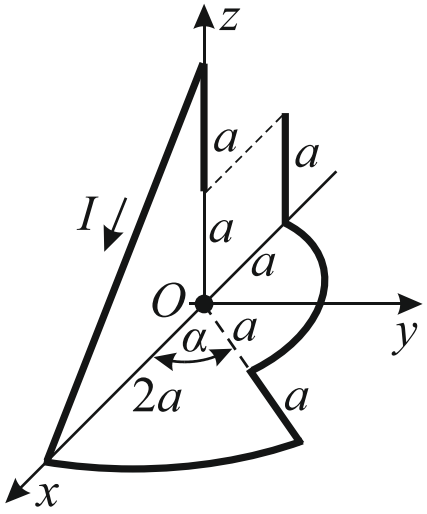
STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (I kolokvijum)	**K1** **18.07.2026.**

ZADACI

Zadatak 1. Na slici 1 je prikazan žičani provodnik, sa vremenski konstantnom strujom jačine I. Provodnik se sastoji od četiri pravolinijska i dva lučna segmenta. Tri pravolinijska segmenta leže u x-z ravni Dekartovog pravouglog koordinatnog sistema, dok preostali četvrti leži u x-y ravni, kao što je prikazano na slici. Lučni segmenti leže u x-y ravni: prvi nad uglom α, poluprečnika $2a$, a drugi nad uglom $(\pi-\alpha)$, poluprečnika a. Sredina je vazduh. Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije koji u tački O (centar koordinatnog sistema) stvara žičani provodnik.

Brojne vrednosti: $\alpha = \pi/4$, $\mu_0 = 4\pi\cdot 10^{-7}$ H/m.

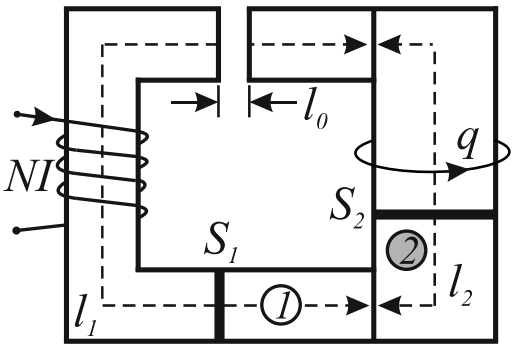


Slika 1.

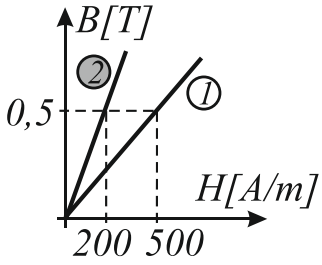
Zadatak 2. Na slici 2a je prikazano tanko magnetsko kolo, načinjeno od dva feromagnetska materijala čije su idealizovane magnetske karakteristike date na slici 2b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa $N = 100$ zavojaka u kojima je uspostavljena struja jačine I. Odrediti:

a) Intenzitet struje u pobudnom namotaju, I, pri kojoj je vektor magnetske indukcije u vazdušnom procepu jednak $B_0 = 1{,}2$ T. Rasipanja zanemariti.
b) Količinu naelektrisanja, q, koja protekne kroz probni zavojak u naznačenom smeru, prilikom uspostavljanja magnetskog polja u kolu. Zavojak ima jedan namotaj otpornosti $R_Z = 15$ Ω.
c) Magnetsku energiju sadržanu u delu kola načinjenog od materijala 1, pri struji određenoj pod a).

Brojni podaci: $l_1 = 20$ cm, $l_2 = 10$ cm, $l_0 = 1$ mm, $S_1 = 1$ cm^2, $S_2 = 2$ cm^2.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 2 (II kolokvijum)

K2

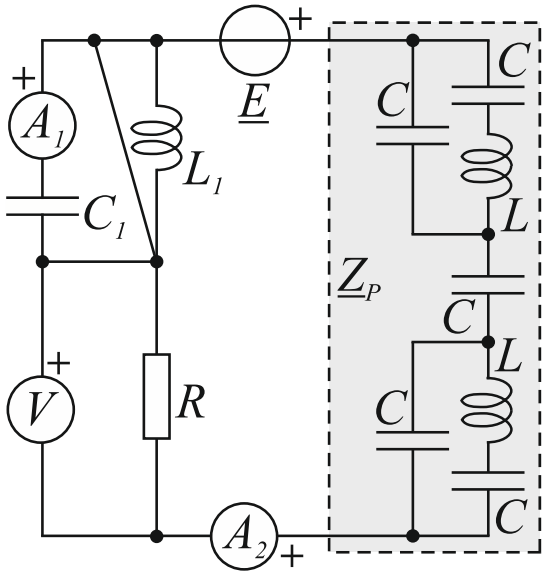
18.07.2026.

ZADACI

Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

a) Odrediti sve rezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.
b) Odrediti pokazivanja idealnih mernih instrumenata, u situaciji kada je ugaona učestanost generatora elektromotorne sile $\underline{E}$ jednaka manjoj rezonantnoj učestanosti prijemnika $\underline{Z}_P$.
c) Odrediti reaktivnu snagu generatora $\underline{E}$, pri ugaonoj učestanosti pod *b*).

Brojni podaci: $C = 100\ \mu F$, $L = 30\ mH$,
$C_1 = 4C$, $L_1 = 2L$, $R = 15\ \Omega$, $\underline{E} = (15 + j5)\ V$.

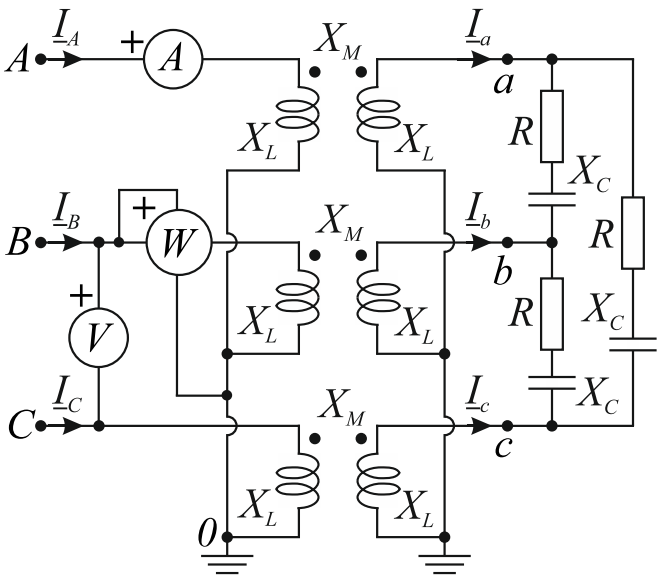


Slika 1.

Zadatak 2. Trofazni prijemnik, sastavljen od otpornika otpornosti R i kondenzatora reaktanse X_C, priključen je na sekundar trofaznog transformatora, kao što je prikazano na slici 2. Primar transformatora je priključen na trofaznu, simetričnu mrežu faznog napona prve faze $\underline{U}_A = 230\ V$.

a) Izračunati kompleksne jačine struja primara i sekundara.
b) Odrediti pokazivanja idealnih mernih instrumenata.
c) Na istom fazorskom dijagramu prikazati fazore faznih napona primara i fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.

Brojne vrednosti: $R = 15\ \Omega$, $X_C = 60\ \Omega$,
$X_L = 20\ \Omega$, $X_M = 10\ \Omega$.



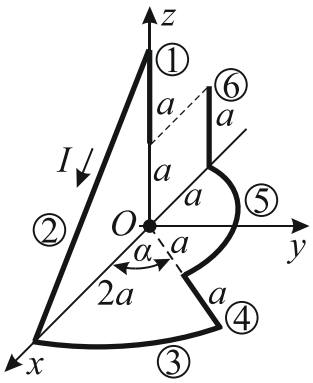
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

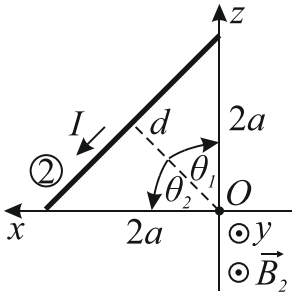
Ispit iz OET2, 18.07.2026.

I-1



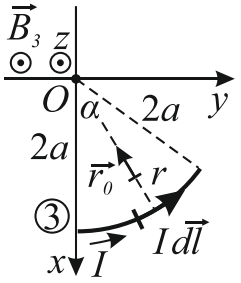
$\boxed{\vec{B}_1 = 0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$

$\boxed{\vec{B}_4 = 0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$



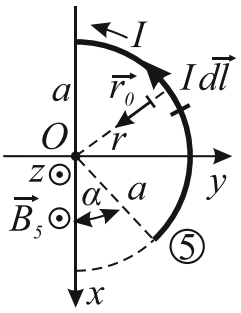
2 $\quad d = a\sqrt{2}, \qquad \theta_1 = -\frac{\pi}{4}, \qquad \theta_2 = \frac{\pi}{4}$

$$B_2 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a\sqrt{2}}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\mu_0 I}{4\pi a} \qquad \boxed{\vec{B}_2 = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_y}$$



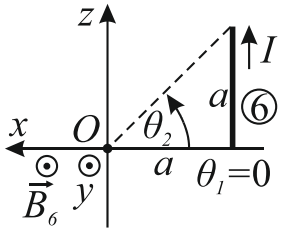
$$\vec{dB}_3 = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_3 = \frac{\mu_0}{4\pi}\frac{I\,dl}{(2a)^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2} \qquad \left(\alpha = \frac{\pi}{4}\right)$$

$$B_3 = \int dB_3 = \frac{\mu_0 I}{16\pi a^2}\int_0^{\frac{1}{8}2\cdot 2a\pi} dl = \frac{\mu_0 I}{16\pi a^2}\frac{a\pi}{2} = \frac{\mu_0 I}{32a} \qquad \boxed{\vec{B}_3 = \frac{\mu_0 I}{32a}\cdot\vec{i}_z}$$



$$\vec{dB}_5 = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_5 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2} \qquad \left(\alpha = \frac{\pi}{4}\right)$$

$$B_5 = \int dB_5 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{3}{8}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}\frac{3a\pi}{4} = \frac{3\mu_0 I}{16a} \qquad \boxed{\vec{B}_5 = \frac{3\mu_0 I}{16a}\cdot\vec{i}_z}$$



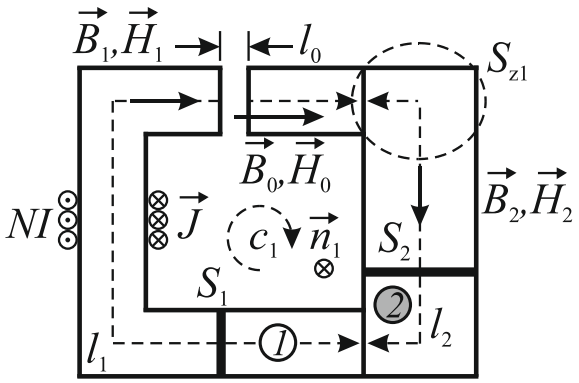
6 $\quad d = a, \qquad \theta_1 = 0, \qquad \theta_2 = \frac{\pi}{4}$

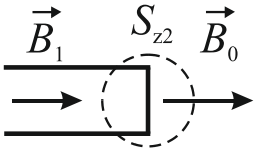
$$B_6 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left(\frac{\sqrt{2}}{2} - 0\right) = \frac{\sqrt{2}\mu_0 I}{8\pi a} \qquad \boxed{\vec{B}_6 = \frac{\sqrt{2}\mu_0 I}{8\pi a}\cdot\vec{i}_y}$$

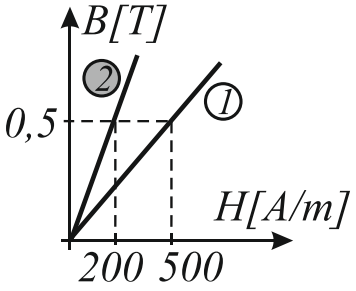
$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 + \vec{B}_6 = \left(\frac{\mu_0 I}{4\pi a} + \frac{\sqrt{2}\mu_0 I}{8\pi a}\right)\cdot\vec{i}_y + \left(\frac{\mu_0 I}{32a} + \frac{3\mu_0 I}{16a}\right)\cdot\vec{i}_z}$$

Ispit iz OET2, 18.07.2026.

I-2







a)

$$\oint_c \vec{H}\cdot\vec{dl} = NI \qquad \oint_{S_Z} \vec{B}\cdot\vec{ds} = 0$$

c_1: $\quad H_1 l_1 + H_0 l_0 + H_2 l_2 = NI \qquad (1)$

S_{z1}: $\quad -B_1 S_1 + B_2 S_2 = 0 \qquad (2)$

S_{z2}: $\quad -B_1 S_1 + B_0 S_0 = 0$

$(S_1 = S_0) \quad B_1 = B_0 \qquad (3)$

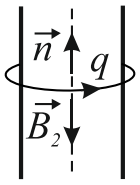
$$B_0 = 1{,}2\ T \quad \Rightarrow \quad H_0 = \frac{B_0}{\mu_0} = \frac{1{,}2}{4\pi\cdot 10^{-7}} = 955{,}41\ kA/m$$

Iz (3) sledi: $\quad B_1 = B_0 = 1{,}2\ T \quad \Rightarrow \quad H_1 = \frac{B_1}{\mu_1} = \frac{1{,}2}{\frac{0{,}5}{500}} = 1200\ A/m$

Iz (2) sledi: $\quad B_2 = B_1\frac{S_1}{S_2} = 1{,}2\cdot\frac{1\cdot 10^{-4}}{2\cdot 10^{-4}} = 0{,}6\ T \quad \Rightarrow \quad H_2 = \frac{B_2}{\mu_2} = \frac{0{,}6}{\frac{0{,}5}{200}} = 240\ A/m$

Iz (1) sledi: $\quad I = \frac{H_1 l_1 + H_0 l_0 + H_2 l_2}{N} = \frac{1200\cdot 0{,}2 + 955{,}41\cdot 10^3\cdot 1\cdot 10^{-3} + 240\cdot 0{,}1}{100}$ $\qquad \boxed{I = 12{,}2\ A}$

b)



$$q = -\frac{1}{R_Z}\left(\Phi_2^{KR} - \cancel{\Phi_2^{POC}}^{\,0}\right) = -\frac{1}{R_Z}\left(\int_{S_2}\vec{B_2}\cdot\vec{ds}\right) = -\frac{1}{R_Z}\left(-B_2 S_2\right) \qquad \sphericalangle\left(\vec{B}_2,\vec{n}\right) = \pi$$

$$q = \frac{1}{R_Z} B_2 S_2 = \frac{1}{15}\cdot 0{,}6\cdot 2\cdot 10^{-4}$$

$\boxed{q = 8\ \mu C}$

c)

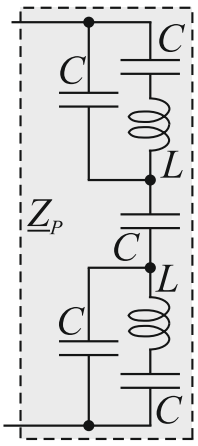
$$W_{m1} = w_{m1} V_1 = \frac{1}{2} B_1 H_1 l_1 S_1 = \frac{1}{2}\cdot 1{,}2\cdot 1200\cdot 0{,}2\cdot 1\cdot 10^{-4}$$

$\boxed{W_{m1} = 14{,}4\ mJ}$

Ispit iz OET2, 18.07.2026.

II-1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \,\|\, \frac{1}{j\omega C} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{1-\omega^2 LC}{j\omega C}+\dfrac{1}{j\omega C}} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{2-\omega^2 LC}{j\omega C}} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

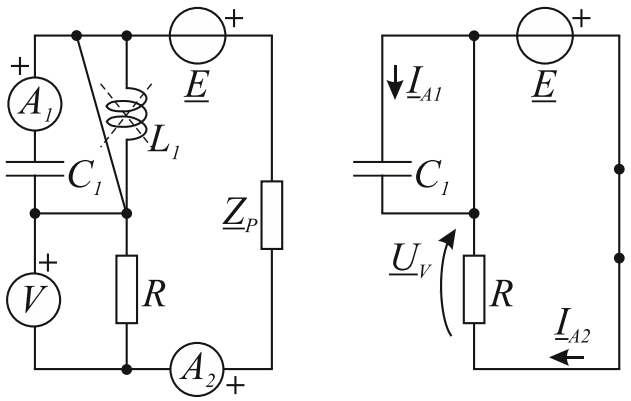
$$\underline{Z}_P = \underline{Z}_2 + \frac{1}{j\omega C} + \underline{Z}_2 = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{1}{j\omega C} + \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

$$\underline{Z}_P = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{2-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} \qquad \boxed{\underline{Z}_P = -j\frac{4-3\omega^2 LC}{\omega C\left(2-\omega^2 LC\right)}}$$

$$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad \boxed{\omega_{r1} = \sqrt{\frac{4}{3LC}}} \qquad \boxed{\omega_{r2} \to \infty}$$

$$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad \boxed{\omega_{ar1} = 0} \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$$

b)



$$\omega = \omega_{r1} = \sqrt{\frac{4}{3LC}} = \sqrt{\frac{4}{3\cdot 30\cdot 10^{-3}\cdot 100\cdot 10^{-6}}} = \frac{2000}{3}\ \frac{rad}{s}$$

$$\underline{Z}_P = 0$$

$$\underline{I}_{A1} = \frac{\underline{U}_{C1}^{\,0}}{\underline{Z}_{C1}} = 0\ A \qquad \boxed{I_{A1} = 0\ A}$$

$$\underline{I}_{A2} = \frac{\underline{E}}{R} = \frac{15+j5}{15} = \left(1+j\frac{1}{3}\right)A$$

$$I_{A2} = \left|\underline{I}_{A2}\right| = \sqrt{1^2+\left(\frac{1}{3}\right)^2} \qquad \boxed{I_{A2} = 1{,}05\ A}$$

$$\underline{U}_V = -R\,\underline{I}_{A2} = -\underline{E} = -(15+j5) = (-15-j5)\,V$$

$$U_V = \left|\underline{U}_V\right| = \sqrt{(-15)^2+(-5)^2} \qquad \boxed{U_V = 15{,}81\,V}$$

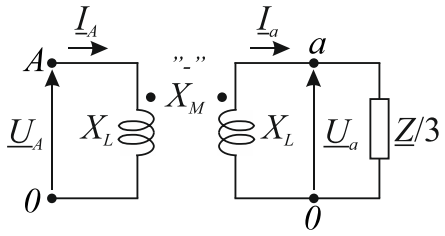
c)

$$\underline{S}_E = \underline{E}\,\underline{I}_{A2}^{\,*} = (15+j5)\cdot\left(1-j\frac{1}{3}\right) = (16{,}67+j0)\,VA \qquad \boxed{Q_E = \mathrm{Im}\{\underline{S}_E\} = 0\,VAr}$$

Ispit iz OET2, 18.07.2026.

II-2

a)



$$\underline{Z} = R - jX_C = (15 - j60)\,\Omega \qquad \frac{\underline{Z}}{3} = (5 - j20)\,\Omega$$

(1) $\underline{U}_A = jX_L\underline{I}_A - jX_M\underline{I}_a$

(2) $\underline{U}_a = -jX_L\underline{I}_a + jX_M\underline{I}_A$

(3) $\underline{U}_a = \frac{\underline{Z}}{3}\underline{I}_a$

(3) $\Rightarrow$ (2) $\quad -jX_L\underline{I}_a + jX_M\underline{I}_A = \frac{\underline{Z}}{3}\underline{I}_a \quad \Rightarrow \quad \underline{I}_a = \frac{jX_M}{\frac{\underline{Z}}{3} + jX_L}\underline{I}_A = \frac{j10}{(5 - j20) + j20}\underline{I}_A = j2\,\underline{I}_A$

(1) $\Rightarrow \quad \underline{U}_A = jX_L\underline{I}_A - jX_M \cdot j2\,\underline{I}_A = jX_L\underline{I}_A + 2X_M\underline{I}_A$

$$\underline{I}_A = \frac{\underline{U}_A}{jX_L + 2X_M} = \frac{230\,e^{j0°}}{20 + j20} = \frac{230\,e^{j0°}}{20\sqrt{2}\,e^{j45°}}$$

$$\boxed{\underline{I}_A = 8{,}13\,e^{-j45°}\ A}$$

$$\boxed{\underline{I}_B = 8{,}13\,e^{-j165°}\ A}$$
$$\boxed{\underline{I}_C = 8{,}13\,e^{-j285°}\ A}$$

$$\underline{I}_a = j2\,\underline{I}_A = 2\,e^{j90°} \cdot 8{,}13\,e^{-j45°}$$

$$\boxed{\underline{I}_a = 16{,}26\,e^{j45°}\ A}$$

$$\boxed{\underline{I}_b = 16{,}26\,e^{-j75°}\ A}$$
$$\boxed{\underline{I}_c = 16{,}26\,e^{-j195°}\ A}$$

b)

$$\boxed{I_{amp} = |\underline{I}_A| = 8{,}13\ A}$$

$$\boxed{U_V = |\underline{U}_{BC}| = \sqrt{3}\,|\underline{U}_A| = \sqrt{3} \cdot 230 = 398{,}4\ V}$$

$$P_W = \mathrm{Re}\{\underline{U}_{B0}\,\underline{I}_B^{\,*}\} = \mathrm{Re}\{\underline{U}_B\,\underline{I}_B^{\,*}\} = U_B\,I_B\cos\sphericalangle(\underline{U}_B, \underline{I}_B) = U_A\,I_A\cos(165° - 120°) = 230 \cdot 8{,}13 \cdot \cos 45°$$

$$\boxed{P_W = 1322{,}22\ W}$$

c)