STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

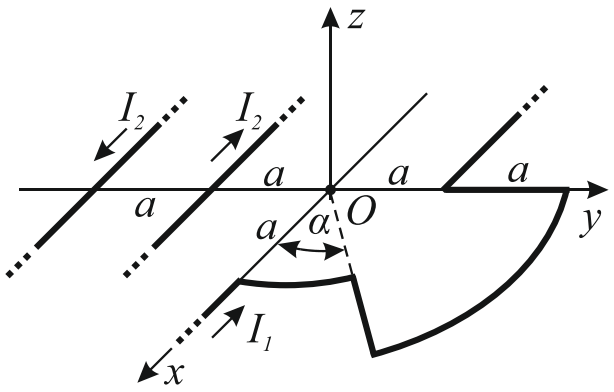
12.07.2026.

ZADACI

Zadatak 1. Na slici 1 je prikazan veoma dugačak žičani provodnik, sa vremenski konstantnom strujom jačine I_1, i veoma dugačak dvožični vod, sa strujom jačine I_2. Obe strukture leže u x-y ravni zadatog koordinatnog sistema. Žičani provodnik se sastoji od četiri pravolinijska segmenta i dva lučna segmenta, poluprečnika a i $2a$.

a) Odrediti u opštim brojevima vektor magnetske indukcije, B_1, koji u tački O (centar koordinatnog sistema), stvara žičani provodnik.
b) Odrediti u opštim brojevima vektor magnetske indukcije, B_2, koji u tački O stvara dvožični vod.
c) Izračunati intenzitet struje dvožičnog voda, I_2, tako da rezultantni vektor magnetske indukcije u tački O bude jednak nuli.

Brojne vrednosti: $I_1 = 5\ A$, $a = 1\ cm$, $\alpha = 30^o$, $\mu_0 = 4\pi \cdot 10^{-7}\ H/m$.

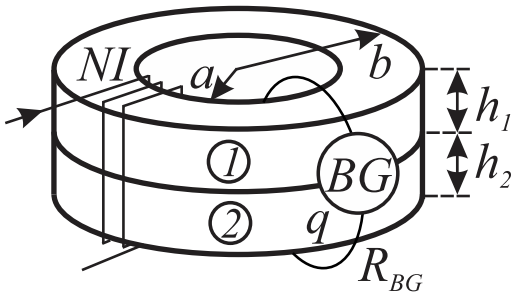


Slika 1.

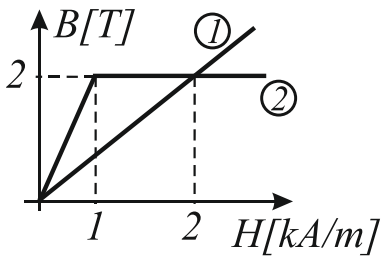
Zadatak 2. Debelo torusno jezgro, prikazano na slici 2a, načinjeno je od dva materijala čije su magnetske karakteristike date na slici 2b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa $N = 200$ zavojaka u kojima je uspostavljena vremenski konstantna struja jačine I.

a) Izračunati jačinu struje I kroz pobudni namotaj, pri kojoj će deo jezgra načinjen od materijala 2 biti do pola u zasićenju.
b) Izračunati fluks kroz poprečni presek jezgra, pri jačini struje izračunatoj pod *a*).
c) Odrediti smer (naznačiti na slici) i izračunati količinu proteklog naelektrisanja, q, kroz namotaj kola balističkog galvanometra otpornosti $R_{BG} = 20\ \Omega$, prilikom uključivanja struje. Pre uključivanja struje, jezgro je bilo nenamagnetisano.

Dimenzije jezgra su: $a = 5\ cm$, $b = 9\ cm$, $h_1 = h_2 = 4\ cm$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 2 (II kolokvijum)

K2

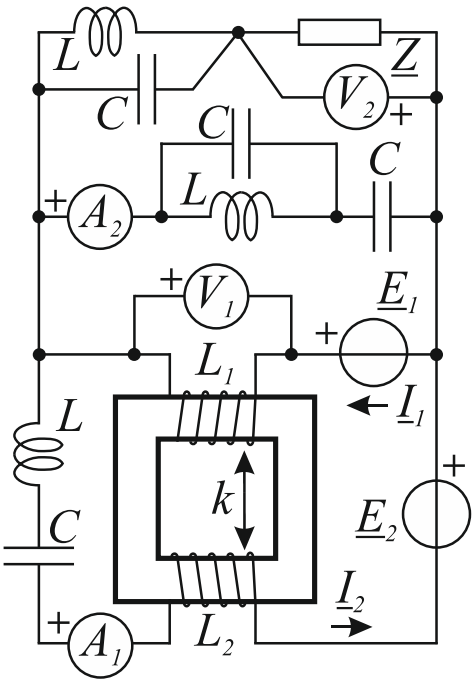
12.07.2026.

ZADACI

Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

a) Nacrtati pojednostavljenu električnu šemu kola i u njoj označiti tip magnetske sprega koja postoji između namotaja induktivnosti L_1 i L_2.
b) Odrediti pokazivanja idealnih mernih instrumenata.
c) Izračunati aktivnu snagu naponskog generatora *ems* $\underline{E}_1$.

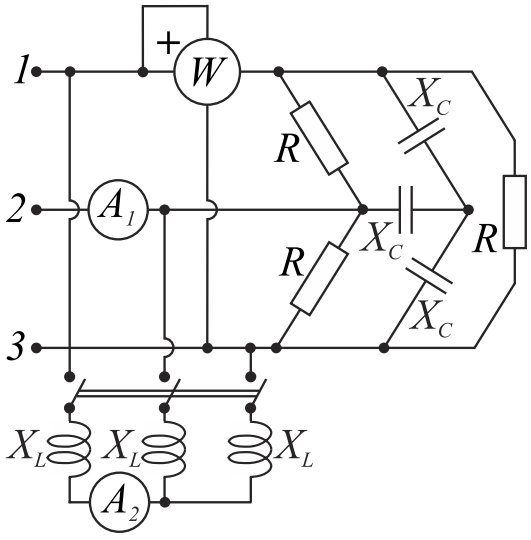
Brojni podaci su: $C = 25\ \mu F$, $L = 40\ mH$, $\omega = 1000\ rad/s$, $\underline{Z} = (3 + j5)\ \Omega$, $\underline{E}_1 = (20 + j100)\ V$, $\underline{E}_2 = (28 - j28)\ V$, $L_1 = L_2 = L$, $k = 0{,}7$.



Slika 1.

Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik sastavljen od otpornika otpornosti $R = 30\ \Omega$ i kondenzatora reaktanse $X_C = 10\ \Omega$, priključen na mrežu faznog napona $\underline{U}_1 = 230\ V$.

a) Odrediti reaktanse kalemova koje je potrebno povezati sa prijemnikom da bi mu se faktor snage popravio na jedinicu.
b) Odrediti pokazivanja idealnih mernih instrumenata pre i posle priključivanja kalemova.
c) Skicirati fazorske dijagrame faznih napona mreže i fazora svih veličina od kojih zavise pokazivanja instrumenata.



Slika 2.

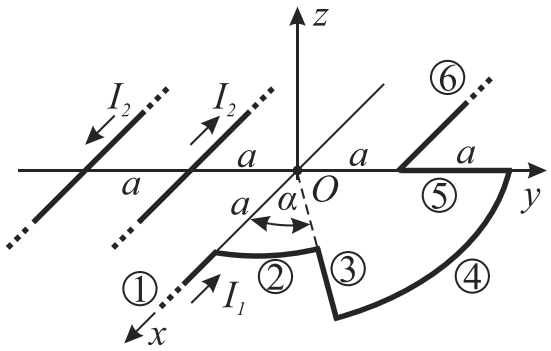
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET2, 12.07.2026.

I-1

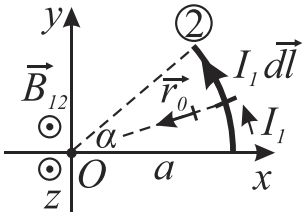
a)



$\boxed{\vec{B}_{11}=0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right)=0$

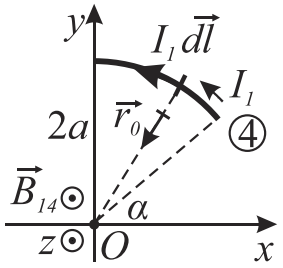
$\boxed{\vec{B}_{13}=0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right)=0$

$\boxed{\vec{B}_{15}=0}$ $\quad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right)=0$



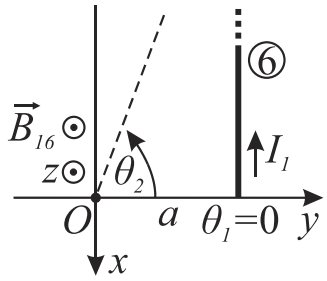
$$\vec{dB}_{12}=\frac{\mu_0}{4\pi}\frac{I_1\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_{12}=\frac{\mu_0}{4\pi}\frac{I_1\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right)=\frac{\pi}{2}$$

$$B_{12}=\int dB_{12}=\frac{\mu_0 I_1}{4\pi a^2}\int_0^{a\alpha} dl=\frac{\mu_0 I_1}{4\pi a^2}a\frac{\pi}{6}=\frac{\mu_0 I_1}{24a} \qquad \boxed{\vec{B}_{12}=\frac{\mu_0 I_1}{24a}\cdot\vec{i}_z}$$



$$\vec{dB}_{14}=\frac{\mu_0}{4\pi}\frac{I_1\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_{14}=\frac{\mu_0}{4\pi}\frac{I_1\,dl}{(2a)^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right)=\frac{\pi}{2}$$

$$B_{14}=\int dB_{14}=\frac{\mu_0 I_1}{16\pi a^2}\int_0^{2a\left(\frac{\pi}{2}-\alpha\right)} dl=\frac{\mu_0 I_1}{16\pi a^2}2a\frac{\pi}{3}=\frac{\mu_0 I_1}{24a} \qquad \boxed{\vec{B}_{14}=\frac{\mu_0 I_1}{24a}\cdot\vec{i}_z}$$

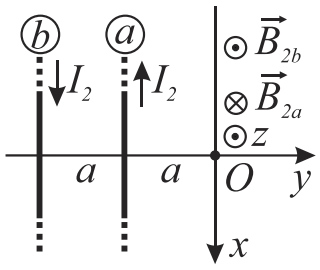


6 $\quad d=a, \qquad \theta_1=0, \qquad \theta_2\to\frac{\pi}{2}$

$$B_{16}=\frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2-\sin\theta_1\right)=\frac{\mu_0 I_1}{4\pi a}(1-0)$$

$$B_{16}=\frac{\mu_0 I_1}{4\pi a} \qquad \boxed{\vec{B}_{16}=\frac{\mu_0 I_1}{4\pi a}\cdot\vec{i}_z}$$

b)



$$B_{2a}=\frac{\mu_0 I_2}{2\pi a} \qquad \boxed{\vec{B}_{2a}=\frac{\mu_0 I_2}{2\pi a}\cdot\left(-\vec{i}_z\right)}$$

$$B_{2b}=\frac{\mu_0 I_2}{2\pi 2a} \qquad \boxed{\vec{B}_{2b}=\frac{\mu_0 I_2}{4\pi a}\cdot\vec{i}_z}$$

$$\vec{B}_2=\vec{B}_{2a}+\vec{B}_{2b}=\left(\frac{\mu_0 I_2}{4\pi a}-\frac{\mu_0 I_2}{2\pi a}\right)\cdot\vec{i}_z \qquad \boxed{\vec{B}_2=\frac{\mu_0 I_2}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

c)

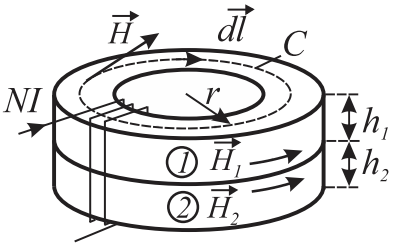
$$\vec{B}_0=\vec{B}_1+\vec{B}_2=\left(2\cdot\frac{\mu_0 I_1}{24a}+\frac{\mu_0 I_1}{4\pi a}\right)\cdot\vec{i}_z+\frac{\mu_0 I_2}{4\pi a}\cdot\left(-\vec{i}_z\right)$$

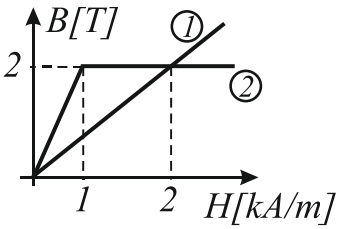
$$\vec{B}_0=0 \quad\Rightarrow\quad \frac{\mu_0 I_1}{12a}+\frac{\mu_0 I_1}{4\pi a}=\frac{\mu_0 I_2}{4\pi a} \quad\Rightarrow\quad I_2=\pi\left(\frac{1}{3}+\frac{1}{\pi}\right)I_1 \qquad \boxed{I_2=10{,}24\ A}$$

Ispit iz OET2, 12.07.2026.

I-2

a)





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_C \vec{H} \cdot \vec{dl} = NI$$

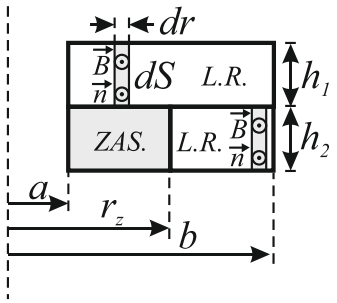
$$H \cdot 2\pi r = NI$$

$$\boxed{H = \frac{NI}{2\pi r}} \qquad a \le r \le b$$

$$r_z = a + \frac{1}{2}(b-a) = 7\ cm$$

$$\frac{NI}{2\pi r_z} = H_{K2} \quad \rightarrow \quad I = \frac{H_{K2}\, 2\pi r_z}{N} = \frac{1000 \cdot 2\pi \cdot 0{,}07}{200} \qquad \boxed{I = 2{,}19\ A}$$

b)



Deo torusa načinjen od materijala 1 je uvek u linearnom režimu rada.

$$\Phi_1 = \int_{S_1} \vec{B_1} \cdot \vec{dS}$$

$$\Phi_1 = \int_{S_1} B_1\, dS = \int_{S_1} \mu_1 H\, dr\, h_1 = \int_a^b \frac{2}{2000}\frac{NI}{2\pi r}\, dr\, h_1 = \frac{2}{2000}\frac{NI}{2\pi} h_1 \ln\frac{b}{a}$$

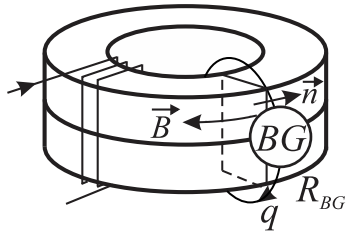
$$\boxed{\Phi_1 = 1{,}64\ mWb}$$

$$\Phi_2 = \int_{S_2} \vec{B_2} \cdot \vec{dS} = \int_{S_2} B_2\, dS = B_{K2}(r_z - a)\, h_2 + \int_{r_z}^b \frac{2}{1000}\frac{NI}{2\pi r}\, dr\, h_2$$

$$\Phi_2 = B_{K2}(r_z - a)\, h_2 + \frac{2}{1000}\frac{NI}{2\pi} h_2 \ln\frac{b}{r_z} \qquad \boxed{\Phi_2 = 3\ mWb}$$

$$\Phi_J = \Phi_1 + \Phi_2 \qquad \boxed{\Phi_J = 4{,}64\ mWb}$$

c)



$$\Phi = \int_S \vec{B} \cdot \vec{dS} = -\int_S B\, dS = -\Phi_J \qquad \sphericalangle(\vec{B}, \vec{n}) = \pi$$

$$q = -\frac{N_{BG}}{R_Z}\left(\Phi_{KR} - \cancel{\Phi_{POČ}}^{\,0}\right) \qquad (N_{BG} = 1)$$

$$q = -\frac{1}{R_Z}\Phi_{KR} = -\frac{1}{R_Z}(-\Phi_J) = \frac{1}{R_Z}\Phi_J$$

$$q = \frac{1}{20} \cdot 4{,}64 \cdot 10^{-3}$$

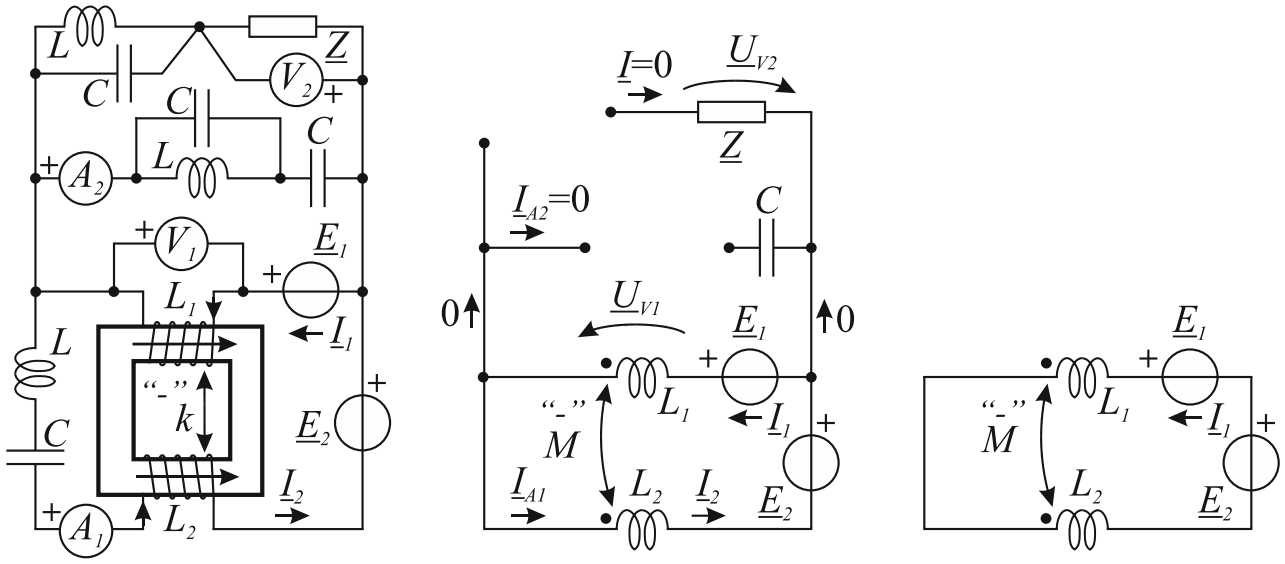
$$\boxed{q = 232\ \mu C}$$

Ispit iz OET2, 12.07.2026.

II-1

a)

$$\omega = 1000\,\frac{rad}{s} \qquad X_L = \omega L = 40\,\Omega \qquad X_C = \frac{1}{\omega C} = 40\,\Omega$$



$$L_1 = L_2 = L \quad \Rightarrow \quad X_{L1} = X_{L2} = X_L = 40\,\Omega$$

$$M = k\sqrt{L_1 L_2} = k\sqrt{L \cdot L} = 0{,}7 \cdot \sqrt{40 \cdot 10^{-3} \cdot 40 \cdot 10^{-3}} = 28\,mH \quad \Rightarrow \quad X_M = \omega M = 28\,\Omega$$

b)

$$-\underline{E}_1 + jX_{L1}\,\underline{I}_1 - jX_M\,\underline{I}_1 + jX_{L2}\,\underline{I}_1 - jX_M\,\underline{I}_1 - \underline{E}_2 = 0$$

$$\underline{I}_1 = \frac{\underline{E}_1 + \underline{E}_2}{jX_{L1} + jX_{L2} - j2X_M} = \frac{(20 + j100) + (28 - j28)}{j40 + j40 - j56} = \frac{48 + j72}{j24} = (3 - j2)\,A$$

$$\underline{I}_{A1} = \underline{I}_1 = (3 - j2)\,A$$

$$I_{A1} = |\underline{I}_{A1}| = \sqrt{3^2 + (-2)^2} \qquad \boxed{I_{A1} = 3{,}6\,A}$$

$$\boxed{I_{A2} = 0\,A}$$

$$\underline{U}_{V1} = -jX_{L1}\,\underline{I}_1 + jX_M\,\underline{I}_1 = (-jX_{L1} + jX_M)\,\underline{I}_1 = (-j40 + j28) \cdot (3 - j2) = -j12 \cdot (3 - j2) = (-24 - j36)\,V$$

$$U_{V1} = |\underline{U}_{V1}| = \sqrt{(-24)^2 + (-36)^2} \qquad \boxed{U_{V1} = 43{,}3\,V}$$

$$\boxed{U_{V2} = 0\,V}$$

c)

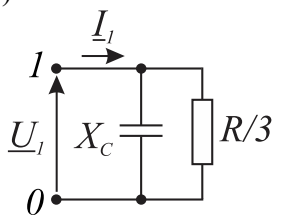
$$\underline{S}_{E1} = \underline{E}_1\,\underline{I}_1^{\,*} = (20 + j100) \cdot (3 + j2) = (-140 + j340)\,VA$$

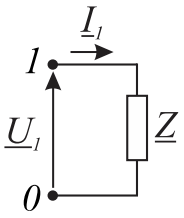
$$\boxed{P_{E1} = \mathrm{Re}\{\underline{S}_{E1}\} = -140\,W}$$

Ispit iz OET2, 12.07.2026.

II-2

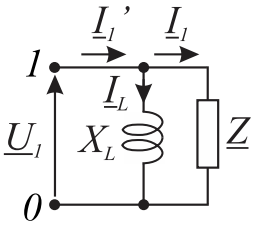
a)





$$\underline{Z} = \frac{R}{3} \| \left(-jX_C\right) = 10 \| \left(-j10\right) = \left(5 - j5\right)\Omega = 5\sqrt{2}\, e^{-j45^\circ}\ \Omega$$

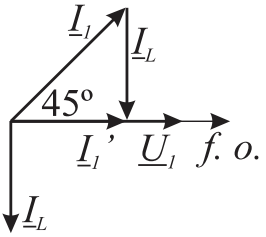
$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{230\, e^{j0^\circ}}{5\sqrt{2}\, e^{-j45^\circ}} = 32{,}53\, e^{j45^\circ}\ A$$



$$\underline{I}_L = \frac{U_1\, e^{j0}}{jX_L} = \frac{U_1\, e^{j0}}{X_L\, e^{j90^\circ}} = \frac{U_1}{X_L}\, e^{-j90^\circ}\ A$$

I KZ: $\underline{I}_1' = \underline{I}_L + \underline{I}_1$

$$I_L = I_1 \sin 45^\circ = 32{,}53 \cdot \sin 45^\circ = 23\ A$$



$$\frac{U_1}{X_L} = \frac{U_1}{Z}\sin 45^\circ \quad \Rightarrow \quad \frac{1}{X_L} = \frac{\sin 45^\circ}{Z} \quad \Rightarrow \quad X_L = \frac{Z}{\sin 45^\circ} = \frac{5\sqrt{2}}{\frac{\sqrt{2}}{2}}$$

$$\boxed{X_L = 10\,\Omega}$$

$$I_1' = I_1 \cos 45^\circ = 32{,}53 \cdot \cos 45^\circ = 23\ A \qquad \underline{I}_1' = 23\, e^{j0^\circ}\ A$$

b)

$$\boxed{I_{A1}^{\ PRE} = \left|\underline{I}_2\right| = 32{,}53\ A}$$

$$\boxed{I_{A1}^{\ POSLE} = \left|\underline{I}_2'\right| = 23\ A}$$

$$\boxed{I_{A2}^{\ PRE} = 0\ A}$$

$$\boxed{I_{A2}^{\ POSLE} = \left|\underline{I}_L\right| = 23\ A}$$

$$P_W^{\ PRE} = P_W^{\ POSLE} = \mathrm{Re}\left\{\underline{U}_{13}\, \underline{I}_1^{\,*}\right\} = U_{13}\, I_1 \cos \sphericalangle\left(\underline{U}_{13}, \underline{I}_1\right) = \sqrt{3}\, U_1\, I_1 \cos\left(45^\circ + 30^\circ\right) = \sqrt{3} \cdot 230 \cdot 32{,}53 \cdot \cos 75^\circ$$

$$\boxed{P_W^{\ PRE} = P_W^{\ POSLE} = 3354\ W}$$

c)