

Zaštita životne sredine i zaštita na radu

Elektrotehnika, okolina i zaštita

Prvi deo kursa - 7 nedelja

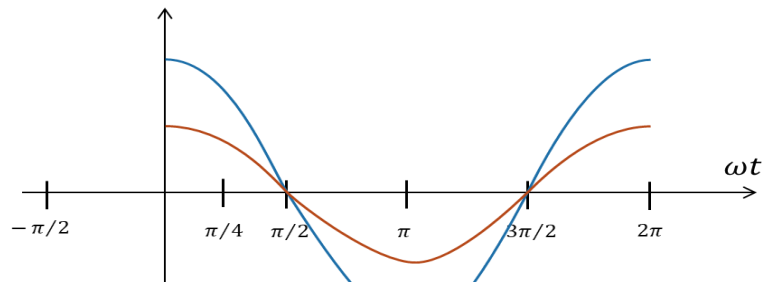
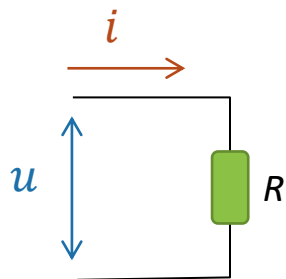
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Drugi deo kursa – 7 nedelja

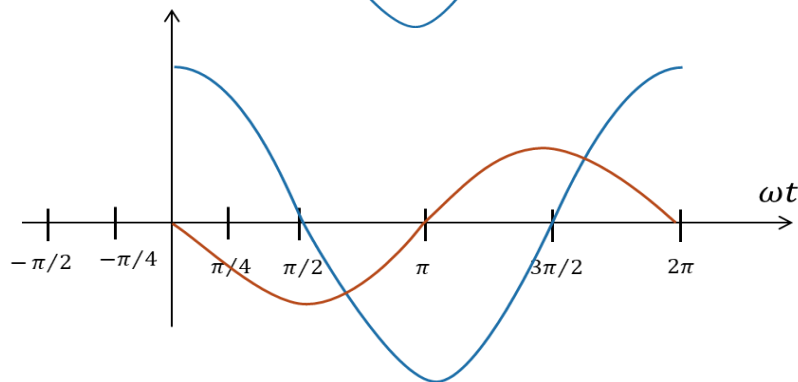
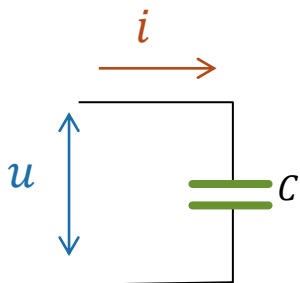
dr Marko Vekić

Podsećanje sa prethodnog časa ...



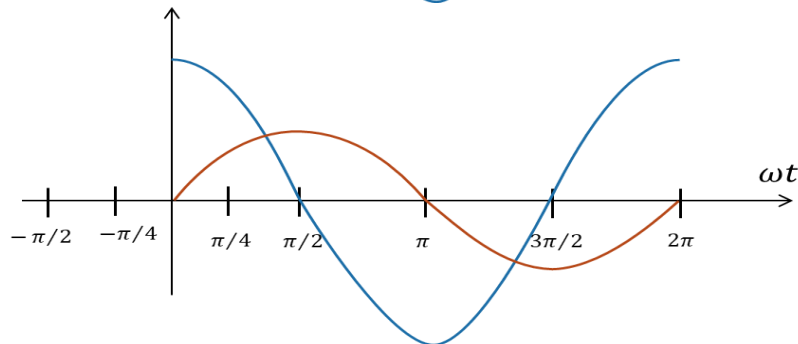
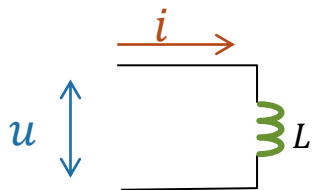
$$I = \frac{U}{R}$$

$$\psi = \theta$$



$$I = U\omega C$$

$$\psi = \theta + \frac{\pi}{2}$$



$$I = \frac{U}{\omega L}$$

$$\psi = \theta - \frac{\pi}{2}$$

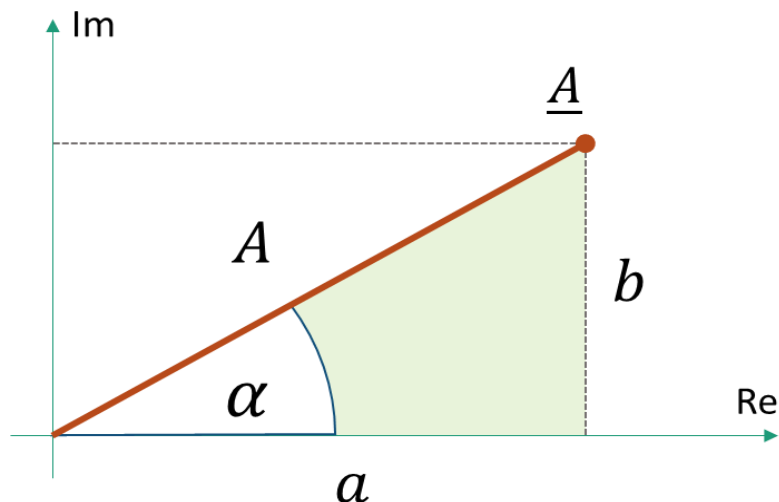
Kompleksni brojevi

Eksponencijalni oblik

$$\underline{A} = A e^{j\alpha}$$

A – moduo

α – argument



Algebarski oblik

$$\underline{A} = a + jb$$

a – realni deo

b – imaginarni deo

$$a = A \cos \alpha, \quad b = A \sin \alpha$$

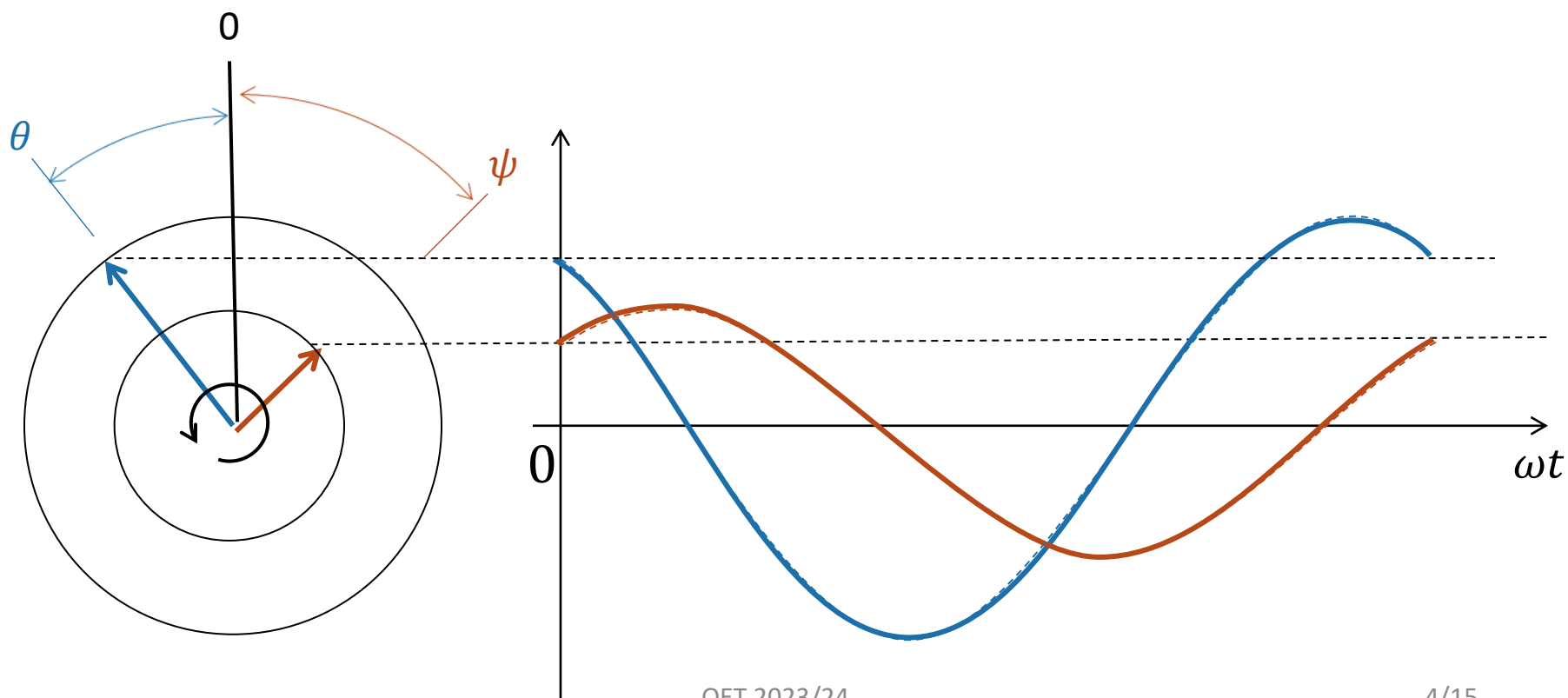
$$A = \sqrt{a^2 + b^2}, \quad \alpha = \tan^{-1} \frac{b}{a}$$

Kompleksni predstavnici napona i struje

Fazori – obrtni vektori

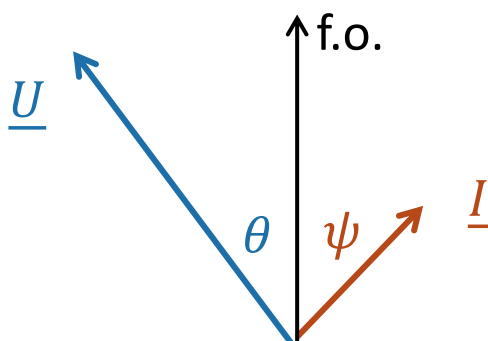
$$u = U_m \cos(\omega t + \theta)$$

$$i = I_m \cos(\omega t + \psi)$$

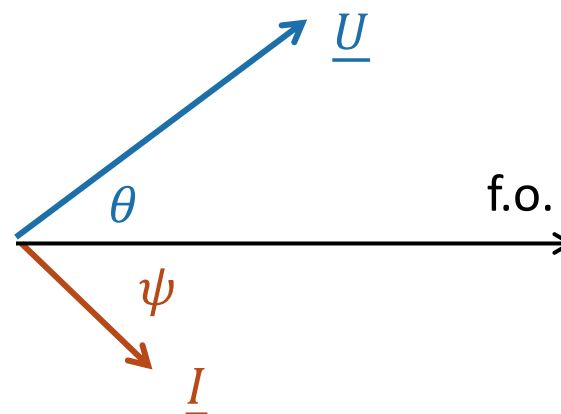


Kompleksni predstavnici napona i struje

Vremenski oblik	$u = U_m \cos(\omega t + \theta)$	$i = I_m \cos(\omega t + \psi)$
Kompleksni predstavnik moduo, argument	$\underline{U} = U e^{j\theta}$	$\underline{I} = I e^{j\psi}$
Moduo = efektivna vrednost	$U = U_m / \sqrt{2}$	$I = I_m / \sqrt{2}$
Argument = početna faza	θ	ψ



ili



Kompleksni predstavnici napona i struje

PRIMER

$$u = 28,2 \cos(\omega t + \pi/3) \text{ V}$$

$$U_m = 28,2 \text{ V} \quad U = 20 \text{ V}$$

$$\theta = +\pi/3$$

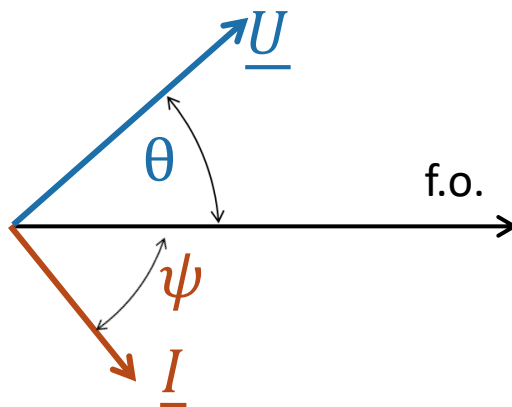
$$\underline{U} = 20e^{+j\pi/3} \text{ V}$$

PRIMER

$$i = 10\sqrt{2}\cos(\omega t - \pi/4) \text{ A}$$

$$I_m = 10\sqrt{2} \text{ A} \quad I = 10 \text{ A}$$

$$\psi = -\pi/4 \quad \underline{I} = 10e^{-j\pi/4} \text{ A}$$



Kompleksni predstavnici napona i struje

$$\underline{U} = U e^{j\theta}$$

$$\underline{U} = U \cos \theta + jU \sin \theta$$

$$\underline{I} = I e^{j\psi}$$

$$\underline{I} = I \cos \psi + jI \sin \psi$$

PRIMER

$$\begin{aligned}\underline{U} &= 20e^{+j\pi/3} \text{ V} = \\ &= 20 \left(\cos \left(\frac{\pi}{3} \right) + j \sin \left(\frac{\pi}{3} \right) \right) \text{ V}\end{aligned}$$

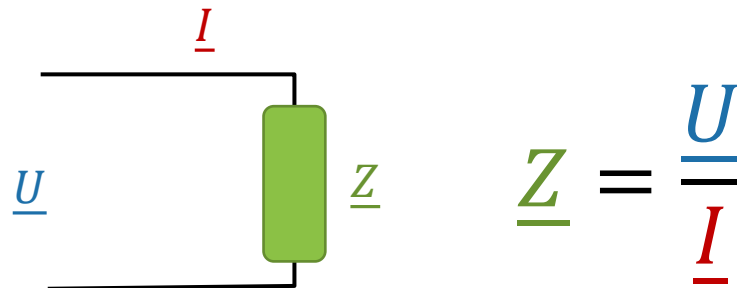
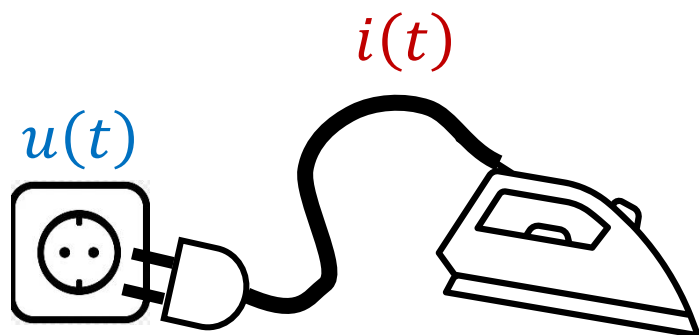
$$\underline{U} = (10 + j10\sqrt{3}) \text{ V}$$

PRIMER

$$\begin{aligned}\underline{I} &= 10e^{-j\pi/4} \text{ A} = \\ &= 10 \left(\cos \left(-\frac{\pi}{4} \right) + j \sin \left(-\frac{\pi}{4} \right) \right) \text{ A}\end{aligned}$$

$$\underline{I} = (5\sqrt{2} - j5\sqrt{2}) \text{ A}$$

Impedansa prijemnika



Prelaskom u kompleksni domen koristimo kompleksne predstavnike napona i struje.

- Količnik **kompleksnog napona** i **kompleksne struje** predstavlja **IMPEDANSU** prijemnika.
- Oznaka $\underline{Z}$, jedinica om (Ω).

Impedansa prijemnika

$$\underline{Z} = \frac{U e^{j\theta}}{I e^{j\psi}} = \frac{U}{I} e^{j(\theta - \psi)} = Z e^{j\varphi}$$

$$Z = \frac{U}{I} \quad \varphi = \theta - \psi$$

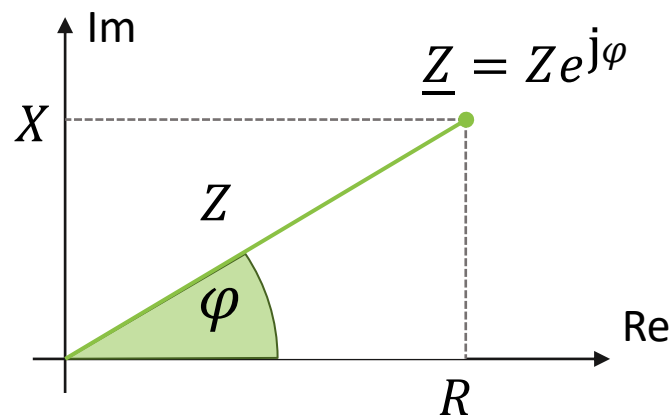
$$\underline{Z} = Z \cos(\varphi) + jZ \sin(\varphi) = R + jX$$

Z - Moduo $Z > 0$

φ - Argument $\varphi \in \left\{-\frac{\pi}{2}, +\frac{\pi}{2}\right\}$

R - Rezistansa $R \geq 0$

X - Reaktansa $X \in \mathbb{R}$



Prijemnik:

$\varphi > 0$ ($X > 0$) – induktivni

$\varphi < 0$ ($X < 0$) – kapacitivni

$\varphi = 0$ ($X = 0$) – rezistivni

Impedansa prijemnika

Odrediti impedansu prijemnika ako su napon i struja prijemnika:

$$u = 220\sqrt{2} \cos(\omega t + \pi/3) \text{ V} \quad i = 10\sqrt{2} \cos(\omega t + \pi/6) \text{ A}$$

$$\underline{U} = 220e^{j\pi/3} \text{ V}$$

$$\underline{Z} = \frac{220e^{j\pi/3}}{10e^{j\pi/6}} = 22e^{j\pi/6} \Omega$$

$$\underline{I} = 10e^{j\pi/6} \text{ A}$$

$$\varphi = \theta - \psi = \pi/6 > 0 \text{ – induktivni prijemnik}$$

$$\theta > \varphi$$

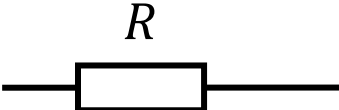
$$\underline{Z} = 22 \cos(\pi/6) + j22 \sin(\pi/6)$$

$$\pi/3 > \pi/6$$

$$\underline{Z} = (19 + j11) \Omega$$

Prednjači napon

Impedanse osnovnih elemenata

Otpornost 

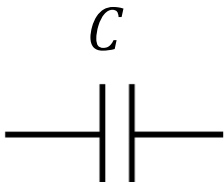
$$\underline{Z}_R = R + j0$$

$$\underline{Z} = Re^{j0}$$

Induktivnost 

$$\underline{Z}_L = 0 + j\omega L$$

$$\underline{Z} = \omega L e^{j\frac{\pi}{2}}$$

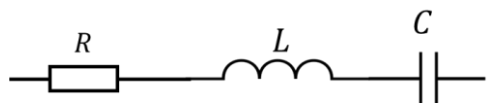
Kapacitivnost 

$$\underline{Z}_C = 0 - j\frac{1}{\omega C}$$

$$\underline{Z} = \frac{1}{\omega C} e^{-j\frac{\pi}{2}}$$

Impedanse osnovnih elemenata

Ako je struja kroz sva tri elementa $i = 10\sqrt{2}\cos(1000t - \pi/4)\text{A}$
odrediti napone na svim elementima.



$$R = 100\Omega \quad L = 50 \text{ mH} \quad C = 50 \mu\text{F}$$

$$\underline{I} = 10e^{-j\pi/4}\text{A}$$

$$\underline{Z}_L = j\omega L = j1000 \cdot 50 \cdot 10^{-3} = j50\Omega$$

$$\underline{Z}_L = 50e^{j\frac{\pi}{2}}\Omega$$

$$\underline{Z}_C = -j\frac{1}{\omega C} = -j\frac{1}{1000 \cdot 50 \cdot 10^{-6}} = -j\frac{10^6}{5 \cdot 10^4} = -j20\Omega$$

$$\underline{Z}_C = 20e^{-j\frac{\pi}{2}}\Omega$$

$$\underline{U}_R = R\underline{I} = 1000e^{-j\pi/4}\text{V}$$

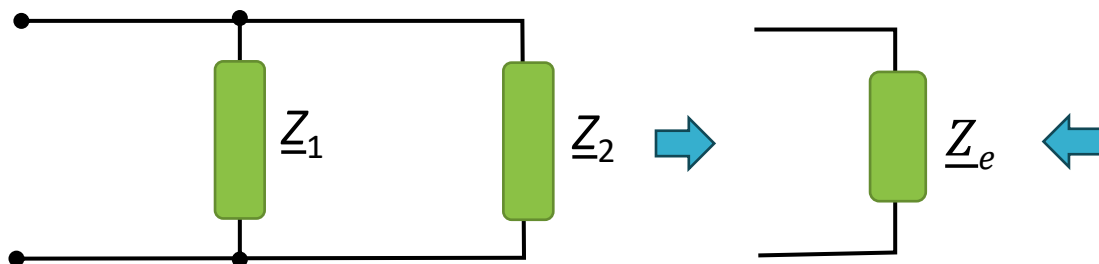
$$\underline{U}_L = \underline{Z}_L\underline{I} = 500e^{+j\pi/4}\text{V}$$

$$\underline{U}_C = \underline{Z}_C\underline{I} = 200e^{-j3\pi/4}\text{V}$$

Modelovanje prijemnika

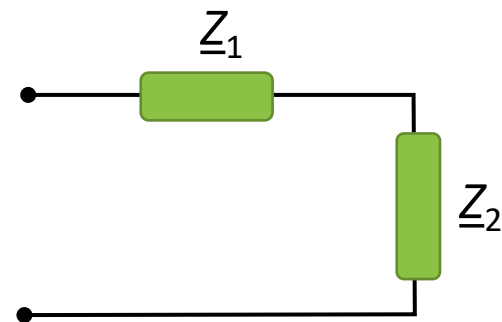
Prijemnici se u kolima prostoperiodične struje modeluju rednom, paralelenom i mešovitom vezom osnovnih elemenata.

Paralelna veza



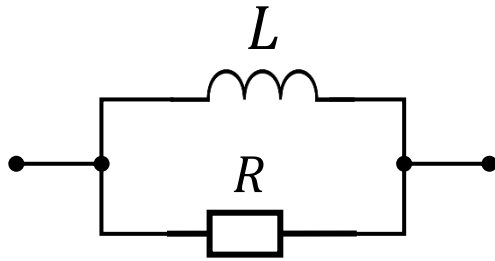
$$\frac{1}{\underline{Z}_e} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} \Rightarrow \underline{Z}_e = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

Redna veza



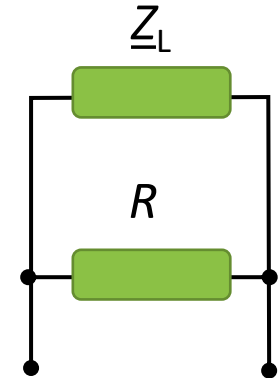
$$\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2$$

Primer



$$L = 30\text{mH}, \quad R = 40\Omega,$$

$$\omega = 1000\text{rad/s}, \quad \underline{Z}_{ek} = ?$$

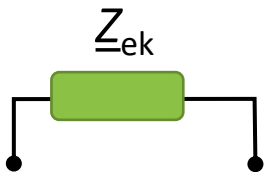


$$\underline{Z}_L = j\omega L = j1000 \cdot 30 \cdot 10^{-3} = j30\Omega$$

$$\underline{Z}_{ek} = \frac{\underline{Z}_L R}{\underline{Z}_L + R} = \frac{j30 \cdot 40}{j30 + 40} = \frac{j1200}{40 + j30} \cdot \frac{40 - j30}{40 - j30} = \frac{36000 + j48000}{40^2 + 30^2}$$

$$\underline{Z}_{ek} = (14,4 + j19,2)\Omega$$

$$\underline{Z}_{ek} = |\underline{Z}_{ek}|e^{j\varphi}$$



$$|\underline{Z}_{ek}| = \sqrt{R^2 + X^2} = 24\Omega$$

$$\varphi = \arg\{\underline{Z}_{ek}\}$$

$$\varphi = 53,13^\circ$$

Pitanja

1. Objasniti vezu između eksponencijalnog i algebarskog oblika kompleksnog broja.
2. Da li moduo kompleksnog broja može imati negativne vrednosti?
3. Objasniti vezu između vremenskog oblika napona/struje i njihovih kompleksnih predstavnika.
4. Nacrtati fazore sledećih veličina $u = 120 \cos(200t + \pi/6)V$ i $i = 12 \cos(200t - \pi/6)A$
5. Koja je veza efektivnih vrednosti napona i struje na otporniku, kalemu, kondenzatoru?
6. Kolike su fazne razlike napona i struje na otporniku, kalemu, kondenzatoru?
7. Šta je impedansa prijemnika?
8. Šta predstavlja moduo impedanse a šta njen argument?
9. Šta je rezistansa i kao se dobija? Šta je reaktansa i kako se dobija?
10. Koliko iznosi reaktansa otpornika, kalema i kondenzatora?