**STUDIJSKI PROGRAM MEHATRONIKA**

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

# Osnovi elektrotehnike 1 (I kolokvijum)

**K1**

14. 7. 2026.

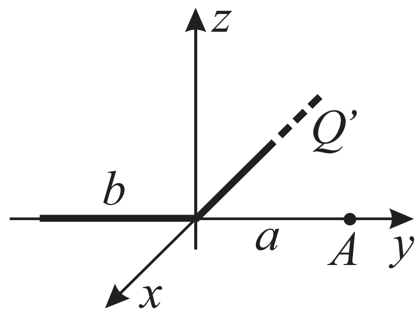
## ZADACI

**Zadatak 1.** Tanak, polubeskonačan štap, naelektrisan podužnom gustinom naelektrisanja $Q'$, savijen je i postavljen u $x$-$y$ ravan Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Prvi, polubeskonačni segment štapa leži na $x$ osi, dok drugi segment, dužine $b$, leži na $y$ osi. Sistem se nalazi u vazduhu.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački $A$ stvara štap. Tačka $A$ se nalazi na $y$ osi, na udaljenosti $a$ od centra koordinatnog sistema.

a) Izračunati brojno potencijal tačke $A$ u odnosu na referentnu tačku u beskonačnosti, koji potiče samo od pravolinijskog segmenta štapa konačne dužine.

Brojni podaci su:
$a$ = 1 cm, $b$ = 1,2 cm, $Q'$ = 5 nC/m, $\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m.




Slika 1.

**Zadatak 2.** Na slici 2 je prikazan koaksijalni kabl poluprečnika unutrašnje elektrode $a$ = 5 mm i spoljašnje $b = 2{,}7a$, dužine $l$ = 1 m. Kabl je naelektrisan podužnim naelektrisanjima $Q'$ i $-Q'$ i ispunjen sa dva čvrsta dielektrika, čije su permitivnosti $\varepsilon_1 = 3\varepsilon_0$ i $\varepsilon_2 = 4\varepsilon_0$.

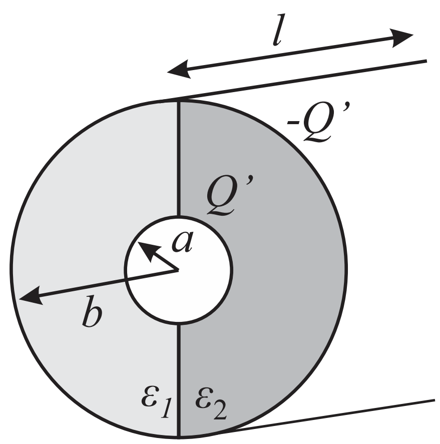
a) Odrediti zavisnost intenziteta vektora jačine električnog polja, električnog pomeraja i polarizacije u funkciji rastojanja od ose kabla.

b) Izračunati podužnu kapacitivnost kabla.

c) Odrediti maksimalni napon na koji sme da se priključi kabl.

d) Izračunati podužnu energiju sadržanu u dielektriku permitivnosti $\varepsilon_1$ kada se kabl priključi na napon koja odgovara jednoj polovini maksimalnog napona.

Električne čvrstine dielektrika iznose:
$E_{č1}$ = 35 kV/cm i $E_{č2}$ = 50 kV/cm.




Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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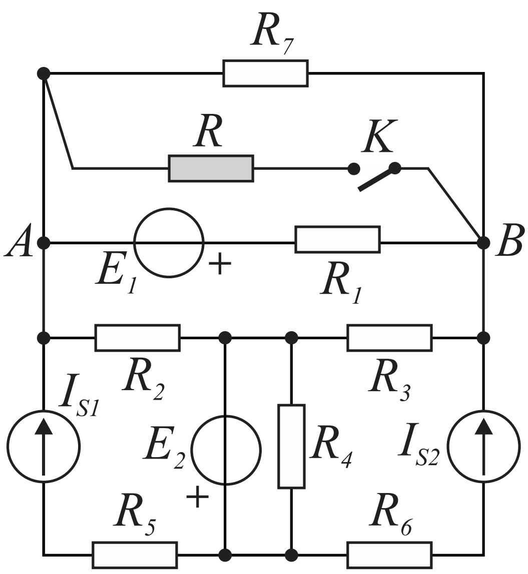
## ZADACI

**Zadatak 1.** Zatvaranjem prekidača $K$, otpornik otpornosti $R$ = 4 Ω, maksimalne snage $P_{max}$ = 200 mW, priključuje se u mrežu prikazanu na slici 1. Primenjujući Tevenenovu teoremu i metodu potencijala čvorova, proveriti da li će otpornik $R$ pregoreti nakon zatvaranja prekidača $K$.

Ostali brojni podaci su:
$R_1$ = 30 Ω, $R_2$ = $R_3$ = 15 Ω,
$R_4$ = $R_5$ = $R_6$ = 5 Ω,
$R_7$ = 30 Ω, $I_{S1}$ = 0,5 A, $I_{S2}$ = 0,4 A,
$E_1$ = 12 V, $E_2$ = 5 V.

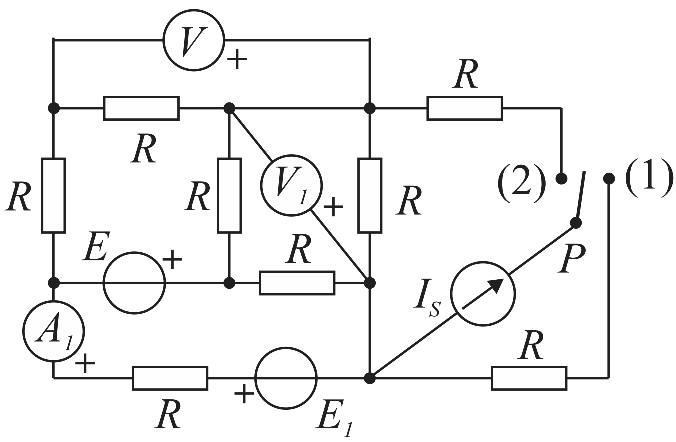



Slika 1.

**Zadatak 2.** U kolu vremenski konstantnih struja, sa slike 2, posle prebacivanja preklopnika $P$ iz položaja (1) u položaj (2), pokazivanje voltmetra se promeni za 1 V.

a) Primenom teoreme superpozicije, odrediti jačinu struje strujnog generatora $I_S$.
b) Izračunati pokazivanje idealnih mernih instrumenata kada je preklopnik u položaju (1).

Ostali brojni podaci su:
$E = E_1$ = 6,5 V, $R$ = 6,5 Ω.




Slika 2.

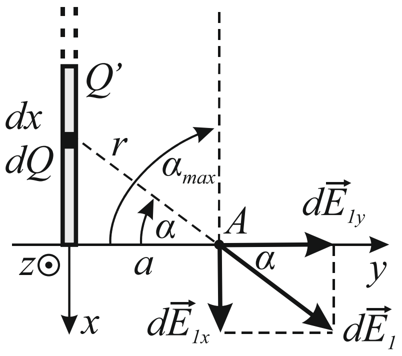
## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

# MH ISPIT, K1, 14. 7 .2026.

## K1 Z1

a)




$$\alpha_{\max}=90^\circ$$

$$dE_1=\frac{dQ}{4\pi\varepsilon_0 r^2}=\frac{Q'\,dx}{4\pi\varepsilon_0 r^2}$$

$$dE_{1x}=dE_1\sin\alpha=\frac{Q'\,dx}{4\pi\varepsilon_0 r^2}\sin\alpha=\frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\sin\alpha=\frac{Q'\dfrac{d\alpha}{\cos\alpha}}{4\pi\varepsilon_0\dfrac{a}{\cos\alpha}}\sin\alpha$$

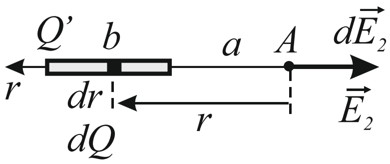
$$E_{1x}=\int dE_{1x}=\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\alpha_{\max}}\sin\alpha\,d\alpha=\frac{Q'}{4\pi\varepsilon_0 a}\left(\cos 0^\circ-\cos 90^\circ\right)=\frac{Q'}{4\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_{1x}=\frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_x}$$

$$dE_{1y}=dE_1\cos\alpha=\frac{Q'\,dx}{4\pi\varepsilon_0 r^2}\cos\alpha=\frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha=\frac{Q'\,d\alpha}{4\pi\varepsilon_0\dfrac{a}{\cos\alpha}}$$

$$E_{1y}=\int dE_{1y}=\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\alpha_{\max}}\cos\alpha\,d\alpha=\frac{Q'}{4\pi\varepsilon_0 a}\left(\sin 90^\circ-\sin 0^\circ\right)=\frac{Q'}{4\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_{1y}=\frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_y}$$




$$dE_2=\frac{dQ}{4\pi\varepsilon_0 r^2}=\frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_2=\int\limits_{\substack{po\\ štapu}}dE_2=\frac{Q'}{4\pi\varepsilon_0}\int_a^{a+b}\frac{dr}{r^2}=\frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{a+b}\right)=\frac{Q'}{4\pi\varepsilon_0}\frac{b}{a(a+b)}$$

$$\boxed{\vec{E}_2=\frac{Q'}{4\pi\varepsilon_0}\frac{b}{a(a+b)}\cdot\vec{i}_y}$$

$$\vec{E}_A=\vec{E}_1+\vec{E}_2=\frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_x+\frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_y+\frac{Q'}{4\pi\varepsilon_0}\frac{b}{a(a+b)}\cdot\vec{i}_y$$

$$\boxed{\vec{E}_A=\frac{Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_x+\left(\frac{Q'}{4\pi\varepsilon_0 a}\left(1+\frac{b}{a(a+b)}\right)\right)\cdot\vec{i}_y}$$

b)

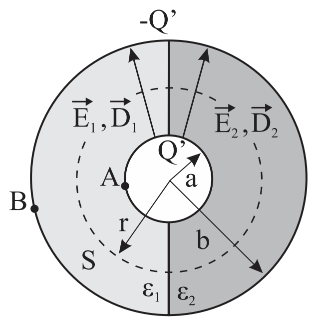
$$dV=\frac{dQ}{4\pi\varepsilon_0 r}=\frac{Q'dr}{4\pi\varepsilon_0 r}$$

$$V_A=\int\limits_{po\ štapu}dV_A=\frac{Q'}{4\pi\varepsilon_0}\int_a^{a+b}\frac{dr}{r}=\frac{Q'}{4\pi\varepsilon_0}\ln\frac{a+b}{a}$$

$$\boxed{V_A=35{,}5\,\mathrm{V}}$$

K1 Z2

a)




Granični uslov:

$$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$$
$$D_{n1} = D_{n2} = 0$$

$$D_1 = \varepsilon_1 \cdot E$$
$$D_2 = \varepsilon_2 \cdot E$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{B1}} \cancel{\vec{D} \cdot \vec{ds}}^{\,0} + \int_{S_{B2}} \cancel{\vec{D} \cdot \vec{ds}}^{\,0} + \int_{S_{OM}} \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{OM1}} D_1 \cdot ds + \int_{S_{OM2}} D_2 \cdot ds = Q' \cdot l$$

$$D_1 \cdot r\pi l + D_2 \cdot r\pi l = Q' \cdot l$$

$$\varepsilon_1 \cdot E \cdot r\pi l + \varepsilon_2 \cdot E \cdot r\pi l = Q' \cdot l$$

$$\boxed{E = \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b}$$

$$\boxed{\begin{aligned} D_1 &= \varepsilon_1 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b \\ D_2 &= \varepsilon_2 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b \end{aligned}}$$

$$P_1 = D_1 - \varepsilon_0 E = \varepsilon_1 E - \varepsilon_0 E = (\varepsilon_1 - \varepsilon_0) E = (\varepsilon_1 - \varepsilon_0) \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi} = 2\varepsilon_0 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b$$

$$P_2 = D_2 - \varepsilon_0 E = \varepsilon_2 E - \varepsilon_0 E = (\varepsilon_2 - \varepsilon_0) E = (\varepsilon_2 - \varepsilon_0) \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi} = 3\varepsilon_0 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b$$

$$\boxed{P_1 = 2\varepsilon_0 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b \qquad P_2 = 3\varepsilon_0 \frac{Q'}{\varepsilon_1 \cdot r\pi + \varepsilon_2 \cdot r\pi}; \quad a \le r \le b}$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E \cdot dr = \int_a^b \frac{Q'}{(\varepsilon_1 + \varepsilon_2) \cdot r\pi} \cdot dr = \frac{Q'}{(\varepsilon_1 + \varepsilon_2) \cdot \pi} \cdot \ln\frac{b}{a}$$

$$C' = \frac{Q'}{U_{AB}} \qquad C' = \frac{(\varepsilon_1 + \varepsilon_2) \cdot \pi}{\ln\frac{b}{a}} \qquad \boxed{C' = 194{,}52\ \frac{\mathrm{pF}}{\mathrm{m}}}$$

c)

$$E_{\max 1} = E_{\max 2} = E_{\max(r=a)} = \frac{Q_{\max}'}{(\varepsilon_1 + \varepsilon_2) \cdot a\pi} = \min\{E_{\check{c}1}, E_{\check{c}2}\} = E_{\check{c}1} = 35\ \frac{\mathrm{kV}}{\mathrm{cm}} \qquad \frac{C' \cdot U_{\max}}{(\varepsilon_1 + \varepsilon_2) \cdot a\pi} = E_{\check{c}1}$$

$$U_{\max} = \frac{E_{\check{c}1} \cdot (\varepsilon_1 + \varepsilon_2) \cdot a\pi}{C'} = \frac{E_{\check{c}1} \cdot (\varepsilon_1 + \varepsilon_2) \cdot a\pi}{\frac{(\varepsilon_1 + \varepsilon_2) \cdot \pi}{\ln\frac{b}{a}}} = E_{\check{c}1} \cdot a \cdot \ln\frac{b}{a} \qquad \boxed{U_{\max} = 17{,}5\ \mathrm{kV}}$$

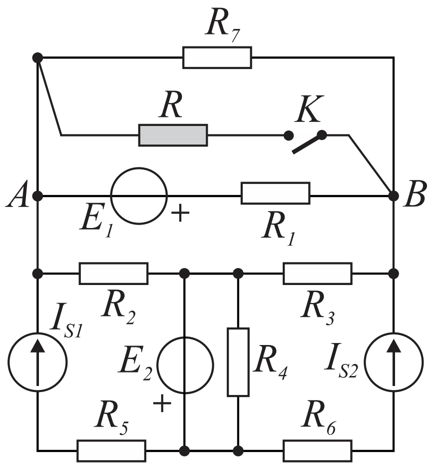
d)

$$C_1' = \frac{\varepsilon_1 \pi}{\ln\frac{b}{a}} = 83{,}37\ \frac{\mathrm{pF}}{\mathrm{m}} \qquad W_{e1}' = \frac{1}{2} C_1' \left(\frac{U_{\max}}{2}\right)^2 \qquad \boxed{W_{e1}' = 3{,}17\ \frac{\mathrm{mJ}}{\mathrm{m}}}$$

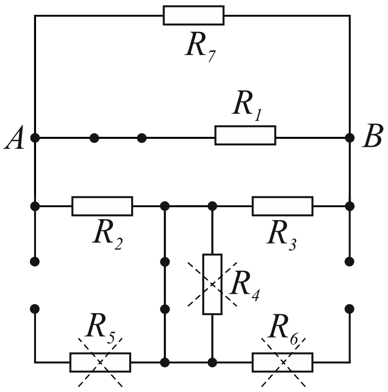
# MH ISPIT, K2, 14. 7. 2026.

K2 Z1

a)




$R_T$:



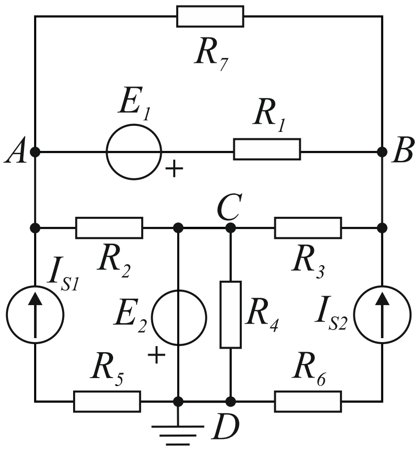

$$R_T = R_{AB} = R_7 \parallel R_1 \parallel \left(R_2 + R_3\right)$$

$$R_T = 30 \parallel 30 \parallel \left(15 + 15\right)$$

$$R_T = 30 \parallel 30 \parallel 30$$

$$\boxed{R_T = 10\,\Omega}$$

$E_T$:




$$V_D = 0\text{ V}, \qquad V_C = -E_2 = -5\text{ V}$$

$$V_A\left(\frac{1}{R_7}+\frac{1}{R_1}+\frac{1}{R_2}+\cancel{\frac{1}{R_5+\infty}}^{0}\right)-V_B\left(\frac{1}{R_7}+\frac{1}{R_1}\right)-V_C\left(\frac{1}{R_2}\right)=-\frac{E_1}{R_1}+I_{S1}$$

$$V_B\left(\frac{1}{R_7}+\frac{1}{R_1}+\frac{1}{R_3}+\cancel{\frac{1}{R_6+\infty}}^{0}\right)-V_A\left(\frac{1}{R_7}+\frac{1}{R_1}\right)-V_C\left(\frac{1}{R_3}\right)=\frac{E_1}{R_1}+I_{S2}$$

$$V_A\left(\frac{1}{30}+\frac{1}{30}+\frac{1}{15}\right)-V_B\left(\frac{1}{30}+\frac{1}{30}\right)-(-5)\cdot\left(\frac{1}{15}\right)=-\frac{12}{30}+0{,}5 \qquad /\cdot 30$$

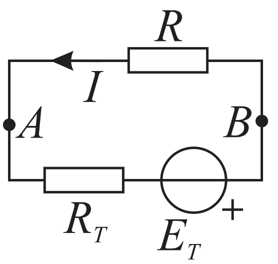
$$V_B\left(\frac{1}{30}+\frac{1}{30}+\frac{1}{15}\right)-V_A\left(\frac{1}{30}+\frac{1}{30}\right)-(-5)\cdot\left(\frac{1}{15}\right)=\frac{12}{30}+0{,}4 \qquad /\cdot 30$$

$$4V_A - 2V_B = -7 \qquad V_A = 0\text{ V}$$

$$-2V_A + 4V_B = 14 \qquad V_B = 3{,}5\text{ V}$$

$$E_T = U_{BA} = V_B - V_A$$

$$\boxed{E_T = 3{,}5\text{ V}}$$




$$I_{\max} = \sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{0{,}2}{4}} = 0{,}224\text{ A}$$

$$I = \frac{E_T}{R_T + R} = \frac{3{,}5}{10+4} = 0{,}25\text{ A} \;>\; 0{,}224\text{ A}$$

Otpornik otpornosti $R$ će pregoreti.

K2 Z2

1. Na osnovu uslova teksta zadatka: $U_V^{(2)} = U_V^{(1)} + \Delta U_V$

2. Na osnovu teoreme superpozicije:

$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_S\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_S\end{matrix}}$$

(2) (1)

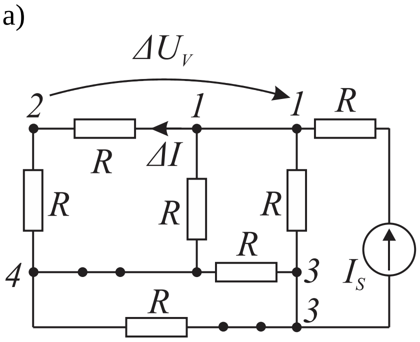
Preklopnik u položaju (2)
(deluju svi generatori)
$U_V^{(2)}$

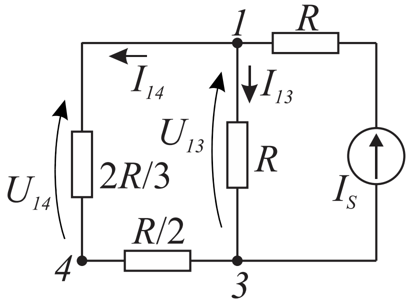
Preklopnik u položaju (1)
(svi generatori osim strujnog generotora $I_S$)
$U_V^{(1)}$

Deluje samo strujni generator $I_S$
$U_V'$

Na osnovu (1) i (2) sledi da je $U_V' = \Delta U_V$

a)




$$\Delta I = \frac{\Delta U_V}{R} = \frac{1}{6,5} = 0,154\ \text{A}$$

$$U_{14} = (R+R)I' = 13 \cdot 0,154 = 2\ \text{V}$$

$$I_{14} = \frac{U_{14}}{2R/3} = \frac{2}{13/3} = 0,462\ \text{A}$$

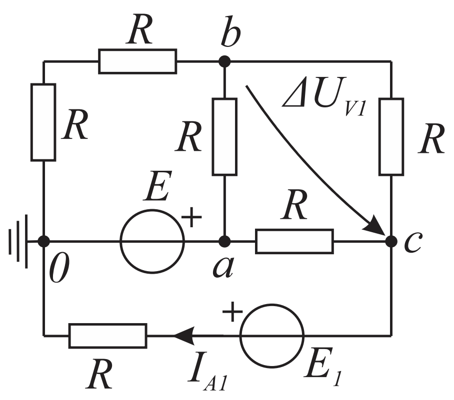
$$U_{13} = (R/2 + 2R/3)I_{14} = 3,503\ \text{V}$$

$$I_{13} = \frac{3,503}{6,5} = \frac{2}{13/3} = 0,538\ \text{A}$$

$$I_S = I_{13} + I_{14} = 0,538 + 0,462$$

$$\boxed{I_S = 1\ \text{A}}$$

b)




$n_č = 4,\quad n_{i.n.g.} = 1$

$MPČ:\quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$

$V_0 = 0\ \text{V},\qquad V_a = E = 6,5\ \text{V}$

$$V_b\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{2R}\right) - V_a\left(\frac{1}{R}\right) - V_c\left(\frac{1}{R}\right) = 0 \qquad / \cdot 2R$$

$$V_c\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R+0}\right) - V_a\left(\frac{1}{R}\right) - V_b\left(\frac{1}{R}\right) = -\frac{E_1}{R} \qquad / \cdot R$$

$$5V_b - 2V_a - 2V_c = 0$$

$$3V_c - V_a - V_b = -E_1$$

$$\left.\begin{matrix}5V_b - 2V_c = 13\\ 3V_c - V_b = 0\end{matrix}\right\} \Rightarrow \quad V_b = 3\ \text{V} \qquad V_c = 1\ \text{V}$$

$$I_{A1} = \frac{V_c - V_0 + E_1}{R} = 1{,}15\ \text{A}$$

$$\boxed{I_{A1} = 1{,}15\ \text{A}}$$

$$U_{V1} = V_c - V_b$$

$$\boxed{U_{V1} = -2\ \text{V}}$$