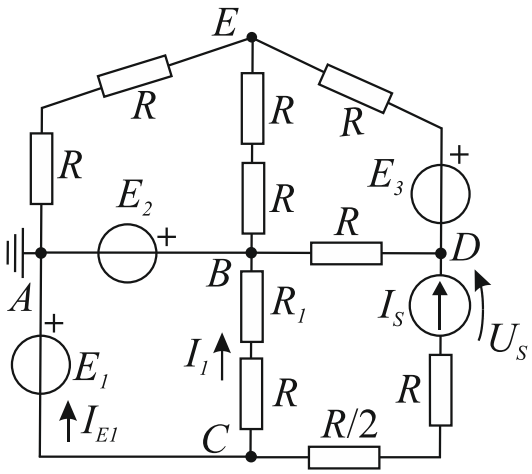


$$E_2 = I_{E1}'\left(R_1 + R\right)$$

$$E_2 = 1\cdot\left(10+10\right)$$

$$\boxed{E_2 = 20\,V}$$

b)



$$V_A = 0\,V, \qquad V_B = E_2 = 20\,V, \qquad V_C = -E_1 = -9\,V$$

$$V_D\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{\infty}\right) - V_B\left(\frac{1}{R}\right) - V_C\left(\frac{1}{\infty}\right) - V_E\left(\frac{1}{R}\right) = I_S - \frac{E_3}{R}$$

$$V_E\left(\frac{1}{R}+\frac{1}{2R}+\frac{1}{2R}\right) - V_B\left(\frac{1}{2R}\right) - V_D\left(\frac{1}{R}\right) = \frac{E_3}{R}$$

$$V_D\left(\frac{1}{10}+\frac{1}{10}\right) - 20\cdot\frac{1}{10} - V_E\left(\frac{1}{10}\right) = 1 - \frac{10}{10}$$

$$V_E\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{20}\right) - 20\cdot\frac{1}{20} - V_D\left(\frac{1}{10}\right) = \frac{10}{10}$$

$$\left.\begin{aligned} 2V_D - V_E &= 20\\ -2V_D + 4V_E &= 40\end{aligned}\right\} \qquad V_D = 20V, \qquad V_E = 20\,V$$

$$I_1 = \frac{V_C - V_B}{R_1 + R} = \frac{-9-20}{10+10} = -1{,}45\,A$$

$$I_{E1} = -I_1 - I_S = 1{,}45 - 1 = 0{,}45\,A$$

$$P_{E1} = E_1 I_{E1} = 9\cdot 0{,}45 \qquad \boxed{P_{E1} = 4{,}05\,W}$$

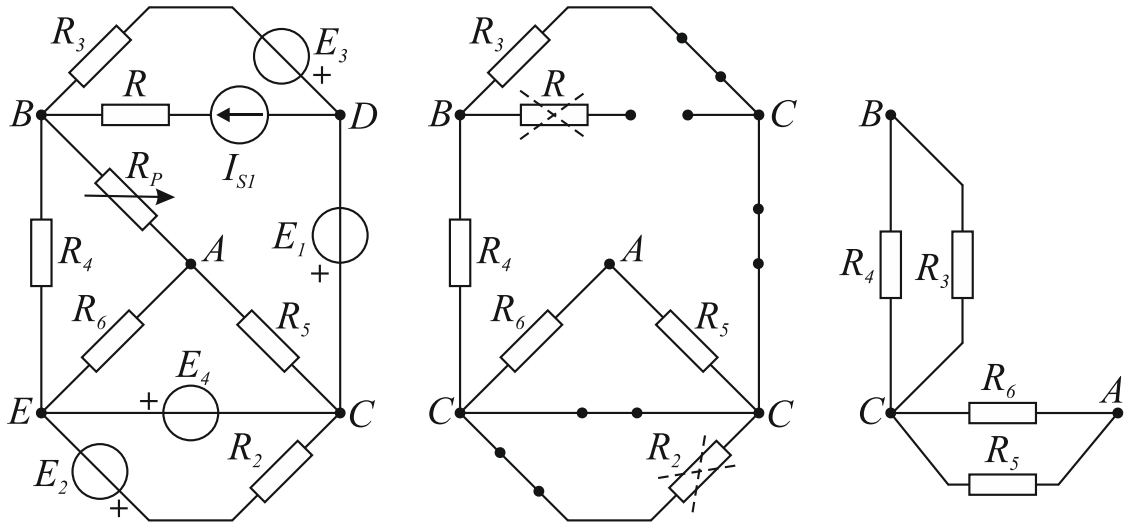
$$U_S = V_D - V_C + \left(\frac{R}{2}+R\right)I_S = 20 - (-9) + (5+10)\cdot 1 = 44\,V$$

$$P_S = U_S I_S = 44\cdot 1 \qquad \boxed{P_S = 44\,W}$$

Ispit iz OET1, 25.08.2026.

II-2

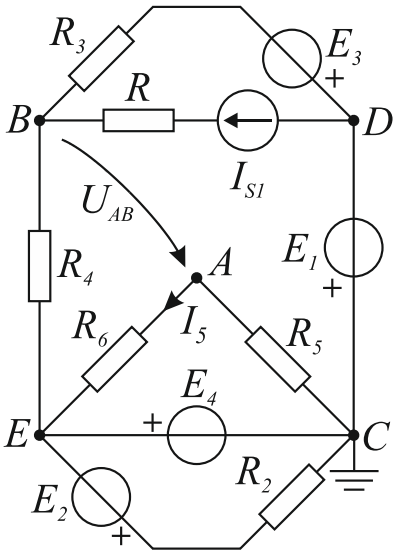
a)



$$R_T = R_{AB} = (R_5 \,||\, R_6) + (R_3 \,||\, R_4) = (60 \,||\, 60) + (40 \,||\, 40) = 30 + 20 = 50\ \Omega$$

$$\boxed{R_P = R_T = 50\ \Omega}$$

b)



$$V_C = 0\,V, \qquad V_D = -E_1 = -12\,V, \qquad V_E = E_4 = 3\,V$$

$$V_B\left(\frac{1}{R_3+0}+\frac{1}{R+\infty}+\frac{1}{R_4}\right) - V_D\left(\frac{1}{R_3+0}+\frac{1}{R+\infty}\right) - V_E\left(\frac{1}{R_4}\right) = -\frac{E_3}{R_3} + I_{S1}$$

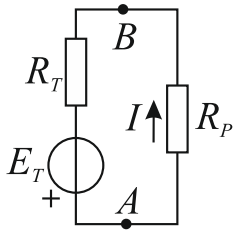
$$V_B\left(\frac{1}{40}+\frac{1}{40}\right) - (-12)\cdot\left(\frac{1}{40}\right) - 3\cdot\left(\frac{1}{40}\right) = -\frac{8}{40} + 0{,}2 \qquad /\cdot 40$$

$$2\,V_B = -9 \qquad \Rightarrow \qquad V_B = -4{,}5\,V$$

$$I_5 = \frac{V_C - V_E}{R_5 + R_6} = \frac{0-3}{60+60} = -0{,}025\,A$$

$$E_T = U_{AB} = U_{AE} + U_{EB} = R_6 I_5 + V_E - V_B = 60\cdot(-0{,}025) + 3 - (-4{,}5) = 6\,V$$

$$\boxed{E_T = 6\,V}$$



$$I = \frac{E_T}{R_T + R_P} = \frac{6}{50+50} = 0{,}06\ A$$

$$P_{RP\max} = R_P\, I^2 = 50\cdot(0{,}06)^2$$

$$\boxed{P_{RP\max} = 0{,}18\,W}$$

c)