STUDIJSKI PROGRAM MEHATRONIKA

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

30.10.2025.

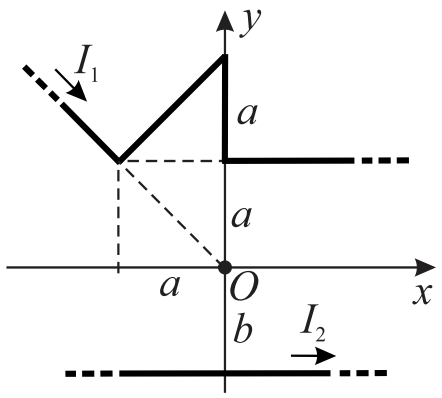
ZADACI

Zadatak 1. Veoma dugačak žičani provodnik, sa vremenski konstantnom strujom jačine I_1, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od četiri pravolinijska segmenta i nalazi se u x-y ravni Dekartovog pravouglog koordinatnog sistema. Sredina je vazduh.

a) Odrediti vektor magnetske indukcije koji u koordinatnom početku (tačka O) stvara provodnik.

b) Odrediti rastojanje, b, beskonačno dugačkog provodnika, sa vremenski konstantnom strujom jačine I_2, od tačke O, tako da ukupni vektor magnetske indukcije u tački O bude jednak nuli.

Brojne vrednosti:
$a = 6$ cm, $I_1 = 5$ A, $I_2 = 3{,}2$ A, $\mu_0 = 4\pi\cdot10^{-7}$ H/m.



Slika 1.

Zadatak 2. Za magnetsko kolo prikazano na slici, odrediti:

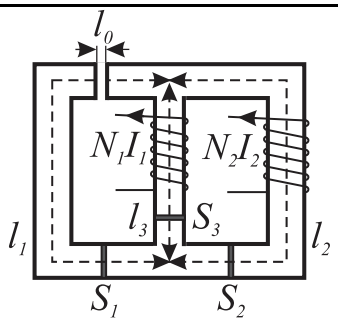
a) jačinu struje u namotaju sa $N_2 = 400$ zavojaka, da bi fluks vektora magnetske indukcije u procepu imao vrednost $\Phi_0 = 0{,}15$ mWb,

b) samoinduktivnost namotaja sa N_1 zavojaka,

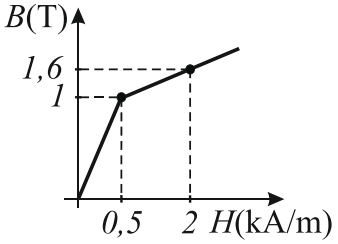
c) energiju uloženu za uspostavljanje magnetskog polja u grani dužine l_2, ako se zna, da je jezgro pre uključivanja struje u namotajima bilo nenamagnetisano.

Brojni podaci su: $N_1 = 200$, $I_1 = 4{,}81$ A,
$l_1 = l_2 = 20$ cm, $l_3 = 8$ cm, $l_0 = 0{,}5$ mm,
$S_1 = S_2 = 1$ cm^2, $S_3 = 0{,}5$ cm^2, $\mu_0 = 4\pi\cdot10^{-7}$ H/m.

Rasipanje magnetskog fluksa zanemariti.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MEHATRONIKA

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (II kolokvijum)	**K2** **30.10.2025.**

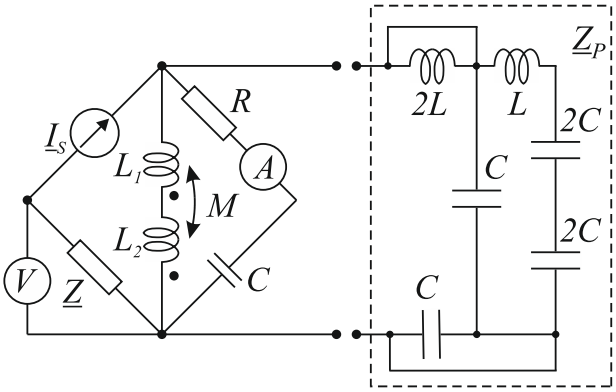
ZADACI

Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Odrediti pokazivanja idealnih mernih instrumenata, kada je prijemnik priključen na ulazni deo kola, pri čemu strujni generator $\underline{I}_S$ radi na manjoj rezonantnoj učestanosti.

Brojni podaci: $L = 10$ mH, $C = 40$ nF, $R = 1$ kΩ, $\underline{Z} = (5 + j3)$ kΩ, $\underline{I}_S = (1 + j)$ mA, $L_1 = L$, $L_2 = 4{,}5L$, $k = 0{,}87$.



Slika 1.

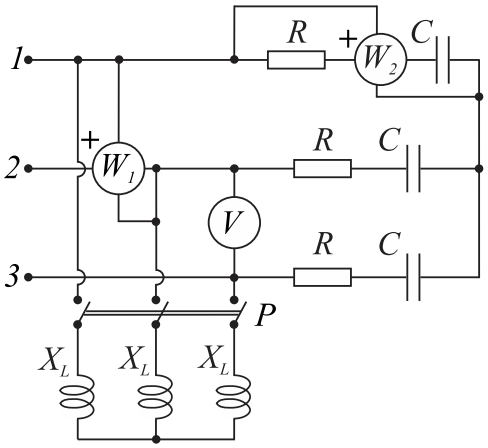
Zadatak 2. U simetričnom trofaznom sistemu, prikazanom na slici 2, prijemnik je sačinjen od jednog otpornika otpornosti $R = 20$ Ω i jednog kondenzatora kapacitivnosti $C = 277$ μF, i priključen je na gradsku mrežu faznog napona $\underline{U}_1 = 230$ V. Pomoću prekidača P u sistem se mogu priključiti kalemovi za popravku faktora snage prijemnika.

a) Odrediti pokazivanja idealnih vatmetara i voltmetra pre priključenja kalemova.

b) Izračunati reaktanse kalemova X_L kojima se faktor snage grupe povećava na jedinicu.

c) Odrediti pokazivanja idealnih vatmetara i voltmetra posle priključenja kalemova.

d) Za svaki vatmetar skicirati fazorski dijagram faznih napona mreže i svih fazora veličina od kojih zavisi njihovo pokazivanje, pre i posle kompenzacije.



Slika 2.

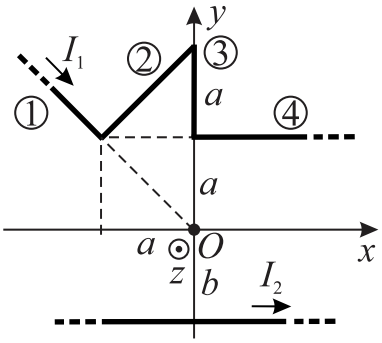
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

ISPIT iz OET2, 30.10.2025.

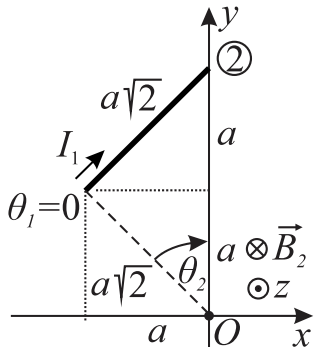
K1 Z1

a)



$\boxed{\vec{B}_1 = 0}$ $\quad \sin\sphericalangle\left(\vec{dl}, \vec{r}_0\right) = 0$

$\boxed{\vec{B}_3 = 0}$ $\quad \sin\sphericalangle\left(\vec{dl}, \vec{r}_0\right) = 0$

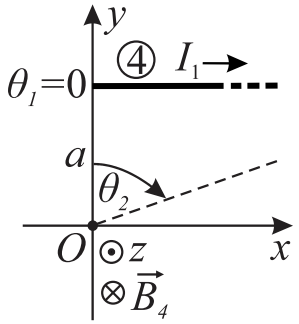


$\boxed{2}$ $\quad d = a\sqrt{2}, \quad \theta_1 = 0, \quad \theta_2 = \frac{\pi}{4}$

$$B_2 = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi a\sqrt{2}}\left(\frac{\sqrt{2}}{2} - 0\right) = \frac{\mu_0 I_1}{4\pi a\sqrt{2}}\frac{\sqrt{2}}{2}$$

$$B_2 = \frac{\mu_0 I_1}{8\pi a}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I_1}{8\pi a}\cdot\left(-\vec{i}_z\right)}$$



$\boxed{4}$ $\quad d = a, \quad \theta_1 = 0, \quad \theta_2 \to \frac{\pi}{2}$

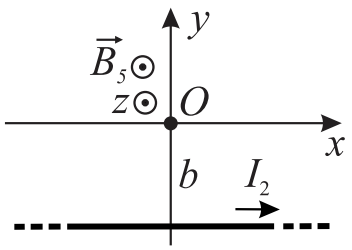
$$B_4 = \frac{\mu_0 I_1}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I_1}{4\pi a}\left(1 - 0\right)$$

$$B_4 = \frac{\mu_0 I_1}{4\pi a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I_1}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I_1}{8\pi a}\cdot\left(-\vec{i}_z\right) + \frac{\mu_0 I_1}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

b)



$$B_5 = \frac{\mu_0 I_2}{2\pi b} \qquad \vec{B}_5 = \frac{\mu_0 I_2}{2\pi b}\cdot\vec{i}_z$$

$$\vec{B}_{Ouk} = 0 \quad \Rightarrow \quad \frac{\mu_0 I_1}{8\pi a}\cdot\left(-\vec{i}_z\right) + \frac{\mu_0 I_1}{4\pi a}\cdot\left(-\vec{i}_z\right) + \frac{\mu_0 I_2}{2\pi b}\cdot\vec{i}_z = 0$$

$$\frac{3\mu_0 I_1}{8\pi a}\cdot\left(-\vec{i}_z\right) = \frac{\mu_0 I_2}{2\pi b}\cdot\left(-\vec{i}_z\right)$$

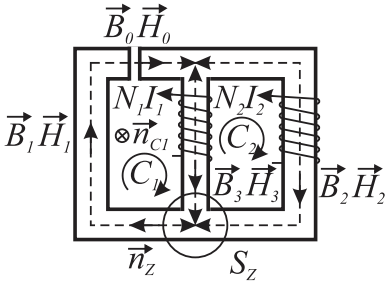
$$\frac{3\mu_0 I_1}{8\pi a} = \frac{\mu_0 I_2}{2\pi b}$$

$$b = \frac{4a}{3}\frac{I_2}{I_1}$$

$$\boxed{b = 5{,}12 \text{ cm}}$$

K1 Z2

a)



$B_1S_1 = B_2S_2 + B_3S_3$

$C_1 : H_1(l_1 - l_0) + H_0l_0 + H_3l_3 = N_1I_1,$

$C_2 : H_2l_2 - H_3l_3 = N_2I_2 - N_1I_1$

$\Phi_0 = B_0S_0 = B_0S_1$

$B_0 = \dfrac{0{,}15\cdot10^{-3}}{10^{-4}}$

$\boxed{B_0 = 1{,}5\,\mathrm{T}}$

$B_0S_0 = B_1S_1 \quad S_0 = S_1$

$\boxed{B_1 = B_0 = 1{,}5\,\mathrm{T}}$

$H_0 = \dfrac{B_0}{\mu_0} = \dfrac{1{,}5}{4\pi\cdot10^{-7}}$

$\boxed{H_0 = 1194\,\dfrac{\mathrm{kA}}{\mathrm{m}}}$

$B - B_{k1} = \dfrac{B_{k2} - B_{k1}}{H_{k2} - H_{k1}}(H - H_{k1}) \quad \Rightarrow \quad B_1 - 1 = \dfrac{1{,}6 - 1}{2000 - 500}(H_1 - 500)$

$\boxed{H_1 = 1{,}75\,\dfrac{\mathrm{kA}}{\mathrm{m}}}$

$H_3 = \dfrac{N_1I_1 - H_1(l_1 - l_0) - H_0l_0}{l_3}$

$\boxed{H_3 = 0{,}2\,\dfrac{\mathrm{kA}}{\mathrm{m}}}$

$B_3 = H_3\dfrac{1}{500}$

$\boxed{B_3 = 0{,}4\,\mathrm{T}}$

$B_2 = \dfrac{B_1S_1 - B_3S_3}{S_2}$

$\boxed{B_2 = 1{,}3\,\mathrm{T}}$

$B_2 - 1 = \dfrac{1{,}6 - 1}{2000 - 500}(H_2 - 500)$

$\boxed{H_2 = 1{,}25\,\dfrac{\mathrm{kA}}{\mathrm{m}}}$

$I_2 = \dfrac{H_2l_2 - H_3l_3 + N_1I_1}{N_2}$

$\boxed{I_2 \approx 3\,\mathrm{A}}$

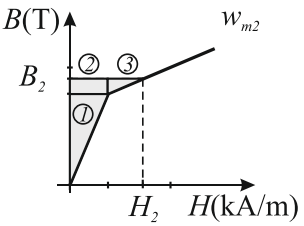
b)

$L = \dfrac{N_1\Phi_3}{I_1} = \dfrac{N_1B_3S_3}{I_1}$

$\boxed{L = 0{,}83\,\mathrm{mH}}$

c)

$W_{m2} = \omega_{m2}l_2S_2$

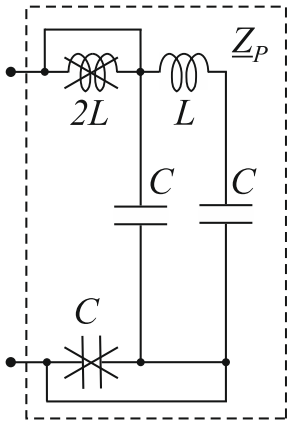


$w_{m2} = \dfrac{1}{2}B_{k1}H_{k1} + (B_2 - B_{k1})H_{k1} + \dfrac{1}{2}(B_2 - B_{k1})(H_2 - H_{k1}) = 512{,}5\,\dfrac{\mathrm{J}}{\mathrm{m}^3},$

$\boxed{W_{m2} = 10{,}25\,\mathrm{mJ}}$

K2 Z1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

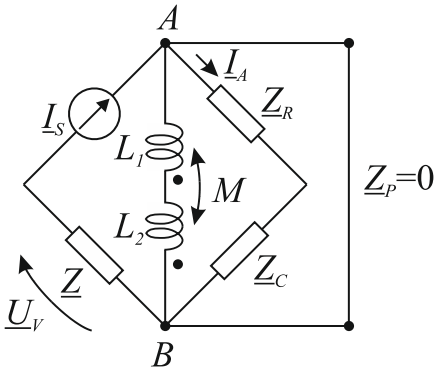
$$\underline{Z}_2 = \underline{Z}_1 \,||\, \frac{1}{j\omega C} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C}+\frac{1}{j\omega C}} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{2-\omega^2 LC}{j\omega C}}$$

$$\boxed{\underline{Z}_P = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}}$$

$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad \boxed{\omega_{r1} = \sqrt{\frac{1}{LC}}} \qquad \boxed{\omega_{r2} \to \infty}$

$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad \boxed{\omega_{ar1} = 0} \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$

b)



$$\omega_{r1} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4 \ \frac{\mathrm{rad}}{\mathrm{s}} = \omega$$

$\underline{Z}_P = 0$

$\underline{U}_{AB} = 0 \quad \Rightarrow \quad \underline{I}_A = 0$

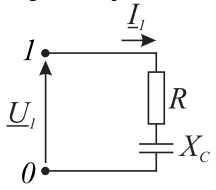
$\boxed{I_A = |\underline{I}_A| = 0\ \mathrm{A}}$

$\underline{U}_V = \underline{Z}\,\underline{I}_S = (5+j3)\cdot(1+j) = 5-3+j3+j5 = (2+j8)\,V$

$U_V = |\underline{U}_V| = \sqrt{2^2+8^2} \qquad \boxed{U_V = 8{,}25\ \mathrm{V}}$

K2 Z2

a) Pre kompenzacije

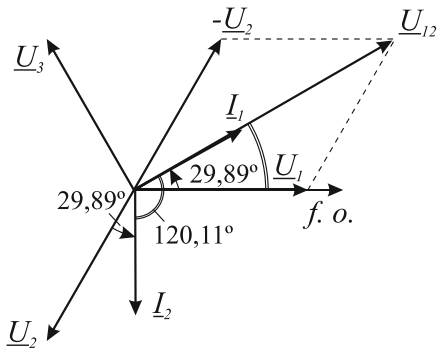


$$\underline{Z} = \underline{Z}_R + \underline{Z}_C = R + \left(\frac{-j}{\omega C}\right) = (20 - j11{,}47)\ \Omega = 23{,}1 \cdot e^{-j29{,}89^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\, e^{j0^\circ}}{Z\, e^{j\varphi}} = \frac{230}{23{,}1\, e^{-j29{,}89^\circ}} = 9{,}96\, e^{j29{,}89^\circ}\ \text{A}$$

$$P_{W1}{}^{pre} = \mathrm{Re}\left\{\underline{U}_{12}\, \underline{I}_2{}^*\right\} = U_{12}\, I_2 \cos\sphericalangle\left(\underline{U}_{12}, \underline{I}_2\right) = \sqrt{3} U_1 I_1 \cos\sphericalangle\left(\underline{U}_{12}, \underline{I}_2\right)$$

$$P_{W1}{}^{pre} = \sqrt{3} \cdot 230 \cdot 9{,}96 \cdot \cos(\underbrace{120 + 30 - 29{,}89^\circ}_{120{,}11^\circ}) = -1990{,}48\,\text{W}$$



$$\boxed{P_{W1}{}^{pre} = -1990{,}48\,\text{W}}$$

$$P_{W2}{}^{pre} = \mathrm{Re}\left\{\underline{U}_1\, \underline{I}_1{}^*\right\} = U_1\, I_1 \cos\sphericalangle\left(\underline{U}_1, \underline{I}_1\right)$$

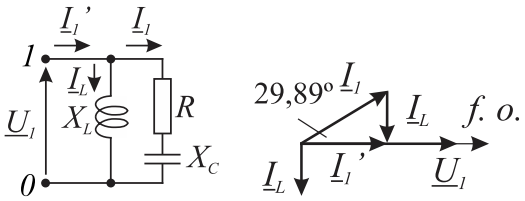
$$P_{W2}{}^{pre} = U_1\, I_1 \cos 29{,}89^\circ = 230 \cdot 9{,}96 \cdot \cos 29{,}89^\circ = 1986{,}07\,\text{W}$$

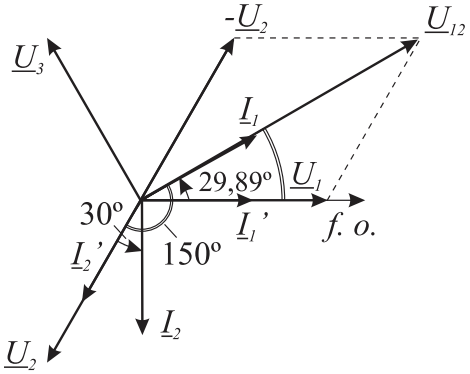
$$\boxed{P_{W2}{}^{pre} = 1986{,}07\,\text{W}}$$

$$U_V{}^{pre} = \left|\underline{U}_{23}\right| = \sqrt{3} \cdot U_1 = \sqrt{3} \cdot 230$$

$$\boxed{U_V{}^{pre} = 398{,}37\,\text{V}}$$

b) Posle kompenzacije





I KZ: $\underline{I}_1' = \underline{I}_1 + \underline{I}_L$

$$\underline{I}_L = \frac{U_1\, e^{j0}}{jX_L} = \frac{U_1\, e^{j0}}{X_L\, e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L} e^{-j\frac{\pi}{2}}\ \text{A}$$

$$I_L = I_1 \sin 29{,}89^\circ$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z} \sin 29{,}89^\circ \rightarrow \frac{1}{X_L} = \frac{\sin 29{,}89^\circ}{Z} \rightarrow X_L = \frac{23}{\sin 29{,}89^\circ}$$

$$\boxed{X_L = 46{,}15\,\Omega}$$

$$I_1' = I_1 \cos 29{,}89^\circ = 9{,}96 \cdot 0{,}87 = 8{,}64\,\text{A}$$

$$P_{W1}{}^{posle} = \mathrm{Re}\left\{\underline{U}_{12}\, \underline{I}_2{}'^*\right\} = U_{12}\, I_2' \cos\sphericalangle\left(\underline{U}_{12}, \underline{I}_2'\right) = \sqrt{3} U_1 I_1' \cos\sphericalangle\left(\underline{U}_{12}, \underline{I}_2'\right)$$

$$P_{W1}{}^{posle} = \sqrt{3} \cdot 230 \cdot 8{,}64 \cdot \cos(\underbrace{30^\circ + 120^\circ}_{150^\circ}) = -2980{,}8\,\text{W}$$

$$\boxed{P_{W1}{}^{posle} = -2980{,}8\,\text{W}}$$

$$P_{W2}{}^{posle} = \mathrm{Re}\left\{\underline{U}_1\, \underline{I}_1{}^*\right\} = P_{W2}{}^{pre}$$

$$\boxed{P_{W2}{}^{posle} = 1986{,}07\,\text{W}}$$

$$U_V{}^{posle} = \left|\underline{U}_{23}\right| = U_V{}^{pre}$$

$$\boxed{U_V{}^{posle} = 398{,}37\,\text{V}}$$