STUDIJSKI PROGRAM MERENJE I REGULACIJA

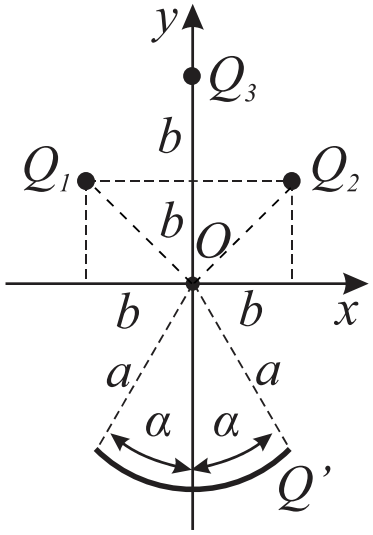
K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **14.7.2026.**

ZADACI

Zadatak 1. Tanak štap od neprovodnog materijala savijen je u obliku kružnog segmenta poluprečnika a, nad uglom 2α i postavljenu je u x-y ravan Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Struktura je ravnomerno naelektrisana linijskom gustinom naelektrisanja Q'. Dodatno, u istoj ravni, postavljena su tri tačkasta naelektrisanja Q_1, Q_2 i Q_3 kao što je prikazano na slici 1.

a) Odrediti, u opštim brojevima, izraz za vektor jačine električnog polja koji u koordinatnom početku stvara naelektrisan luk.
b) Odrediti, u opštim brojevima, izraz za vektor jačine električnog polja koji u koordinatnom početku stvaraju tačkasta naelektrisanja.
c) Odrediti količinu naelektrisanja Q', tako, da rezultantni vektor jačine električnog polja u koordinatnom početku bude jednak nuli.

Brojni podaci su: $a = 1$ cm, $b = 0{,}5a$, $\alpha = 30°$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m, $Q_1 = Q_2 = \sqrt{2}\ \mu$C, $Q_3 = 4\ \mu$C.

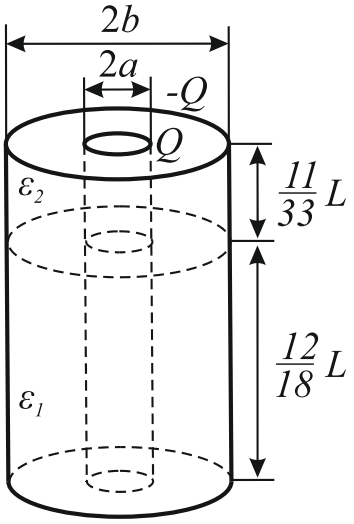


Slika 1.

Zadatak 2. Koaksijalni kabl, poluprečnika elektroda a i b, dužine L, ispunjen je sa dva sloja dielektrika; čvrstim, permitivnosti ε_1 i tečnim, permitivnosti ε_2. Relativna permitivnost, ε_{r1}, jednaka 6, dvostruko je veća od ε_{r2}, dok je električna čvrstina $E_{č2} = 75$ kV/cm dvostruko manja od $E_{č1}$. Koaksijalni kabl postavljen je u vertikalni položaj, kao što je prikazano na slici 2.

a) Odrediti zavisnost intenziteta vektora električnog pomeraja i vektora jačine električnog polja od rastojanja od ose kabla.
b) Izvesti izraz za kapacitivnost kabla.
c) Odrediti maksimalni napon na koji sme da se priključi kondenzator.
d) Odrediti energiju sadržanu u sloju od dielektrika čija je relativna permitivnost ε_{r1}.

Brojni podaci su: $a = 1$ mm, $b = 2{,}7a$, $L = 9$ cm.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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K TET

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

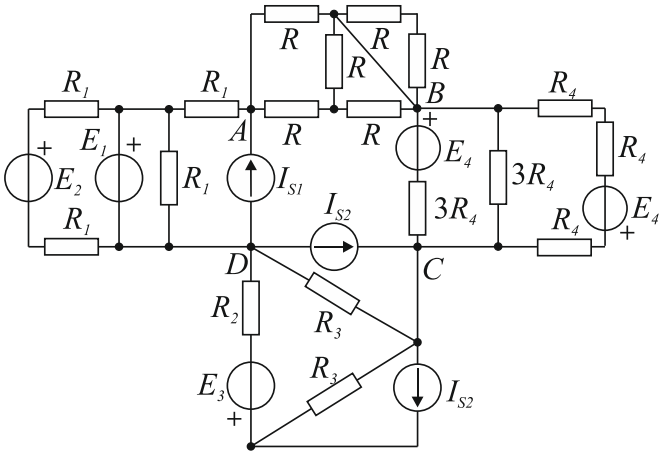
14.7.2026.

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja sa slike 1 transformisati na sledeći način:

a) Deo mreže između tačaka A i D predstaviti ekvivalentnim Tevenenovim generatorom E_{T1}, R_{T1}.
b) Prostom transformacijom generatora, deo mreže između tačaka D i C predstaviti ekvivalentnim realnim naponskim generatorom E_{T2}, R_{T2}.
c) Deo mreže između tačaka B i C predstaviti ekvivalentnim Tevenenovim generatorom E_{T3}, R_{T3}.
d) Odrediti otpornost otpornika R, tako da se na grupi otpornika, između tačaka A i B razvija najveća moguća snaga.

Brojni podaci su:
$R_1 = R_4 = 1\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 2\ \Omega$,
$E_1 = 8$ V, $E_2 = 12$ V, $E_3 = 1$ V, $E_4 = 3$ V,
$I_{S1} = 2$ A, $I_{S2} = 3$ A.

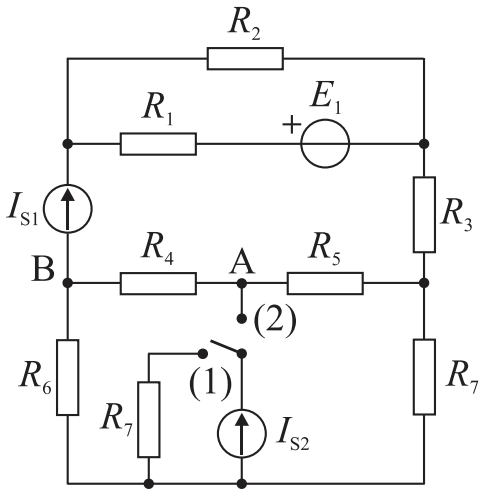


Slika 1.

Zadatak 2. Kada se u mreži vremenski konstantnih struja sa slike 2, preklopnik prebaci iz položaja (1) u položaj (2), napon na otporniku otpornosti R_4 poraste za $\Delta U_{AB} = 3$V.

a) Primenjujući teoremu superpozicije izra-čunati jačinu struje strujnog generatora I_{S2}.
b) Izračunati snage svih generatora kada je preklopnik u položaju (1).

Ostali brojni podaci su:
$R_1 = 100\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 80\ \Omega$,
$R_4 = R_7 = 50\ \Omega$, $R_5 = R_6 = 25\ \Omega$,
$E_1 = 12$ V, $I_{S1} = 80$ mA.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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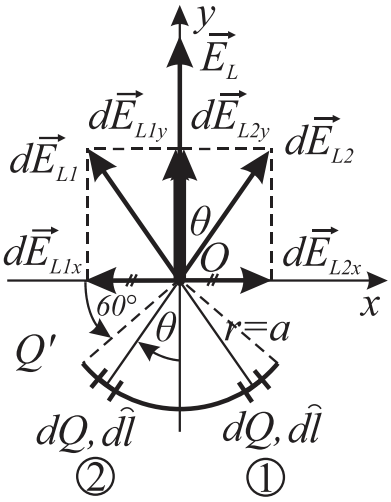
K1 Z1

a)



Zbog simetrije je:

$$d\vec{E}_{L1x} + d\vec{E}_{L2x} = 0 \quad \Rightarrow \quad \boxed{E_{Lx} = 0}$$

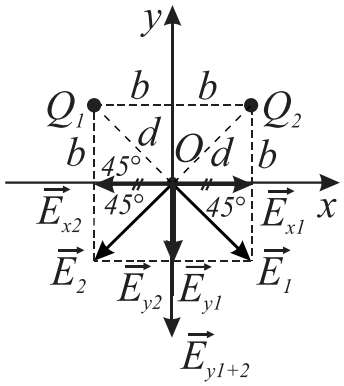
$$dE_{L1y} = dE_{L2y} = dE_{L1}\cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\cos\theta = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta$$

$$dE_{L1y} = \frac{Q'}{4\pi\varepsilon_0 a}\cos\theta\,d\theta$$

$$E_L = \int_{po\ luku} dE_{L1y} = 2\,\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{6}}\cos\theta\,d\theta = 2\,\frac{Q'}{4\pi\varepsilon_0 a}\left(\sin\frac{\pi}{6} - \sin 0\right) = 2\frac{Q'}{4\pi\varepsilon_0 a}\frac{1}{2}$$

$$\boxed{\vec{E}_L = \frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_y}$$

b)



Zbog simetrije je:

$$\vec{E}_{1x} + \vec{E}_{2x} = 0 \quad \Rightarrow \quad \boxed{E_x = 0}$$

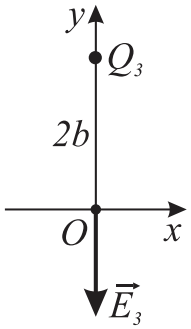
$$d = b\sqrt{2} = \frac{a\sqrt{2}}{2}$$

$$E_1 = E_2 = \frac{Q_1}{4\pi\varepsilon_0 d^2} = \frac{Q_1}{4\pi\varepsilon_0\left(\frac{a\sqrt{2}}{2}\right)^2} = \frac{Q_1}{4\pi\varepsilon_0\frac{2a^2}{4}} = \frac{Q_1}{2\pi\varepsilon_0 a^2}$$

$$E_{1y} = E_{2y} = E_1\cdot\cos\frac{\pi}{4} = \frac{Q_1}{2\pi\varepsilon_0 a^2}\cdot\frac{\sqrt{2}}{2} = \frac{Q_1\sqrt{2}}{4\pi\varepsilon_0 a^2}$$

$$\vec{E}_{y1+2} = 2\cdot\frac{Q_1\sqrt{2}}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)$$

$$\boxed{\vec{E}_{y1+2} = \frac{Q_1\sqrt{2}}{2\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)}$$



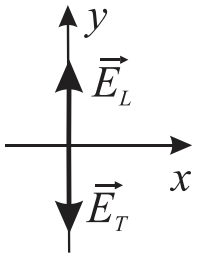
$$E_3 = \frac{Q_3}{4\pi\varepsilon_0(2b)^2}$$

$$E_3 = \frac{Q_3}{4\pi\varepsilon_0 a^2}$$

$$\boxed{\vec{E}_3 = \frac{Q_3}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right)}$$

$$\boxed{\vec{E}_T = \vec{E}_{y1+2} + \vec{E}_3 = \frac{Q_1\sqrt{2}}{2\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right) + \frac{Q_3}{4\pi\varepsilon_0 a^2}\cdot\left(-\vec{i}_y\right) = \frac{1}{2\pi\varepsilon_0 a^2}\left(Q_1\sqrt{2} + \frac{Q_3}{2}\right)\cdot\left(-\vec{i}_y\right)}$$

c)



$$\left|\vec{E}_O\right| = 0 \quad \Rightarrow \quad \vec{E}_T = -\vec{E}_L \quad \Rightarrow \quad \frac{1}{2\pi\varepsilon_0 a^2}\left(Q_1\sqrt{2} + \frac{Q_3}{2}\right) = \frac{Q'}{4\pi\varepsilon_0 a}$$

$$\frac{1}{a}\left(Q_1\sqrt{2} + \frac{Q_3}{2}\right) = \frac{Q'}{2}$$

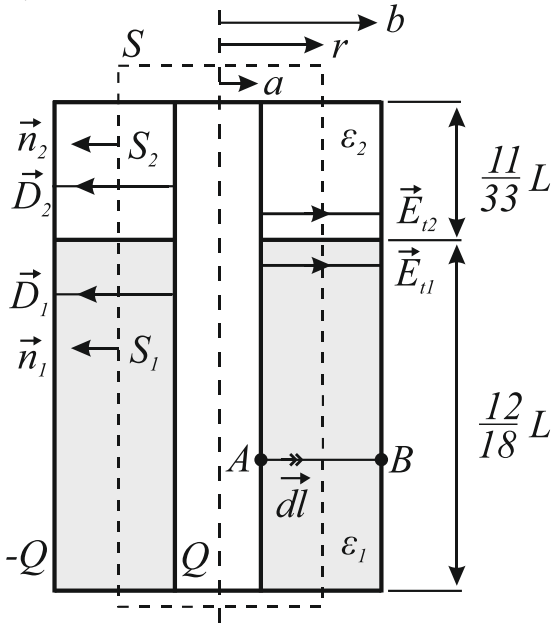
$$Q' = \frac{2}{a}\left(Q_1\sqrt{2} + \frac{Q_3}{2}\right) = \frac{2}{10^{-2}}\left(\sqrt{2}\cdot 10^{-6}\cdot\sqrt{2} + \frac{4\cdot 10^{-6}}{2}\right)$$

$$Q' = \frac{2}{10^{-2}}\left(2\cdot 10^{-6} + 2\cdot 10^{-6}\right) = \frac{2}{10^{-2}}\cdot 4\cdot 10^{-6} = 8\cdot 10^{-4}\,\frac{\mathrm{C}}{\mathrm{m}}$$

$$\boxed{Q' = 0{,}8\,\frac{\mathrm{mC}}{\mathrm{m}}}$$

K1 Z2

a)



Granični uslov:

$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$

$D_1 \neq D_2$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{u\,S}$

$\int_{S_{OM}} D\, ds = Q$

$\int_{OM_2} D_2\, ds + \int_{OM_1} D_1\, ds = Q$

$D_2\, 2\pi r \frac{1}{3}L + D_1\, 2\pi r \frac{2}{3}L = Q \qquad \left(D_1 = \varepsilon_1 E = 6\varepsilon_0 E,\ D_2 = \varepsilon_2 E = 3\varepsilon_0 E\right)$

$\varepsilon_{r2}\varepsilon_0 E\, 2\pi r \frac{1}{3}L + \varepsilon_{r1}\varepsilon_0 E\, 2\pi r \frac{2}{3}L = Q$

$$\boxed{E = \frac{Q}{\varepsilon_0 2r\pi \frac{1}{3}L\left(\varepsilon_{r2} + 2\varepsilon_{r1}\right)} = \frac{Q}{10\varepsilon_0 r\pi L}}, \qquad a \le r \le b$$

$$\boxed{D_1 = \varepsilon_1 \frac{Q}{10\varepsilon_0 r\pi L} = \varepsilon_{r1}\frac{Q}{10r\pi L}}, \qquad a \le r \le b$$

$$\boxed{D_2 = \varepsilon_2 \frac{Q}{10\varepsilon_0 r\pi L} = \varepsilon_{r2}\frac{Q}{10r\pi L}}, \qquad a \le r \le b$$

b)

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{10\varepsilon_0 r\pi L}\, dr = \frac{Q}{10\varepsilon_0\pi L}\ln\frac{b}{a}$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{10\varepsilon_0\pi L}{\ln\frac{b}{a}} = 10\varepsilon_0\pi L} \qquad \boxed{C = 25\ \text{pF}}$$

c)

$E_{č2} = 75\frac{kV}{cm} \rightarrow E_{č1} = 150\frac{kV}{cm}$

$E_{\max}(r=a) = \frac{Q_{\max}}{10\varepsilon_0 a\pi L} \le \min\left\{E_{č1}, E_{č2}\right\} = E_{č2} \rightarrow Q_{\max} = E_{č2}\cdot 10\varepsilon_0 a\pi L = 0{,}188\ \mu\text{C}$

$U_{\max} = \frac{Q_{\max}}{C} = \frac{0{,}188\ \mu\text{C}}{25\ \text{pF}} = 7500\ \text{V}$ ili $U_{\max} = \frac{Q_{\max}}{C} = \frac{E_{č2}\cdot 10\varepsilon_0 a\pi L}{10\varepsilon_0\pi L} = a\cdot E_{č2} = 7500\ \text{V}$

$\boxed{U_{\max} = 7{,}5\text{kV}}$

d)

$W_{e1} = \frac{1}{2}C_1 U_{\max}{}^2$

$C_1 = 8\varepsilon_0\pi L = 20\ \text{pF}$

$W_{e1} = \frac{1}{2}\cdot 20\cdot 10^{-12}\cdot\left(7{,}5\cdot 10^3\right)^3 = 0{,}56\cdot 10^{-3}\ \text{J}$

$\boxed{W_{e1} = 0{,}56\ \text{mJ}}$

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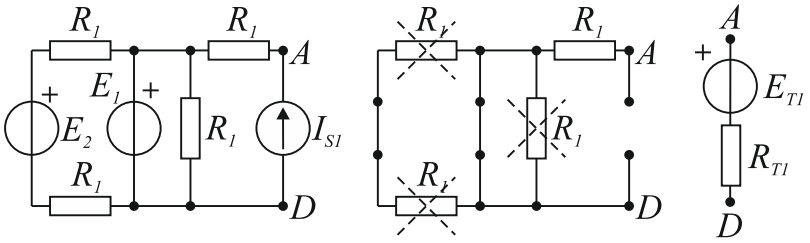
K2 Z1

a)

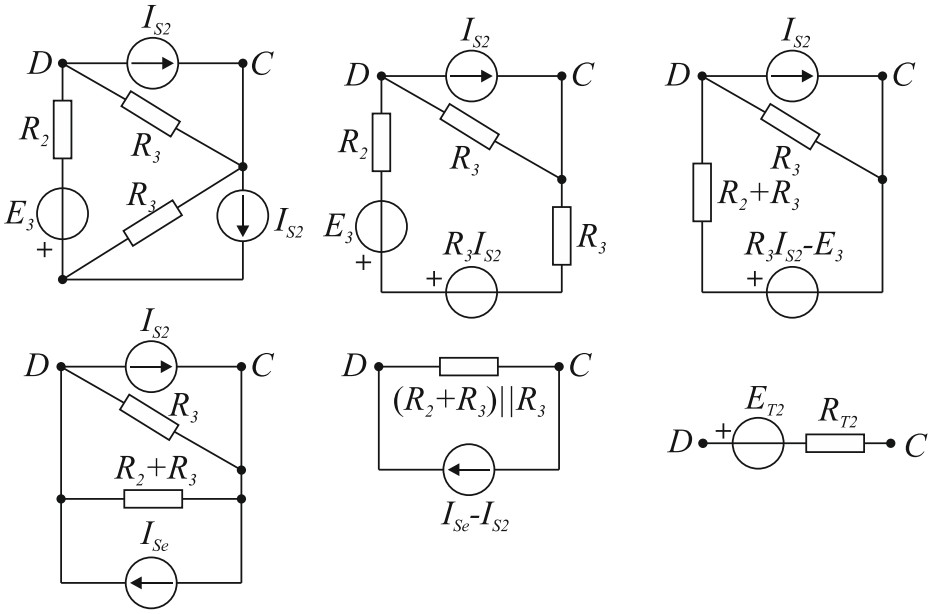


$$E_{T1} = U_{AD} = R_1 \cdot I_{S1} + E_1 = 1 \cdot 2 + 8$$

$$\boxed{E_{T2} = 10\,\text{V}}$$

$$\boxed{R_{T1} = R_1 = 1\,\Omega}$$

b)



$$(R_2 + R_3) \square R_3 = (4+2) \square 2 = \frac{6 \cdot 2}{6+2} = 1{,}5\Omega$$

$$\boxed{R_{T1} = 1{,}5\Omega}$$

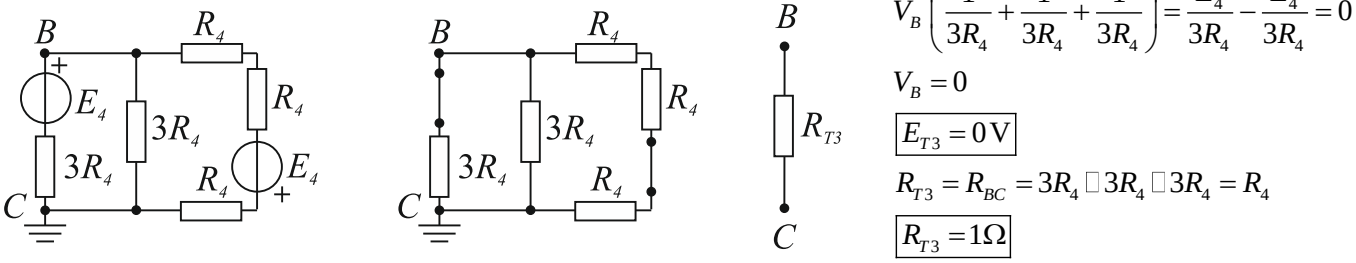
$$I_{Se} = \frac{R_3 I_{S2} - E_3}{R_2 + R_3} = \frac{2 \cdot 3 - 1}{4+2} = \frac{5}{6}\,\text{A}$$

$$I_{Se} - I_{S2} = \frac{5}{6} - 3 = -\frac{13}{6}\,\text{A}$$

$$E_{T1} = (I_{Se} - I_{S2}) \cdot (R_2 + R_3) \square R_3 = -\frac{13}{4}\,\text{V}$$

$$\boxed{E_{T1} = -3{,}25\,\text{V}}$$

c)



$$V_B \left(\frac{1}{3R_4} + \frac{1}{3R_4} + \frac{1}{3R_4} \right) = \frac{E_4}{3R_4} - \frac{E_4}{3R_4} = 0$$

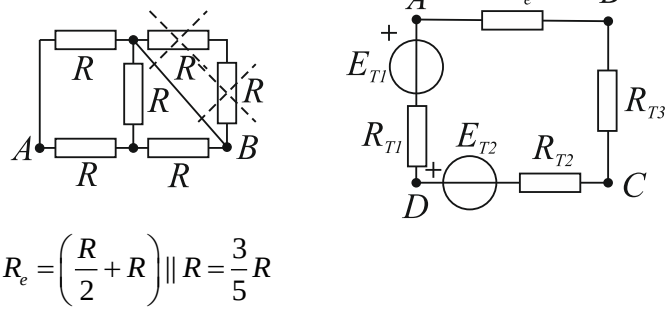
$$V_B = 0$$

$$\boxed{E_{T3} = 0\,\text{V}}$$

$$R_{T3} = R_{BC} = 3R_4 \square 3R_4 \square 3R_4 = R_4$$

$$\boxed{R_{T3} = 1\Omega}$$

d)



$$R_e = \left(\frac{R}{2} + R \right) || \, R = \frac{3}{5} R$$

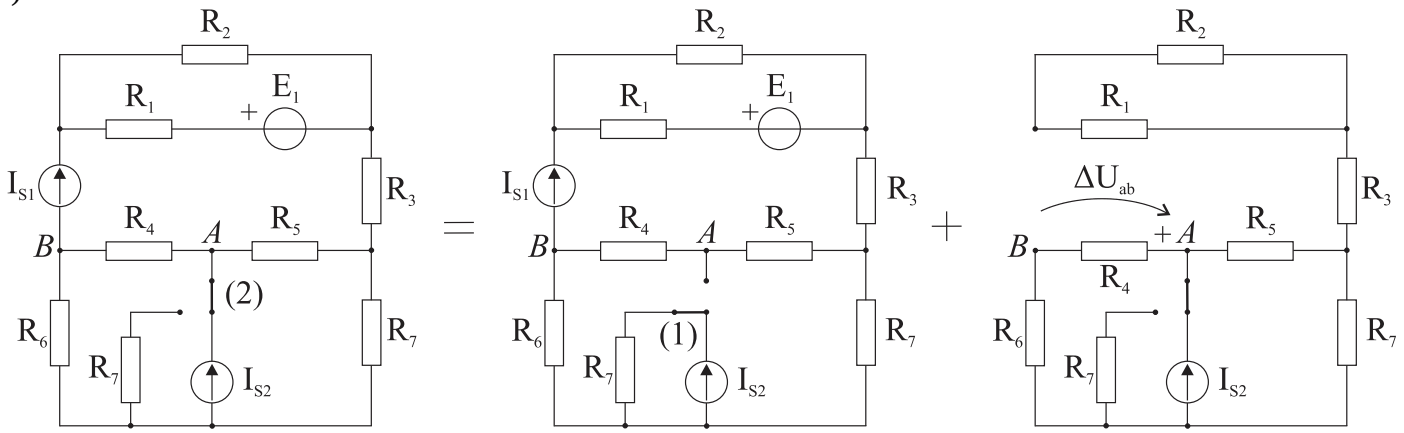
$$R_e = R_{T1} + R_{T2} + R_{T3} = 1 + 1{,}5 + 1$$

$$R_e = 3{,}5\Omega$$

$$\frac{3}{5} R = 3{,}5\Omega \quad \Rightarrow \quad \boxed{R = 5{,}83\Omega}$$

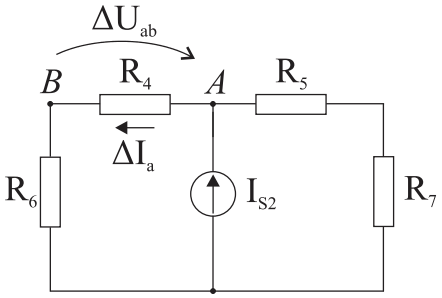
K2 Z2

a)



Položaj (2) *Položaj* (1)

$$U_{AB}^{(2)} = U_{AB}^{(1)} + \Delta U_{AB} \quad \Rightarrow \quad \Delta U_{AB} = U_{AB}^{(2)} - U_{AB}^{(1)} = 3\,\mathrm{V}$$

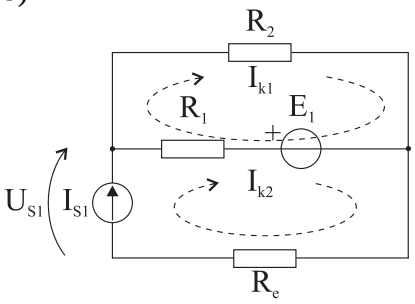


$$\Delta I_a = \frac{\Delta U_{ab}}{R_4} = 60\,mA$$

$$\Delta I_a = \frac{R_5 + R_7}{R_4 + R_5 + R_6 + R_7} I_{S2}$$

$$I_{S2} = \frac{R_4 + R_5 + R_6 + R_7}{R_5 + R_7} \Delta I_a \qquad \boxed{I_{S2} = 120\,mA}$$

b)



$$R_e = \frac{(R_4 + R_5)(R_6 + R_7)}{R_4 + R_5 + R_6 + R_7} + R_3$$

$$R_e = 117{,}5\,\Omega$$

$$(R_1 + R_2) I_{k1} - R_1 I_{k2} = E_1$$

$$I_{k2} = I_{S1}$$

$$I_{k1} = \frac{E_1 + R_1 I_{k2}}{R_1 + R_2} = 166{,}7\,mA$$

$$P_{E1} = E_1 (I_{k1} - I_{k2}) = E_1 (I_{k1} - I_{S1}) \qquad \boxed{P_{E1} = 1{,}04\,\mathrm{W}}$$

$$P_{S1} = U_{S1} I_{S1} = (R_2 I_{k1} + R_e I_{S1}) I_{S1} \qquad \boxed{P_{S1} = 1{,}02\,\mathrm{W}}$$

$$P_{S2} = U_{S2} I_{S2} = R_7 I_{S2} I_{S2} P_{S2} \qquad \boxed{P_{S2} = 0{,}72\,\mathrm{W}}$$