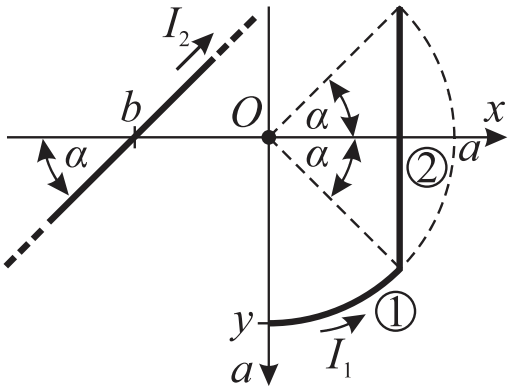
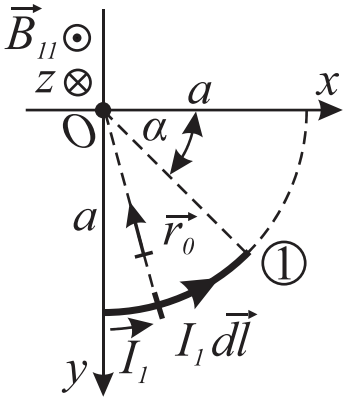
MH ISPIT, K1, 25. 2. 2026.

K1 Z1

a)

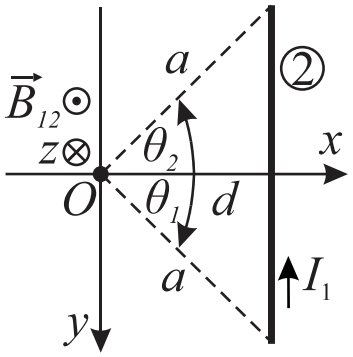




$$\overrightarrow{dB}_{11} = \frac{\mu_0}{4\pi}\frac{I_1 \vec{dl}\times\vec{r}_0}{r^2} \qquad dB_{11} = \frac{\mu_0}{4\pi}\frac{I_1 dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_{11} = \int dB_{11} = \frac{\mu_0 I_1}{4\pi a^2}\int_0^{\frac{1}{8}2a\pi} dl = \frac{\mu_0 I_1}{4\pi a^2}\frac{1}{4}a\pi = \frac{\mu_0 I_1}{16a}$$

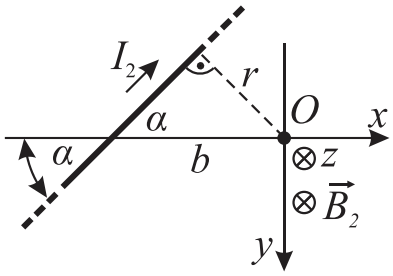
$$\boxed{\vec{B}_{11} = \frac{\mu_0 I_1}{16a}\cdot\left(-\vec{i}_z\right)}$$



$$d = a\cos\alpha = a\cos 45° = \frac{a\sqrt{2}}{2}, \qquad \theta_1 = -\alpha = -45°, \qquad \theta_2 = \alpha = 45°$$

$$B_{12} = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi\frac{a\sqrt{2}}{2}}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\mu_0 I_1}{4\pi\frac{a\sqrt{2}}{2}}\sqrt{2} = \frac{\mu_0 I_1}{2\pi a}$$

$$\boxed{\vec{B}_{12} = \frac{\mu_0 I_1}{2\pi a}\cdot\left(-\vec{i}_z\right)}$$



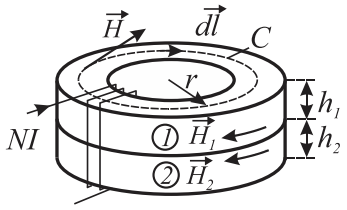
$$r = b\sin\alpha = b\sin 45° = \frac{b\sqrt{2}}{2}$$

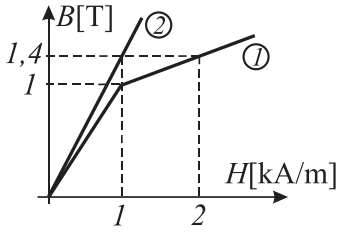
$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{\mu_0 I_2}{2\pi\frac{b\sqrt{2}}{2}} = \frac{\mu_0 I_2}{\pi b\sqrt{2}} \qquad \boxed{\vec{B}_2 = \frac{\mu_0 I_2}{\pi b\sqrt{2}}\cdot\vec{i}_z}$$

$$\boxed{\vec{B}_O = \vec{B}_{11} + \vec{B}_{12} + \vec{B}_2 = \left(\frac{\mu_0 I_2}{\pi b\sqrt{2}} - \frac{\mu_0 I_1}{16a} - \frac{\mu_0 I_1}{2\pi a}\right)\cdot\vec{i}_z} \qquad \boxed{\vec{B}_O = 0{,}43\,\mu\mathrm{T}\cdot\vec{i}_z}$$

K1 Z2

a)





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_C \vec{H} \cdot d\vec{l} = NI$$

$$H\, 2\pi r = NI$$

$$H = \frac{NI}{2\pi r} \qquad a \le r \le b$$

Sa krive magetnisanja uočavamo da je materijal 2 uvek u linearnom režimu rada.

$$B_2 = \frac{1,4}{1000}\frac{NI}{2\pi r}$$

Za materijal 1 ispitujemo sledeće:

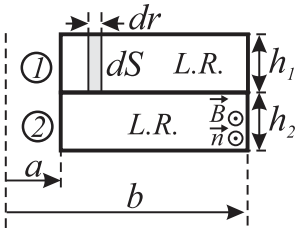
$$H_{1\max} = \frac{NI_a}{2\pi a} \le H_{K1} = 1000\,\frac{\text{A}}{\text{m}} \quad \Rightarrow \quad I_a \le \frac{1000 \cdot 2\pi a}{N} \le \frac{1000 \cdot 2 \cdot 3,14 \cdot 0,04}{626} \le 0,4\,\text{A}$$

$$H_{1\min} = \frac{NI_b}{2\pi b} \le H_{K1}\,\frac{\text{A}}{\text{m}} = 1000\,\frac{\text{A}}{\text{m}} \quad \Rightarrow \quad I_b \le \frac{1000 \cdot 2\pi b}{N} \le \frac{1000 \cdot 2 \cdot 3,14 \cdot 0,12}{626} \le 1,42\,\text{A}$$

$$I = \min\{I_a, I_b\} = I_a = 0,4\,\text{A} \qquad \boxed{I = 0,4\,\text{A}}$$

$$B_1 = \frac{1}{1000}\frac{NI_1}{2\pi r}$$

b)



$$\Phi_1 = \int_{S_1} B_1\, dS = \int_{S_1} \mu_1 H\, dr\, h_1 = \int_a^b \frac{1}{1000}\frac{NI}{2\pi r}\, dr\, h_1 = \frac{1}{1000}\frac{NI}{2\pi}\, h_1 \ln\frac{b}{a}$$

$$\Phi_1 = 1,32\ \text{mWb}$$

$$\Phi_2 = \int_{S_2} B_2\, dS = \int_a^b \frac{1,4}{1000}\frac{NI}{2\pi r}\, dr\, h_2 = \frac{1,4}{1000}\frac{NI}{2\pi}\, h_2 \ln\frac{b}{a}$$

$$\Phi_2 = 1,85\ \text{mWb}$$

$$\Phi_{pp} = \Phi_1 + \Phi_2 = 3,17\ \text{mWb} \qquad \boxed{\Phi_{pp} = 3,17\ \text{mWb}}$$

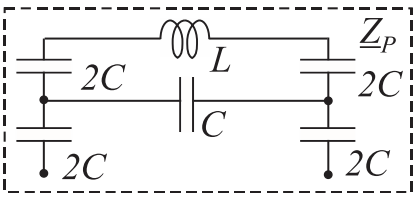
c)

$$L = \frac{N\Phi_{pp}}{I} = \frac{626 \cdot 3,17m}{0,4} = 4,96\ \text{mH}$$

$$\boxed{L = 4,96\ \text{mH}}$$

K2 Z1

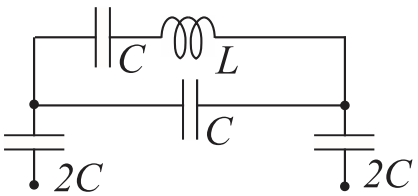
a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel \frac{1}{j\omega C} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{1-\omega^2 LC}{j\omega C}+\dfrac{1}{j\omega C}} = \frac{\dfrac{1-\omega^2 LC}{(j\omega C)^2}}{\dfrac{2-\omega^2 LC}{j\omega C}} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

$$\underline{Z}_e = \underline{Z}_2 + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{1}{j\omega C} = \frac{1-\omega^2 LC+2-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$



$$\boxed{\underline{Z}_e = -j\frac{3-2\omega^2 LC}{\omega C\left(2-\omega^2 LC\right)}}$$

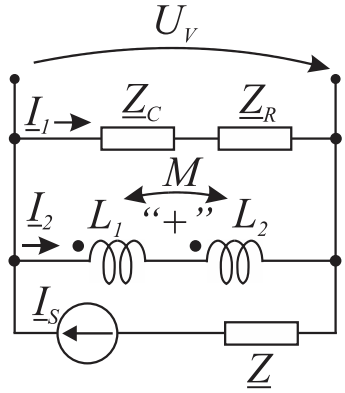
$$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad 2-3\omega^2 LC = 0 \qquad \boxed{\omega_{r1} = \sqrt{\frac{3}{2LC}}}$$

$$\boxed{\omega_{r2} \to \infty}$$

$$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad \omega C = 0 \qquad \boxed{\omega_{ar1} = 0}$$

$$2-\omega^2 LC = 0 \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$$

b)



$$\omega = \omega_{ar2} = \sqrt{\frac{2}{LC}} = \sqrt{\frac{2}{20\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4\ \frac{rad}{s}$$

$$\underline{Z}_P \to \infty$$

$$\underline{Z}_M = j1{,}66\ k\Omega$$

$$\underline{Z}_C = \frac{1}{j\omega C} = -j0{,}5\ k\Omega$$

$$\underline{Z}_{L1} = j\omega L_1 = j2\ k\Omega$$

$$\underline{Z}_{L2} = j\omega L_2 = j3\ k\Omega,$$

$$\underline{I}_S = \underline{I}_1 + \underline{I}_2 \qquad \Rightarrow \qquad \underline{I}_2 = \underline{I}_S - \underline{I}_1$$

$$\underline{Z}_C\underline{I}_1 + \underline{Z}_R\underline{I}_1 - \underline{Z}_{L2}\underline{I}_2 - \underline{Z}_M\underline{I}_2 - \underline{Z}_{L1}\underline{I}_2 - \underline{Z}_M\underline{I}_2 = 0$$

$$\left(\underline{Z}_C + \underline{Z}_R\right)\underline{I}_1 - \left(\underline{Z}_{L1} + \underline{Z}_{L2} + 2\underline{Z}_M\right)\left(\underline{I}_S - \underline{I}_1\right) = 0$$

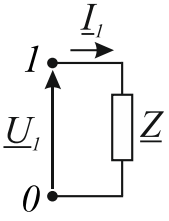
$$\underline{I}_1 = \frac{\underline{Z}_{L1} + \underline{Z}_{L2} + 2\underline{Z}_M}{\underline{Z}_C + \underline{Z}_R + \underline{Z}_{L1} + \underline{Z}_{L2} + 2\underline{Z}_M}\underline{I}_S = \frac{\left(j2 + j3 + j3{,}32\right)k}{\left(-j0{,}5 + 1 + j2 + j3 + j3{,}32\right)k}(1-j)\,m = (1{,}18 - j0{,}91)\ m\mathrm{A}$$

$$\underline{U}_{\underline{Z}_P} = \left(\underline{Z}_C + \underline{Z}_R\right)\underline{I}_1 = (-j0{,}5+1)k\cdot(1{,}18 - j0{,}91)\,m = (0{,}725 - j1{,}5)\ \mathrm{V}$$

$$U_V = \left|\underline{U}_{\underline{Z}_P}\right| = 1{,}67\ \mathrm{V} \qquad \boxed{U_V = 1{,}67\ \mathrm{V}}$$

K2 Z2

a)



$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{230\,e^{j0°}}{20 + j10} = \frac{230\,e^{j0°}}{22,4\,e^{j26,57°}}$$

$$\underline{I}_1 = 10,3\,e^{j-26,57}\ \text{A}$$

$$\underline{I}_2 = \underline{I}_1\,e^{-j120°} = 10,3\,e^{-j146,57°}\ \text{A}$$

$$\underline{I}_3 = \underline{I}_1\,e^{-j240°} = 10,3\,e^{-j266,57°}\ \text{A}$$

$$\boxed{I_A = |\underline{I}_1| = 10,3\ \text{A}}$$

$$\boxed{U_{V1} = |\underline{U}_{12}| = \sqrt{3}\,|\underline{U}_1| = \sqrt{3}\cdot 230 = 398,37\ \text{V}}$$

$$\boxed{U_{V2} = |\underline{U}_{30}| = |\underline{U}_3| = |\underline{U}_1| = 230\ \text{V}}$$

$$P_{W1} = \text{Re}\left\{\underline{U}_{31}\,\underline{I}_2^{\,*}\right\} = U_{31}\,I_2\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_2\right) = \sqrt{3}U_1\,I_1\cos\left(90° - 26,57°\right) = \sqrt{3}\cdot 230\cdot 10,3\cdot\cos 63,43°$$

$$\boxed{P_{W1} = 1835,3\ \text{W}}$$

$$P_{W2} = \text{Re}\left\{\underline{U}_{31}\,\underline{I}_3^{\,*}\right\} = U_{31}\,I_3\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_3\right) = \sqrt{3}U_1\,I_1\cos\left(30° + 26,57°\right) = \sqrt{3}\cdot 230\cdot 10,3\cdot\cos 56,57°$$

$$\boxed{P_{W2} = 2260,5\ \text{W}}$$

b)