STUDIJSKI PROGRAM MERENJE I REGULACIJA

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

12.7.2026.

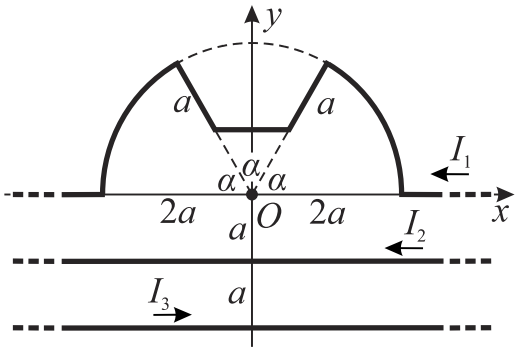
ZADACI

Zadatak 1. Na slici 1 je prikazan tanak, veoma dugačak, žičani provodnik sa vremenski konstantnom strujom jačine I_1. Provodnik se sastoji iz pet pravolinijskih segmenata i dva lučna segmenta, poluprečnika $2a$. Ceo provodnik leži u x-y ravni zadatog koordinatnog sistema. Sredina je vazduh.

a) Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije u tački O (koordinatni početak).

b) Izračunati intenzitete vremenski konstantnih struja I_2 i I_3, kroz beskonačno dugačke žičane provodnike postavljene u x-y ravni kao na slici 1, tako da ukupni vektor magnetske indukcije u tački O bude jednak nuli.

Brojne vrednosti su: $I_1 = 1$ A, $a = 1$ cm, $I_2 = 1{,}5 \cdot I_3$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



Slika 1.

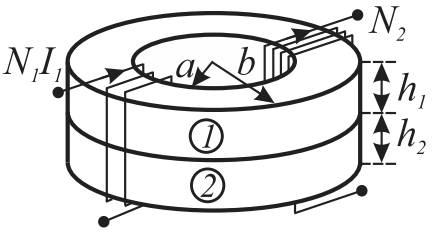
Zadatak 2. Na slici 2a je prikazano debelo torusno jezgro, načinjeno od dva materijala, čije su magnetske karakteristike date na slici 2b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa $N_1 = 628$ zavojaka u kojima je uspostavljena vremenski konstantna struja jačine I_1. Dimenzije jezgra su: $a = 4$ cm, $b = 12$ cm, $h_1 = 3$ cm, $h_2 = 3$ cm. Odrediti:

a) minimalnu jačinu pobudne struje, I_1, tako da deo jezgra načinjen od materijala 1 bude ceo u zasićenju,

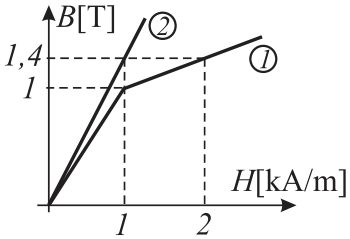
b) režim rada jezgra ako se pobudna struja I_1 smanji 3 puta,

c) fluks kroz poprečni presek jezgra, nakon što se pobudna struja smanjila,

d) **(Bonus – 5p)** elektromotornu silu na krajevima namotaja sa $N_2 = 10$ zavojaka, ako su njegovi krajevi otvoreni, a pobudni namotaj priključen na izvor naizmenične struje $i(t) = I_m \cos\omega t$, gde je amplituda I_m jednaka jačina struje određene pod b). Kružna učestanost izvora je $\omega = 1000$ rad/s.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 2 (II kolokvijum)

K2

12.7.2026.

ZADACI

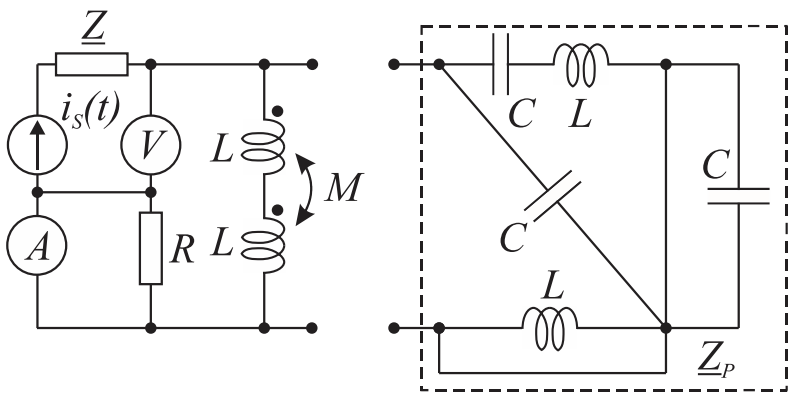
Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Ako je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka manjoj rezonantnoj učestanosti prijemnika, odrediti pokazivanja idealnih mernih instrumenata.

c) **(Bonus – 5p)** Odrediti pokazivanje voltmetra, u slučaju da je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka većoj antirezonantnoj učestanosti prijemnika.

Brojni podaci su: $i_S(t) = 2\cdot\cos(\omega t + \pi/4)$ A,
$R = 50\ \Omega$, $C = 500$ nF,
$L = 20$ mH, $M = 10$ mH,
$\underline{Z} = (50 + j100)\ \Omega$.



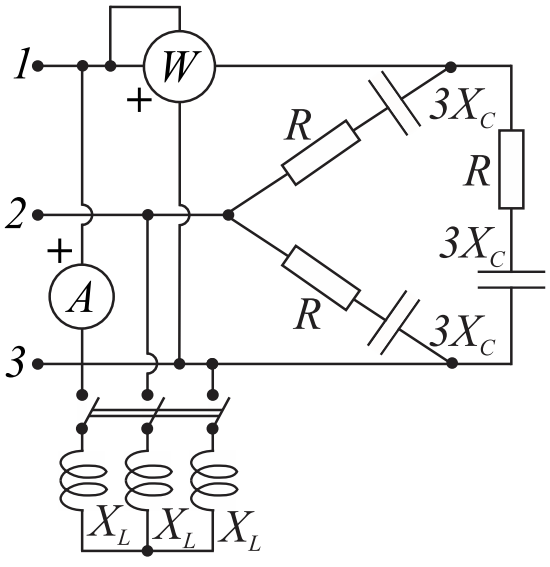
Slika 1.

Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik, sastavljen od otpornika otpornosti $R = 150\ \Omega$ i kondenzatora reaktanse $X_C = 150\ \Omega$. Prijemnik je priključen na mrežu faznog napona $\underline{U}_1 = 230$ V.

a) Odrediti reaktanse kalemova, koje je potrebno povezati, kako bi se faktor snage prijemnika popravio na jedinicu.

b) Odrediti pokazivanja idealnih mernih instrume-nata pre i posle priključivanja kalemova.

c) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instru-menata.



Slika 2.

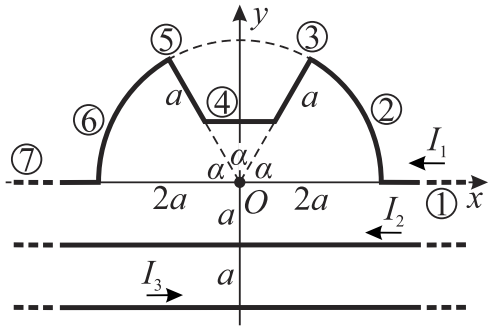
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MiR ISPIT 12.7.2026.

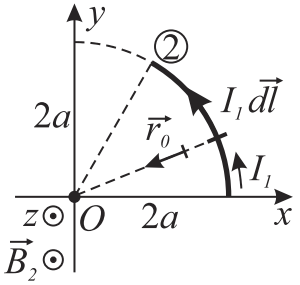
K1 Z1

a)



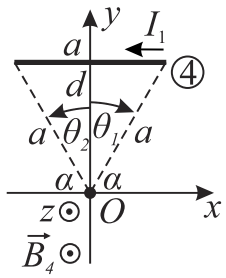
$\boxed{\vec{B}_1 = 0}$ $\boxed{\vec{B}_3 = 0}$ $\boxed{\vec{B}_5 = 0}$ $\boxed{\vec{B}_7 = 0}$ $\sin \sphericalangle\left(\vec{dl}, \vec{r}_0\right) = 0$

$\alpha = \frac{\pi}{3}$



$$\vec{dB}_2 = \frac{\mu_0}{4\pi}\frac{I_1 \vec{dl} \times \vec{r}_0}{r^2} \qquad dB_2 = \frac{\mu_0}{4\pi}\frac{I_1\, dl}{(2a)^2} \qquad \sphericalangle\left(\vec{dl}, \vec{r}_0\right) = \frac{\pi}{2}$$

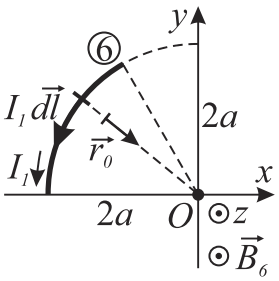
$$B_2 = \int dB_2 = \frac{\mu_0 I_1}{16\pi a^2}\int_0^{\frac{1}{6}2\cdot 2a\pi} dl = \frac{\mu_0 I_1}{16\pi a^2}\frac{2}{3}a\pi = \frac{\mu_0 I_1}{24a} \qquad \boxed{\vec{B}_2 = \frac{\mu_0 I_1}{24a}\cdot\vec{i}_z}$$



4 $\qquad d = \frac{a\sqrt{3}}{2}, \qquad \theta_1 = -30°, \qquad \theta_2 = 30°$

$$B_4 = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi\frac{a\sqrt{3}}{2}}\left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right] = \frac{\sqrt{3}\mu_0 I_1}{6\pi a}$$

$$\boxed{\vec{B}_4 = \frac{\sqrt{3}\mu_0 I_1}{6\pi a}\cdot\vec{i}_z}$$

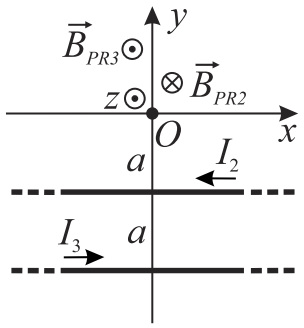


$$\vec{dB}_6 = \frac{\mu_0}{4\pi}\frac{I_1 \vec{dl} \times \vec{r}_0}{r^2} \qquad dB_6 = \frac{\mu_0}{4\pi}\frac{I_1\, dl}{(2a)^2} \qquad \sphericalangle\left(\vec{dl}, \vec{r}_0\right) = \frac{\pi}{2}$$

$$B_6 = \int dB_6 = \frac{\mu_0 I_1}{16\pi a^2}\int_0^{\frac{1}{6}2\cdot 2a\pi} dl = \frac{\mu_0 I_1}{16\pi a^2}\frac{2}{3}a\pi = \frac{\mu_0 I_1}{24a} \qquad \boxed{\vec{B}_6 = \frac{\mu_0 I_1}{24a}\cdot\vec{i}_z}$$

$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 + \vec{B}_6 + \vec{B}_7 = \left(\frac{\mu_0 I_1}{24a} + \frac{\sqrt{3}\mu_0 I_1}{6\pi a} + \frac{\mu_0 I_1}{24a}\right)\cdot\vec{i}_z}$$

b)



$$B_{PR2} = \frac{\mu_0 I_2}{2\pi a} \qquad \boxed{\vec{B}_{PR2} = \frac{\mu_0 I_2}{2\pi a}\cdot\left(-\vec{i}_z\right)} \qquad B_{PR3} = \frac{\mu_0 I_3}{2\pi 2a} \qquad \boxed{\vec{B}_{PR3} = \frac{\mu_0 I_3}{4\pi a}\cdot\vec{i}_z}$$

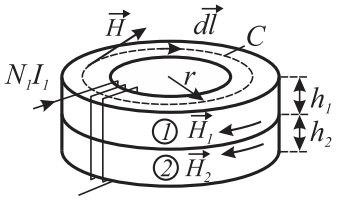
$$\vec{B}_{Ouk} = \left(\frac{\mu_0 I_1}{12a} + \frac{\sqrt{3}\mu_0 I_1}{6\pi a}\right)\cdot\vec{i}_z + \frac{\mu_0 I_2}{2\pi a}\cdot\left(-\vec{i}_z\right) + \frac{\mu_0 I_3}{4\pi a}\cdot\vec{i}_z = 0$$

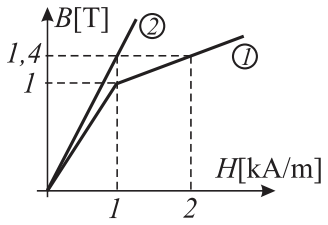
$$\frac{\mu_0 I_1}{12a} + \frac{\sqrt{3}\mu_0 I_1}{6\pi a} = \frac{\mu_0 I_2}{2\pi a} - \frac{\mu_0 I_3}{4\pi a} \qquad \left(I_2 = 1{,}5 I_3\right)$$

$$I_3 = \frac{\pi + 2\sqrt{3}}{6} I_1 = 0{,}81\,A \qquad \boxed{I_3 = 1{,}1\,\text{A}} \qquad \boxed{I_2 = 1{,}65\,\text{A}}$$

K1 Z2

a)





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_C \vec{H} \cdot \vec{dl} = NI$$

$$H\ 2\pi r = NI$$

$$H = \frac{NI}{2\pi r} \qquad a \le r \le b$$

$$H_{1\min} = \frac{N_1 I_1}{2\pi b} \ge H_{K1} \quad \Rightarrow \quad I_1 \ge \frac{H_{K1}\, 2\pi b}{N_1} = \frac{1000 \cdot 2 \cdot 3{,}14 \cdot 0{,}12}{628}$$

$$\boxed{I_1 = 1{,}2 \text{ A}}$$

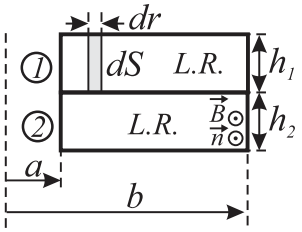
b)

$$I_1' = \frac{I_1}{3} = 0{,}4 \text{ A}$$

$$\left.\begin{aligned} H_{1\max}{}' &= \frac{N_1 I_1'}{2\pi a} = 1000\ \frac{\text{A}}{\text{m}} \\ H_{1\min}{}' &= \frac{N_1 I_1}{2\pi b} = 333{,}3\ \frac{\text{A}}{\text{m}} \end{aligned}\right\} \Rightarrow$$

Materijal 1 i 2 u linearnom režimu rada. $B_1 = \frac{1}{1000}\frac{N_1 I_1}{2\pi r}$, $B_2 = \frac{1{,}4}{1000}\frac{N_1 I_1}{2\pi r}$

c)



$$\Phi_1 = \int_{S_1} B_1\, dS = \int_{S_1} \mu_1 H\, dr\, h_1 = \int_a^b \frac{1}{1000}\frac{N_1 I_1'}{2\pi r}\, dr\, h_1 = \frac{1}{1000}\frac{N_1 I_1'}{2\pi} h_1 \ln\frac{b}{a}$$

$$\Phi_1 = 1{,}32 \text{ mWb}$$

$$\Phi_2 = \int_{S_2} B_2\, dS = \int_a^b \frac{1{,}4}{1000}\frac{N_1 I_1'}{2\pi r}\, dr\, h_2 = \frac{1{,}4}{1000}\frac{N_1 I_1'}{2\pi} h_2 \ln\frac{b}{a}$$

$$\Phi_2 = 1{,}85 \text{ mWb}$$

$$\Phi_{pp} = \Phi_1 + \Phi_2 = 3{,}17 \text{ mWb}$$

$$\Phi_m = N_2 \Phi_{pp} = 31{,}7 \text{ mWb} \quad \Rightarrow \quad \Phi(t) = \Phi_m \cos\omega t$$

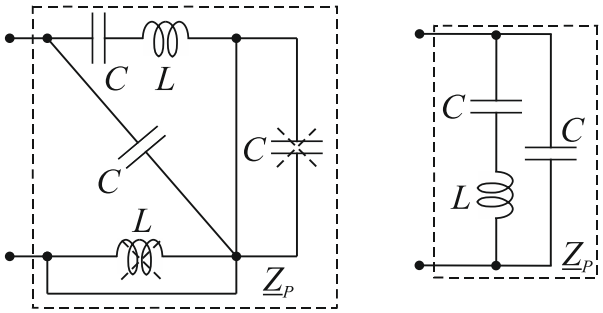
$$e = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\Phi_m \cos\omega t) = \Phi_m \omega \sin\omega t = 31{,}7 \sin 1000t \text{ V}$$

$$\boxed{e = 31{,}7 \sin 1000t \text{ V}}$$

MiR ISPIT 12.7.2026.

K2 Z1

a)



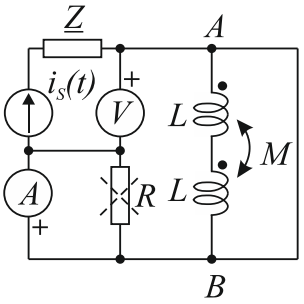
$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

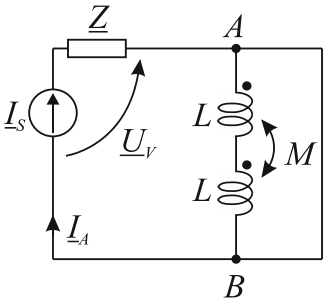
$$\underline{Z}_P = \underline{Z}_1 \parallel \frac{1}{j\omega C} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C}+\frac{1}{j\omega C}} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{2-\omega^2 LC}{j\omega C}} \qquad \boxed{\underline{Z}_P = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}}$$

$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad 1-\omega^2 LC = 0$ $\qquad \boxed{\omega_{r1} = \sqrt{\frac{1}{LC}}}$ $\qquad \boxed{\omega_{r2} \to \infty}$

$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad \omega C = 0$ $\qquad \boxed{\omega_{ar1} = 0}$ $\qquad 2-\omega^2 LC = 0$ $\qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$

b)





$$\omega = \omega_{r1} = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{20\cdot 10^{-3}\cdot 500\cdot 10^{-9}}} = 10^4 \ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$\underline{Z}_L = j\omega L = j10^4 \cdot 20\cdot 10^{-3} = j200\ \Omega$$

$$\underline{Z}_M = j\omega M = j10^4 \cdot 10\cdot 10^{-3} = j100\ \Omega$$

$$\underline{I}_S = \frac{2}{\sqrt{2}}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = (1+j)\ \mathrm{A}$$

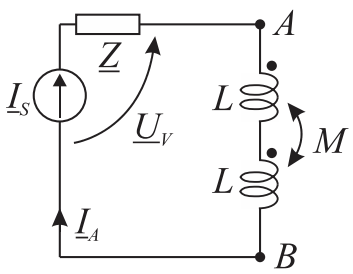
$\underline{U}_V = \underline{U}_{AB}$

$\underline{U}_{AB} = 0 \qquad \Rightarrow \qquad \underline{U}_V = 0$

$\boxed{U_V = |\underline{U}_V| = 0}$

$I_A = |\underline{I}_A| = |\underline{I}_S| = \sqrt{1^2+1^2}$ $\qquad \boxed{I_{A1} = \sqrt{2}\ \mathrm{A}}$

c) **bonus – 5p**



$$\omega_{ar2} = \sqrt{\frac{2}{LC}} = \sqrt{\frac{2}{20m\cdot 500n}} = \sqrt{\frac{2}{2\cdot 5n}} = 1{,}41\cdot 10^4 \ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$\underline{Z}_L = j\omega L = j1{,}41\cdot 10^4 \cdot 20\cdot 10^{-3} = j282\ \Omega$$

$$\underline{Z}_M = j\omega M = j1{,}41\cdot 10^4 \cdot 10\cdot 10^{-3} = j141\ \Omega$$

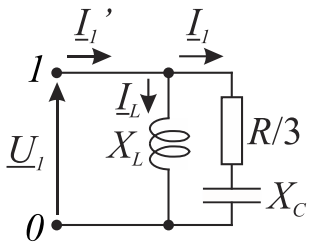
$$\underline{U}_V = \underline{U}_{AB} = (j\omega L + j\omega M + j\omega L + j\omega M)\cdot \underline{I}_S = 2(j\omega L + j\omega M)\cdot \underline{I}_S$$

$$\underline{U}_V = 2\cdot j(282+141)\cdot(1+j1) = 846\cdot(-1+j)\mathrm{V}$$

$\boxed{U_V = |\underline{U}_V| = 846\sqrt{2} = 1{,}196\,\mathrm{kV}}$

K2 Z2

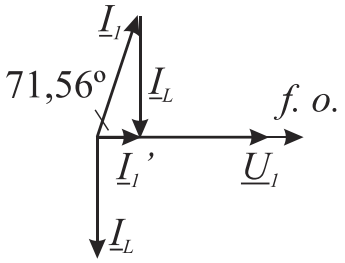
a)



$$\underline{Z} = \frac{R}{3} - jX_C = \left(\frac{150}{3} - j150\right)\Omega = (50 - j150)\,\Omega = 158{,}1\,e^{-j71{,}56^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\,e^{j0^\circ}}{Z\,e^{j\varphi}} = \frac{230\,e^{j0^\circ}}{158{,}1\,e^{-j71{,}56^\circ}} = 1{,}45\,e^{j71{,}56^\circ}\ \text{A}$$

$$\underline{I}_L = \frac{U_1\,e^{j0}}{jX_L} = \frac{U_1\,e^{j0}}{X_L\,e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L}\,e^{-j\frac{\pi}{2}}\ \text{A}$$



$$I_L = I_1 \sin 71{,}56^\circ = 1{,}45 \cdot \sin 71{,}56^\circ = 1{,}38\ \text{A}$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z}\sin 71{,}56^\circ \quad \Rightarrow \quad \frac{1}{X_L} = \frac{\sin 71{,}56^\circ}{Z} \quad \Rightarrow \quad X_L = \frac{158{,}1}{\sin 71{,}56^\circ}$$

$$\boxed{X_L = 166{,}7\,\Omega}$$

I KZ: $\underline{I}_1' = \underline{I}_L + \underline{I}_1$

b)

$$\boxed{I_A^{\,PRE} = 0\ \text{A}}$$

$$P_W^{\,PRE} = \text{Re}\left\{\underline{U}_{31}\,\underline{I}_1^{\,*}\right\} = U_{31}\,I_1'\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_1\right)$$

$$P_W^{\,PRE} = \sqrt{3}U_1\,I_1\cos(150^\circ - 71{,}56^\circ) = \sqrt{3}\cdot 230\cdot 1{,}45\cdot\cos 78{,}44^\circ$$

$$\boxed{P_W^{\,PRE} = 115{,}8\ \text{W}}$$

$$\boxed{I_A^{\,POSLE} = |\underline{I}_L| = 1{,}38\ \text{A}}$$

$$P_W^{\,POSLE} = \text{Re}\left\{\underline{U}_{31}\,\underline{I}_1^{\,*}\right\} = P_W^{\,PRE}$$

$$\boxed{P_W^{\,POSLE} = 115{,}8\ \text{W}}$$

c)