STUDIJSKI PROGRAM MEHATRONIKA

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

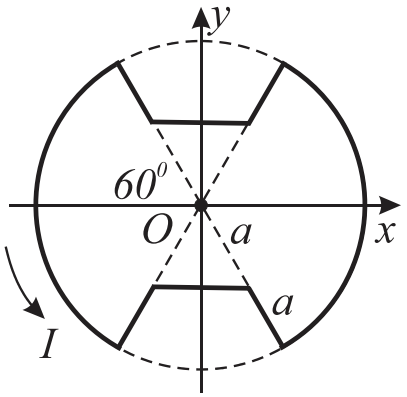
K1

27.10.2025.

ZADACI

Zadatak 1. Žičani provodnik sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Kontura se nalazi u x-y ravni Dekartovog pravouglog koordi-natnog sistema. Odrediti intenzitet vektor magnetske indu-kcije u koordinatnom početku. Sredina je vazduh.

Brojni podaci su: $I = 3$ A, $a = 2$ cm.

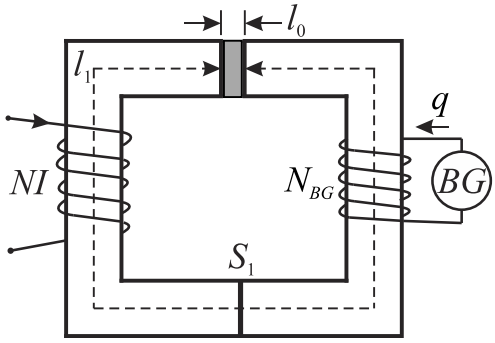


Slika 1.

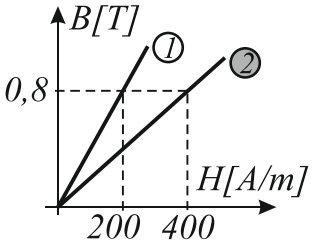
Zadatak 2. Tanko magnetsko kolo sa slike 2a načinjeno je od materijala 1, čija je idealizovana kriva prvobitnog magnetisanja prikazana na slici 2b. Kolo je napravljeno sa uskim procepom, širine l_0, u koji je ubačen komad magnetskog materijala 2, koji savršeno naleže na ivice, i čija je idealizovana kriva prvobitnog magnetisanja takođe prikazana na slici 2b.

a) Izračunati intenzitet struje, I, kroz pobudni namotaj sa N zavojaka, ako prilikom uspostavljanja magnetskog polja u jezgru, kroz balistički galvanometar protekne količina naelektrisanja $q = 50$ μC, u naznačenom referentnom smeru. Pre uključivanja struje, jezgro je bilo nenamagnetisano.
b) Izračunati promenu magnetskog fluksa unutar jezgra, koja bi nastala ukoliko bi stranom, mehaničkom, silom iz procepa izbacili komad načinjen od materijala 2. Rasipanje magnetskog fluksa zanemariti.

Ostali brojni podaci su: $N = 5$, $l_1 = 20$ cm, $l_0 = 0{,}1$ mm, $S_1 = 1$ cm^2, $N_{BG} = 5$, $R_{BG} = 10$ Ω.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 2 (II kolokvijum)

K2

27.10.2025.

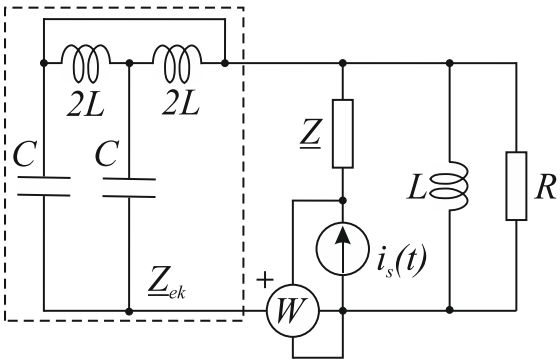
ZADACI

Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_{ek}$.

b) Ako u kolu strujni generator $i_s(t)$ radi na većoj antirezonantnoj učestanosti, odrediti pokazivanje vatmetra i ampitudu napona na otporniku.

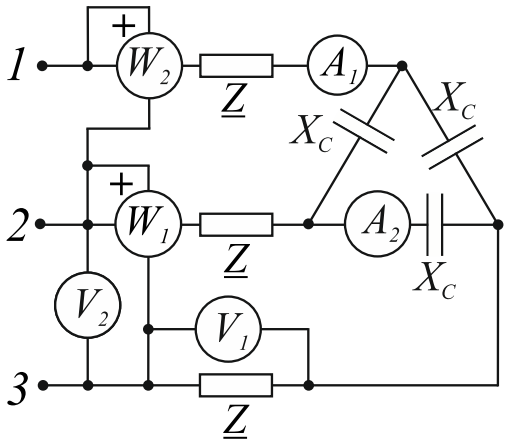
Brojni podaci su: $i_s(t) = 2\sqrt{3}\cos(\omega t + 75^o)$ A,
L = 10 mH, C = 200 μF, R = 5 Ω, $\underline{Z}$ = (5–j3) Ω.



Slika 1.

Zadatak 2. Trofazni prijemnik reaktanse X_C = 30 Ω, priključen je na simetričnu trofaznu mrežu faznog napona prve faze $\underline{U}_1$ = 230 V, preko trofaznog voda impedanse $\underline{Z} = (5 + j15)$ Ω, kao što je prikazano na slici 2. Odrediti:

a) pokazivanje idealnih mernih instrumenata i
b) skicirati fazorski dijagram svih veličina od kojih zavisi pokazivanje mernih instrumenata.



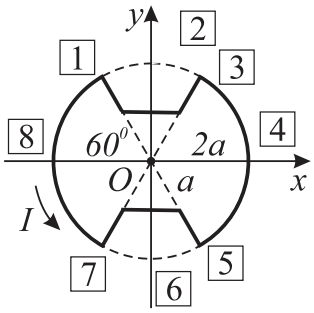
Slika 2.

PRAVILA POLAGANJA

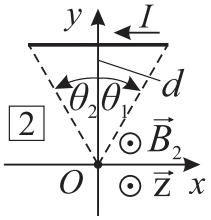
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MH ISPIT, K1, 27.10.2025.

K1 Z1



$$\vec{B}_1=\vec{B}_3=\vec{B}_5=\vec{B}_7=0 \qquad \left(I\,\vec{dl}\times\vec{r}_0=0\right)$$



$$\theta_1=-\frac{\pi}{6}$$

$$\theta_2=\frac{\pi}{6}$$

$$d=\frac{a\sqrt{3}}{2}$$

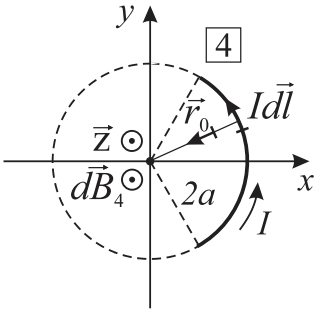
$$B_2=\frac{\mu_0 I}{4\pi d}\left(\sin\theta_2-\sin\theta_1\right)=\frac{\mu_0 I}{4\pi\frac{a\sqrt{3}}{2}}\cdot\left(\sin\frac{\pi}{6}-\sin\left(-\frac{\pi}{6}\right)\right)$$

$$B_2=\frac{\mu_0 I}{4\pi\frac{a\sqrt{3}}{2}}\cdot\left(\frac{1}{2}-\left(-\frac{1}{2}\right)\right)=\boxed{\frac{\mu_0 I}{2\sqrt{3}\pi a}}$$

$$\boxed{B_2=B_6}$$

$$\boxed{\vec{B}_2=B_2\cdot\vec{i}_z}$$

$$\boxed{\vec{B}_6=B_6\cdot\vec{i}_z}$$



$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2}$$

$$dB_4=\frac{\mu_0}{4\pi}\frac{Idl}{(2a)^2}\sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right)=\frac{\pi}{2}$$

$$dB_4=\frac{\mu_0}{4\pi}\frac{Idl}{4a^2}$$

$$B_4=\frac{\mu_0 I}{16\pi a^2}\int_0^{\frac{2\pi}{3}\cdot 2a} dl=\frac{\mu_0 I}{16\pi a^2}\cdot\frac{4\pi a}{3}=\boxed{\frac{\mu_0 I}{12a}}$$

$$\boxed{B_4=B_8}$$

$$\boxed{\vec{B}_4=B_4\cdot\vec{i}_z}$$

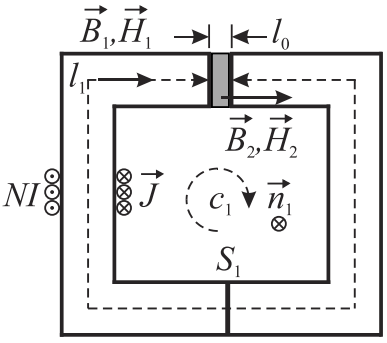
$$\boxed{\vec{B}_8=B_8\cdot\vec{i}_z}$$

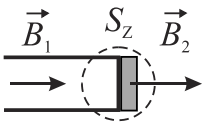
$$\boxed{\vec{B}_o=\vec{B}_2+\vec{B}_4+\vec{B}_6+\vec{B}_8=\left(2\cdot B_2+2\cdot B_4\right)\cdot\vec{i}_z=\left(\frac{\mu_0 I}{\sqrt{3}\pi a}+\frac{\mu_0 I}{6a}\right)\cdot\vec{i}_z}$$

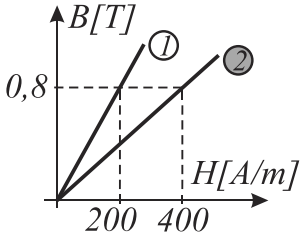
$$\boxed{B_o=66{,}06\,\mu\mathrm{T}}$$

K1 Z2

a)



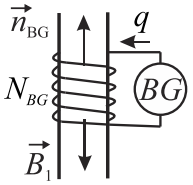




$$\oint_{S_Z}\vec{B}\cdot\vec{ds}=0 \qquad \oint_{C}\vec{H}\cdot\vec{dl}=NI$$

$$S_Z: \quad -B_1S_1+B_2S_2=0 \qquad c_1: \quad H_1l_1+H_2l_0=NI \qquad (2)$$

$$(S_1=S_2) \qquad B_1=B_2 \qquad (1)$$



$$q=-\frac{N_{BG}}{R_{BG}}\left(\Phi_{1KR}-\cancel{\Phi_{1POČ}}^{\,0}\right)=-\frac{N_{BG}}{R_{BG}}\left(\int_{S_1}\vec{B}_1\cdot\vec{ds}\right)=-\frac{N_{BG}}{R_{BG}}\left(-B_1S_1\right)=\frac{N_{BG}}{R_{BG}}B_1S_1 \qquad \sphericalangle\left(\vec{B}_1,\vec{n}_{BG}\right)=\pi$$

$$B_1=\frac{R_{BG}\,q}{N_{BG}\,S_1}=\frac{10\cdot50\cdot10^{-6}}{5\cdot1\cdot10^{-4}}=1\,\mathrm{T} \quad\Rightarrow\quad H_1=\frac{B_1}{\mu_1}=250\ \mathrm{A/m} \qquad \mu_1=\frac{0{,}8}{200}$$

Iz (1) sledi $B_2=B_1=1\,\mathrm{T} \quad\Rightarrow\quad H_2=\frac{B_2}{\mu_2}=500\ \mathrm{A/m} \qquad \mu_2=\frac{0{,}8}{400}$

Iz (2) sledi $I=\frac{H_1l_1+H_2l_0}{N}=\frac{250\cdot0{,}2+500\cdot0{,}0001}{5}$ $\qquad \boxed{I=10{,}01\,\mathrm{A}}$

b)

$$\oint_{S_Z}\vec{B}\cdot\vec{ds}=0 \qquad \oint_{C}\vec{H}\cdot\vec{dl}=NI$$

$$S_Z: \quad -B_1^{\,N}S_1+B_0S_0=0 \qquad c_1: \quad H_1^{\,N}l_1+H_0l_0=NI \qquad (4)$$

$$(S_1=S_0) \qquad B_1^{\,N}=B_0 \qquad (3) \qquad H_0=\frac{B_0}{\mu_0} \qquad (5)$$

Iz (3), (4) i (5) sledi $\frac{B_1^{\,N}}{\mu_1}l_1+\frac{B_0}{\mu_0}l_0=NI$

$$\frac{B_1^{\,N}}{\mu_1}l_1+\frac{B_1^{\,N}}{\mu_0}l_0=NI$$

$$B_1^{\,N}=\frac{NI}{\frac{l_1}{\mu_1}+\frac{l_0}{\mu_0}}=\frac{5\cdot10{,}01}{\frac{0{,}2}{0{,}8/200}+\frac{0{,}0001}{4\pi\cdot10^{-7}}}=\frac{50{,}05}{50+79{,}62}=0{,}39\,\mathrm{T}$$

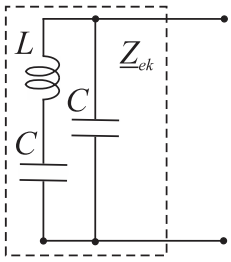
$$\Delta\Phi=\Phi^N-\Phi=B_1^{\,N}S_1-B_1S_1=0{,}39\cdot1\cdot10^{-4}-1\cdot1\cdot10^{-4}$$

$\boxed{\Delta\Phi=-61\,\mu\mathrm{Wb}}$

MH ISPIT, K2, 27.10.2025.

K2 Z1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

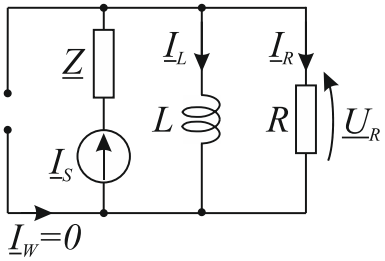
$$\underline{Z}_2 = \underline{Z}_1 \,\|\, \frac{1}{j\omega C} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{1-\omega^2 LC}{j\omega C}+\dfrac{1}{j\omega C}} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{2-\omega^2 LC}{j\omega C}}$$

$$\boxed{\underline{Z}_{ek} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}}$$

$\mathrm{Im}\{\underline{Z}_{ek}\} = 0 \quad \Rightarrow \quad \boxed{\omega_{r1} = \sqrt{\dfrac{1}{LC}}} \qquad \boxed{\omega_{r2} \to \infty}$

$\mathrm{Im}\{\underline{Y}_{ek}\} = 0 \quad \Rightarrow \quad \boxed{\omega_{ar1} = 0} \qquad \boxed{\omega_{ar2} = \sqrt{\dfrac{2}{LC}}}$

b)



$$\omega_{ar2} = \sqrt{\frac{2}{LC}} = \sqrt{\frac{2}{10\cdot 10^{-3}\cdot 200\cdot 10^{-6}}} = \sqrt{\frac{2}{2000\cdot 10^{-9}}} = \sqrt{\frac{1}{1000\cdot 10^{-9}}} = \sqrt{10^6} = 10^3\ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$i_S(t) = 2\sqrt{3}\cos\left(\omega t + 75^\circ\right)\ \mathrm{A}$$

$$\underline{I}_S = \frac{2\sqrt{3}}{\sqrt{2}}e^{j75^\circ}\ \mathrm{A} = 2{,}45\,e^{j75^\circ}\ \mathrm{A} = \left(0{,}63 + j2{,}37\right)\mathrm{A}$$

$\underline{I}_W = 0\ \mathrm{A}$

$\boxed{P_W = 0\ \mathrm{W}}$

$\underline{Z}_L = j\omega L = j\cdot 10^3 \cdot 10\cdot 10^{-3} = j10\ \Omega$

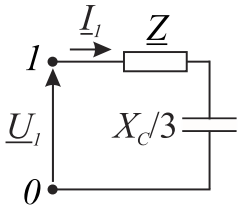
$\underline{I}_R = \dfrac{\underline{Z}_L}{\underline{Z}_L + R}\cdot \underline{I}_S = \dfrac{j10}{j10+5}\cdot\left(0{,}63 + j2{,}37\right) = \left(-0{,}444 + j2{,}148\right)\mathrm{A}$

$\underline{U}_R = R\cdot \underline{I}_R = 50\cdot\left(-0{,}444 + j2{,}148\right) = \left(-22{,}2 + j107{,}4\right)\mathrm{V} = 109{,}67\cdot \mathrm{e}^{j101{,}68^\circ}\ \mathrm{V}$

$\boxed{U_{Rm} = 109{,}67\cdot\sqrt{2} = 155{,}1\ \mathrm{V}}$

K2 Z2

a)



$$\underline{Z}_e = \underline{Z} + \left(-j\frac{X_C}{3}\right) = 5 + j15 - j10 = (5 + j5)\,\Omega = 5\sqrt{2}\,e^{j45^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}_e} = \frac{230\,e^{j0^\circ}}{5\sqrt{2}\,e^{j45^\circ}} = 32{,}53\,e^{-j45^\circ}\ \text{A}$$

$$\boxed{I_{A1} = |\underline{I}_1| = 32{,}53\ \text{A}}$$

$$U_{V1} = |\underline{Z}|\,|\underline{I}_3| = \sqrt{5^2 + 15^2}\cdot 32{,}53 \qquad \boxed{U_{V1} = 514{,}34\ \text{V}}$$

$$U_{V2} = |\underline{U}_{23}| = \sqrt{3}\,|\underline{U}_1| = \sqrt{3}\cdot 230 \qquad \boxed{U_{V2} = 398{,}37\ \text{V}}$$

$$P_{W1} = \text{Re}\left\{\underline{U}_{23}\,\underline{I}_2^{\,*}\right\} = U_{23}\,I_2\cos\sphericalangle\left(\underline{U}_{23}, \underline{I}_2\right) = \sqrt{3}\,U_1\,I_1\cos(45^\circ + 30^\circ) = \sqrt{3}\cdot 230\cdot 32{,}53\cdot\cos 75^\circ$$

$$\boxed{P_{W1} = 3354{,}04\ \text{W}}$$

$$P_{W2} = \text{Re}\left\{\underline{U}_{12}\,\underline{I}_1^{\,*}\right\} = U_{12}\,I_1\cos\sphericalangle\left(\underline{U}_{12}, \underline{I}_1\right) = \sqrt{3}\,U_1\,I_1\cos(45^\circ + 30^\circ) = \sqrt{3}\cdot 230\cdot 32{,}53\cdot\cos 75^\circ$$

$$\boxed{P_{W2} = 3354{,}04\ \text{W}}$$

$$\boxed{I_{A2} = \frac{|\underline{I}_1|}{\sqrt{3}} = 18{,}8\ \text{A}}$$

b)