STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

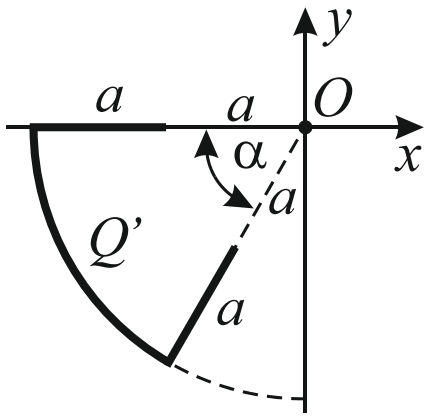
K1

14.07.2026.

ZADACI

Zadatak 1. Tanak štap, od izolacionog materijala, savijen kao što je prikazano na slici 1, naelektrisan je ravnomerno podužnom gustinom naelektrisanja Q'. Štap se sastoji od dela u obliku luka, poluprečnika $2a$, i dva pravolinijska segmenta dužina a. Struktura se nalazi u x-y ravni zadatog koordinatnog sistema. Izvesti u opštim brojevima izraz za vektor jačine električnog polja u tački O (centar koordinatnog sistema), koji potiče od štapa.

Brojni podaci su: $\alpha = \pi/3$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m.

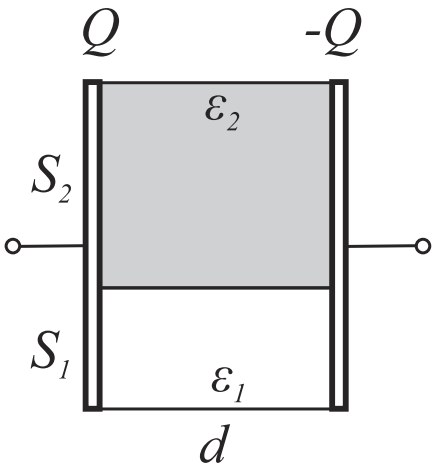


Slika 1.

Zadatak 2. Na slici 2 je prikazan pločasti kondenzator ispunjen sa dva sloja dielektrika, relativnih permitivnosti ε_{r1} i $\varepsilon_{r2} = 7{,}5$. Rastojanje između elektroda je $d = 0{,}55$ mm, a njihova površina $S = 6$ cm^2.

a) Izvesti, u opštim brojevima, izraz za kapacitivnost kondenzatora.
b) Odrediti relativnu permitivnost drugog dielektrika, ε_{r1}, tako da kapacitivnost prvog sloja bude tri puta manja od kapacitivnosti drugog sloja.
c) Izračunati maksimalni napon na koji sme da se priključi kondenzator.
d) Izračunati količinu vezanog naelektrisanja na razdvojnoj površi dva dielektrika.

Brojni podaci su: $S_2 = 2S_1$, $E_{Č1} = 43$ kV/cm, $E_{Č2} = 64$ kV/cm.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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K T E T Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **14.07.2026.**

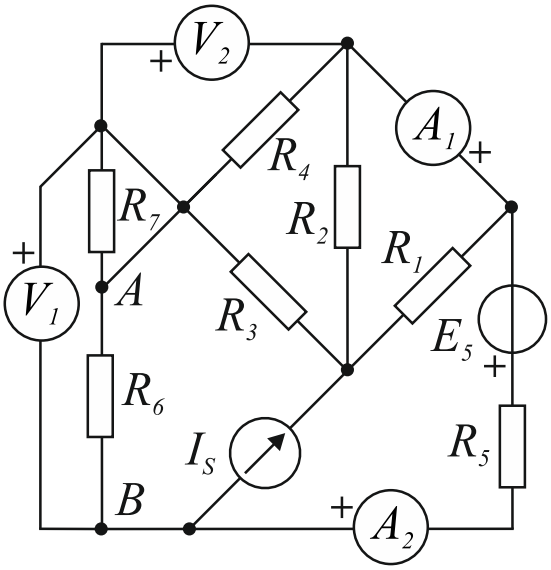
ZADACI

Zadatak 1. U kolu vremenski konstantnih struja, sa slike 1, poznate su brojne vrednosti svih elemenata, osim otpornosti otpornika R_6.

a) Primenjujući teoremu o kompenzaciji, izračunati otpornost otpornika R_6, ako se zna da je pokazivanje prvog voltmetra $U_{V1} = 6\ V$. Prilikom rešavanja kola primeniti metodu potencijala čvorova.

b) Izračunati pokazivanja ostalih mernih instrumenata (ampermetri A_1 i A_2 i voltmetar V_2), kao i snagu naponskog generatora E_5, kada otpornik R_6 ima otpornost izračunatu pod *a*).

Brojni podaci su: $R_1 = R_2 = 10\ \Omega$, $R_3 = R_4 = R_5 = R_7 = 20\ \Omega$, $E_5 = 12\ V$, $I_S = 2\ A$.



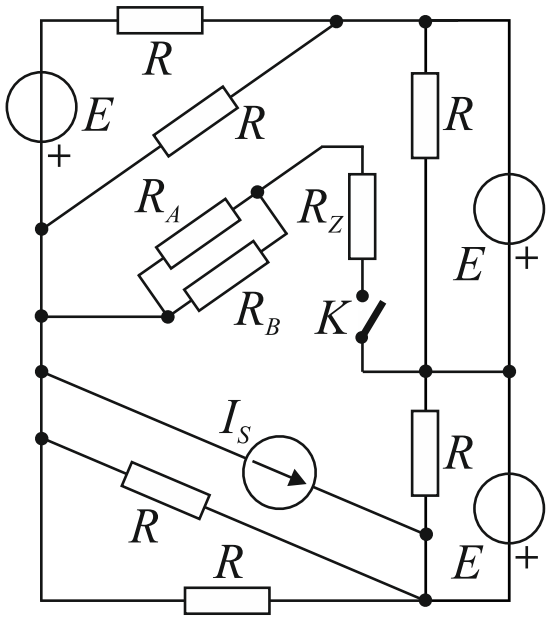
Slika 1.

Zadatak 2. Otpornici otpornosti $R_A = 1\ k\Omega$ i $R_B = 2\ k\Omega$, maksimalnih snaga $P_{Amax} = P_{Bmax} = 1\ mW$, se zatvaranjem prekidača K priključuju u kolo vremenski konstantnih struja, prikazano na slici 2.

a) Primenjujući Tevenenovu teoremu, proveriti da li će neki od otpornika R_A i R_B pregoreti nakon zatvaranja prekidača K.

b) Ukoliko se desi da neki od otpornika pregori, odrediti otpornost zaštitnog otpornika, R_Z, koji treba vezati na red sa njima, kako bi otpornike zaštitio od pregorevanja.

Brojni podaci su: $E = 10\ V$, $I_S = 1\ mA$, $R = 1\ k\Omega$.



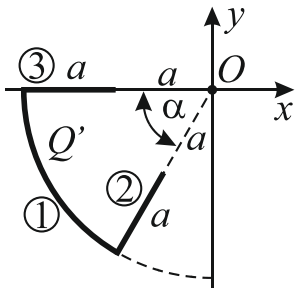
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 14.07.2026.

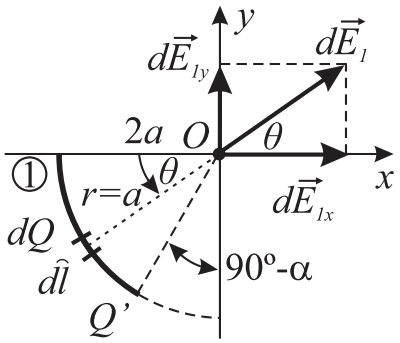
I-1



$$dE_{1x} = dE_1 \cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 (2a)^2}\cos\theta = \frac{Q'\,2a\,d\theta}{16\pi\varepsilon_0 a^2}\cos\theta$$

$$E_{1x} = \int\limits_{\substack{po\\luku\,1}} dE_{1x} = \frac{Q'}{8\pi\varepsilon_0 a}\int_0^{\alpha}\cos\theta\,d\theta = \frac{Q'}{8\pi\varepsilon_0 a}\left(\sin\frac{\pi}{3} - \sin 0\right) = \frac{Q'}{8\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}$$

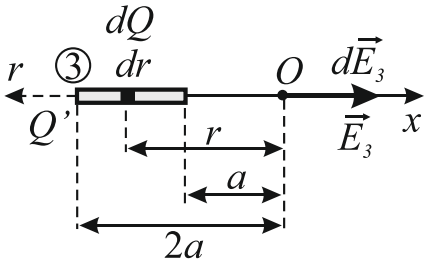
$$\boxed{\vec{E}_{1x} = \frac{\sqrt{3}\,Q'}{16\pi\varepsilon_0 a}\cdot\vec{i}_x}$$



$$dE_{1y} = dE_1 \sin\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 (2a)^2}\sin\theta = \frac{Q'\,2a\,d\theta}{16\pi\varepsilon_0 a^2}\sin\theta$$

$$E_{1y} = \int\limits_{\substack{po\\luku\,1}} dE_{1y} = \frac{Q'}{8\pi\varepsilon_0 a}\int_0^{\alpha}\sin\theta\,d\theta = \frac{Q'}{8\pi\varepsilon_0 a}\left(\cos 0 - \cos\frac{\pi}{3}\right) = \frac{Q'}{8\pi\varepsilon_0 a}\left(1 - \frac{1}{2}\right)$$

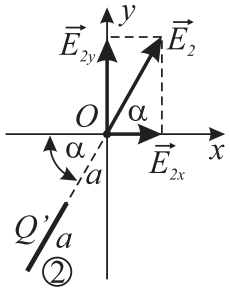
$$\boxed{\vec{E}_{1y} = \frac{Q'}{16\pi\varepsilon_0 a}\cdot\vec{i}_y}$$



$$dE_3 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

$$E_3 = \int\limits_{\substack{po\\štapu}} dE_3 = \frac{Q'}{4\pi\varepsilon_0}\int_a^{2a}\frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{Q'}{4\pi\varepsilon_0}\frac{1}{2a} = \frac{Q'}{8\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_3 = \frac{Q'}{8\pi\varepsilon_0 a}\cdot\vec{i}_x}$$



$$E_2 = E_3 = \frac{Q'}{8\pi\varepsilon_0 a}$$

$$E_{2x} = E_2\cos\alpha = \frac{Q'}{8\pi\varepsilon_0 a}\frac{1}{2} = \frac{Q'}{16\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_{2x} = \frac{Q'}{16\pi\varepsilon_0 a}\cdot\vec{i}_x}$$

$$E_{2y} = E_2\sin\alpha = \frac{Q'}{8\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,Q'}{16\pi\varepsilon_0 a}$$

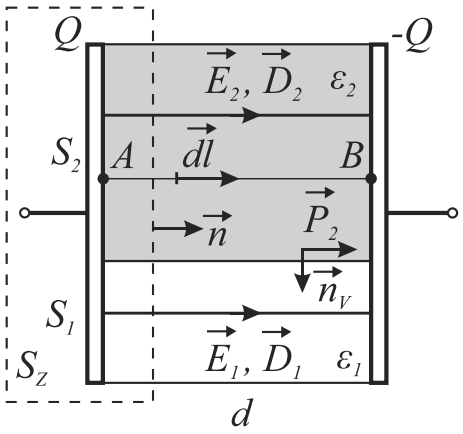
$$\boxed{\vec{E}_{2y} = \frac{\sqrt{3}\,Q'}{16\pi\varepsilon_0 a}\cdot\vec{i}_y}$$

$$\boxed{\vec{E}_O = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \left(\frac{\sqrt{3}\,Q'}{16\pi\varepsilon_0 a} + \frac{Q'}{16\pi\varepsilon_0 a} + \frac{Q'}{8\pi\varepsilon_0 a}\right)\cdot\vec{i}_x + \left(\frac{Q'}{16\pi\varepsilon_0 a} + \frac{\sqrt{3}\,Q'}{16\pi\varepsilon_0 a}\right)\cdot\vec{i}_y}$$

Ispit iz OET1, 14.07.2026.

I-2

a)



Granični uslov:

$E_{t1} = E_{t2} = E, \qquad E_1 = E_2 = E$

$D_{n1} = D_{n2} = 0$

$$\oint_S \vec{D} \cdot \vec{dS} = Q_{slobodno\ u\ S}$$

$$\int_{S_1} D_1 \cdot dS + \int_{S_2} D_2 \cdot dS = Q$$

$$D_1 S_1 + D_2 S_2 = Q$$

$$\varepsilon_1 E S_1 + \varepsilon_2 E S_2 = Q$$

$$\boxed{E = \frac{Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2}}$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = E\,d = \frac{Q}{\varepsilon_1 S_1 + \varepsilon_2 S_2} d$$

$$C = \frac{Q}{U_{AB}} \qquad \boxed{C = \frac{\varepsilon_1 S_1 + \varepsilon_2 S_2}{d} = \frac{\varepsilon_1 S_1}{d} + \frac{\varepsilon_2 S_2}{d} = C_1 + C_2}$$

b)

$$C_1 = \frac{\varepsilon_1 S_1}{d}, \qquad C_2 = \frac{\varepsilon_2 S_2}{d}$$

$$C_1 = \frac{C_2}{3}, \qquad C_2 = 3C_1$$

$$\frac{\varepsilon_2 S_2}{d} = 3\frac{\varepsilon_1 S_1}{d} \qquad (S_2 = 2S_1)$$

$$\varepsilon_1 = \frac{2}{3}\varepsilon_2 = \frac{2}{3} \cdot 7{,}5\varepsilon_0 = 5\varepsilon_0 \qquad \boxed{\varepsilon_{r1} = 5}$$

c)

$$E_{\max} = \frac{Q_{\max}}{\varepsilon_1 S_1 + \varepsilon_2 S_2} \le \min\left\{E_{Č1}, E_{Č2}\right\} = E_{Č1} \qquad \Rightarrow \qquad Q_{\max} = E_{Č1}\left(\varepsilon_1 S_1 + \varepsilon_2 S_2\right)$$

$$U_{\max} = \frac{Q_{\max}}{\varepsilon_1 S_1 + \varepsilon_2 S_2} d = \frac{E_{Č1}\left(\varepsilon_1 S_1 + \varepsilon_2 S_2\right)}{\varepsilon_1 S_1 + \varepsilon_2 S_2} d = E_{Č1}\,d \qquad \boxed{U_{\max} = 2{,}37\ kV}$$

d)

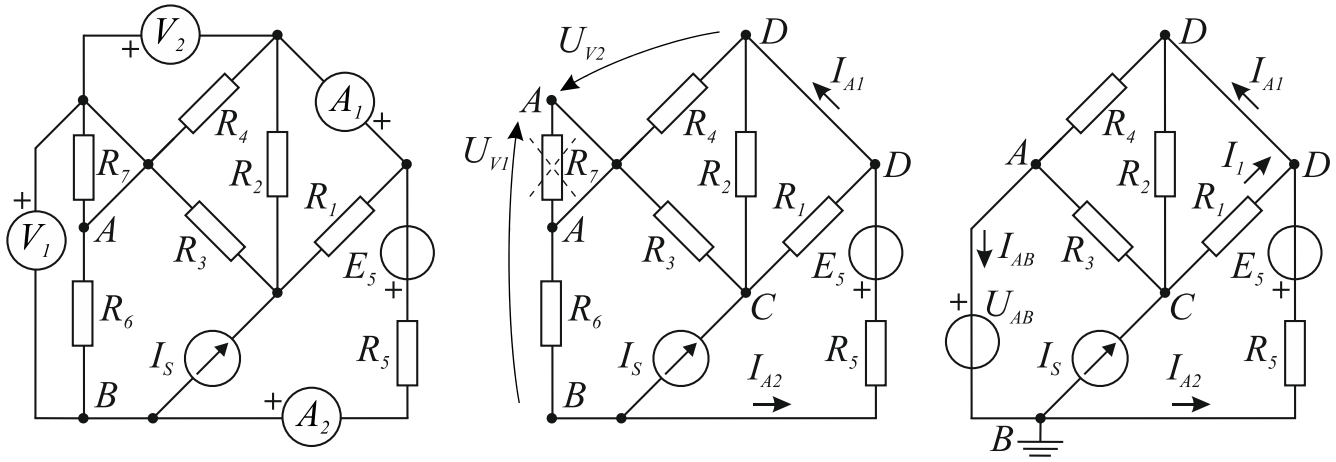
$$\sigma_V = \vec{P}_2 \cdot \vec{n}_V = P_2 \cdot 1 \cdot \cos 90° = 0\ \frac{C}{m^2}$$

$$\boxed{Q_V = 0\ C}$$

Ispit iz OET1, 14.07.2026.

II-1

a)



$$V_B = 0\,V, \qquad V_A = U_{AB} = U_{V1} = 6\,V$$

$$V_C\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{\infty}\right)-V_A\left(\frac{1}{R_3}\right)-V_D\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=I_S$$

$$V_D\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5+0}\right)-V_A\left(\frac{1}{R_4}\right)-V_C\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=-\frac{E_5}{R_5}$$

$$V_C\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{20}\right)-6\cdot\left(\frac{1}{20}\right)-V_D\left(\frac{1}{10}+\frac{1}{10}\right)=2$$

$$V_D\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{20}+\frac{1}{20}\right)-6\cdot\left(\frac{1}{20}\right)-V_C\left(\frac{1}{10}+\frac{1}{10}\right)=-\frac{12}{20}$$

$$\left.\begin{aligned}5V_C-4V_D&=46\\-4V_C+6V_D&=-6\end{aligned}\right\} \quad\Rightarrow\quad V_C = 18\,V \qquad V_D = 11\,V$$

$$I_{A2} = \frac{V_B - V_D - E_5}{R_5} = \frac{0-11-12}{20} = -1{,}15\ A$$

$$I_{AB} = I_S + I_{A2} = 2 + (-1{,}15) = 0{,}85\ A$$

$$R_6 = \frac{U_{AB}}{I_{AB}} = \frac{6}{0{,}85} \qquad \boxed{R_6 = 7{,}1\,\Omega}$$

b)

$$U_{V2} = U_{AD} = V_A - V_D = 6 - 11 \qquad \boxed{U_{V2} = -5\,V}$$

$$\boxed{I_{A2} = -1{,}15\ A}$$

$$I_1 = \frac{V_C - V_D}{R_1} = \frac{18-11}{10} = 0{,}7\ A$$

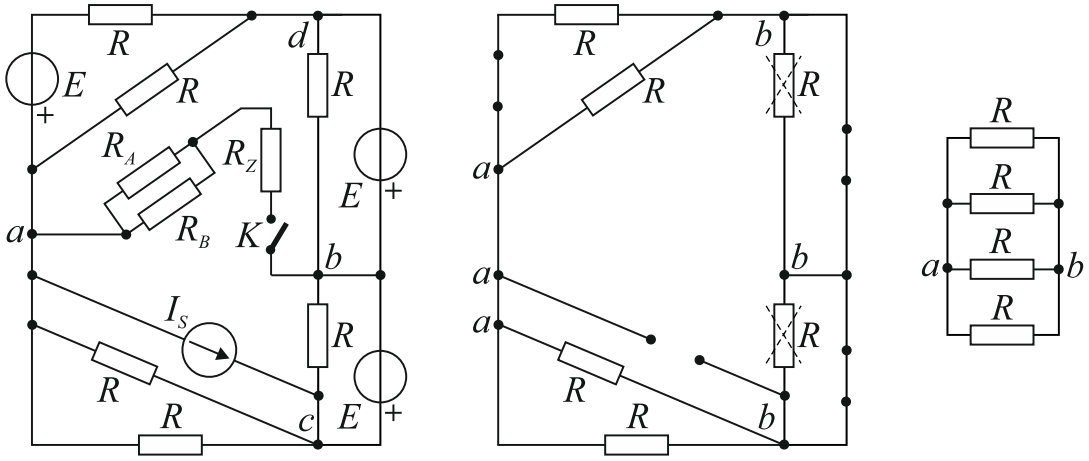
$$I_{A1} = I_1 + I_{A2} = 0{,}7 + (-1{,}15) \qquad \boxed{I_{A1} = -0{,}45\ A}$$

$$P_{E5} = -E_5\ I_{A2} = -12\cdot(-1{,}15) \qquad \boxed{P_{E5} = 13{,}8\,W}$$

Ispit iz OET1, 14.07.2026.

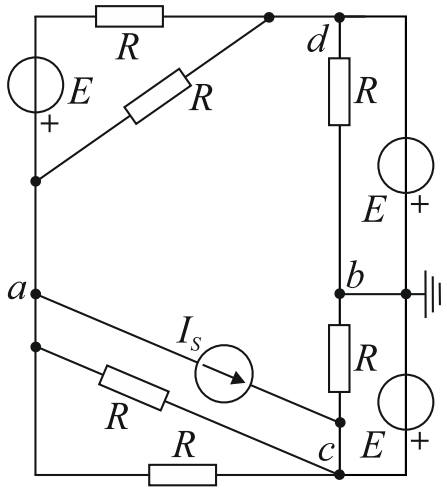
II-2

a)



$$R_T = R_{ab} = R \,||\, R \,||\, R \,||\, R = 1000 \,||\, 1000 \,||\, 1000 \,||\, 1000$$

$$\boxed{R_T = 250\,\Omega}$$



$$V_b = 0\,V, \qquad V_c = E = 10\,V, \qquad V_d = -E = -10\,V$$

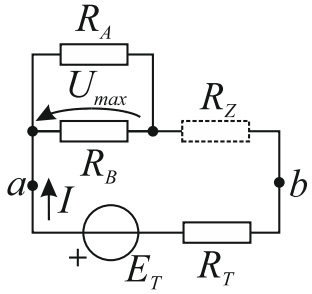
$$V_a\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{\infty}+\frac{1}{R}+\frac{1}{R}\right) - V_c\left(\frac{1}{\infty}+\frac{1}{R}+\frac{1}{R}\right) - V_d\left(\frac{1}{R}+\frac{1}{R}\right) = -I_S + \frac{E}{R}$$

$$V_a\left(\frac{1}{1k}+\frac{1}{1k}+\frac{1}{1k}+\frac{1}{1k}\right) - 10\cdot\left(\frac{1}{1k}+\frac{1}{1k}\right) - (-10)\cdot\left(\frac{1}{1k}+\frac{1}{1k}\right) = -1m + \frac{10}{1k}$$

$$4V_a = 9 \quad \Rightarrow \quad V_a = 2{,}25\,V$$

$$E_T = U_{ab} = V_a - V_b = 2{,}25 - 0$$

$$\boxed{E_T = 2{,}25\,V}$$



$$\left.\begin{aligned} U_{A\max} &= \sqrt{P_{A\max}\, R_A} = \sqrt{1m \cdot 1k} = 1\,V \\ U_{B\max} &= \sqrt{P_{B\max}\, R_B} = \sqrt{1m \cdot 2k} = 1{,}41\,V \end{aligned}\right\} \quad \Rightarrow \quad U_{\max} = U_{A\max} = 1\,V$$

$$I_{\max} = \frac{U_{\max}}{R_{AB}} = \frac{U_{\max}}{R_A \,||\, R_B} = \frac{1}{2k/3} = 1{,}5\,mA$$

$$I = \frac{E_T}{R_T + R_{AB}} = \frac{2{,}25}{250 + 2000/3} = 2{,}45\,mA \;>\; I_{\max} = 1{,}5\,mA$$

Treba dodati R_Z

b)

$$I' = \frac{E_T}{R_T + R_{AB} + R_Z} \le I_{\max}$$

$$R_Z \ge \frac{E_T}{I_{\max}} - R_T - R_{AB} = \frac{2{,}25}{1{,}5\,m} - 250 - \frac{2000}{3}$$

$$\boxed{R_Z \ge 583{,}33\,\Omega}$$