

Industrijsko inženjerstvo

Osnovi elektrotehnike

Prostoperiodične struje 2

- Impedansa
- Snaga

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Impedansa prijemnika

$$i = I_m \cos(\omega t + \psi)$$

$$u = U_m \cos(\omega t + \theta)$$

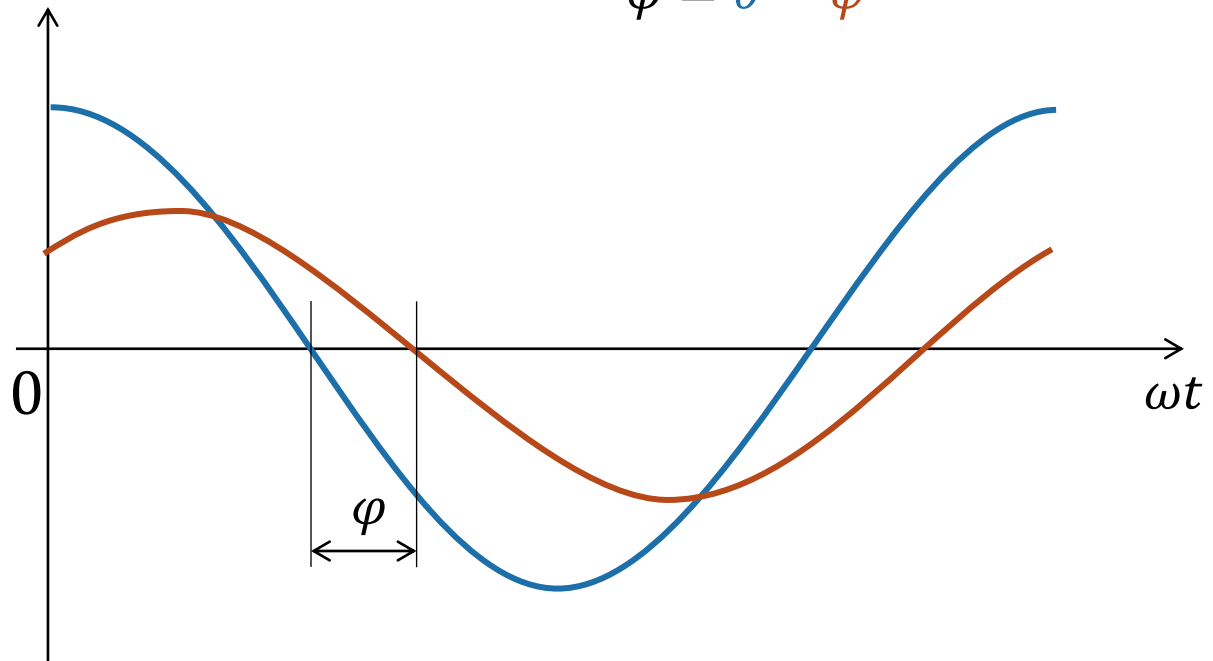
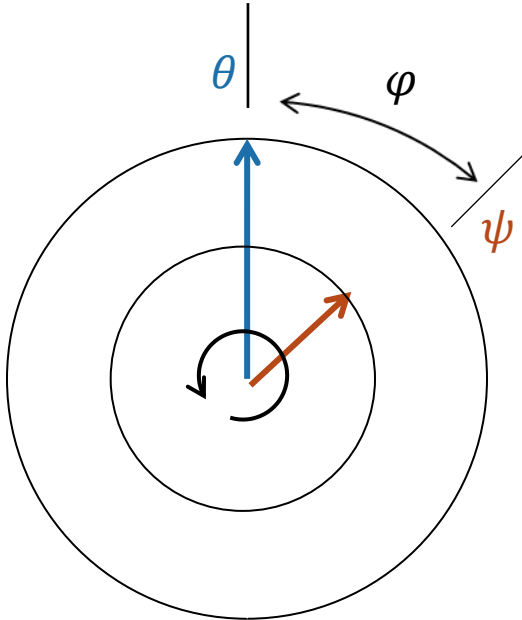
$$I = I_m / \sqrt{2}$$

$$U = U_m / \sqrt{2}$$

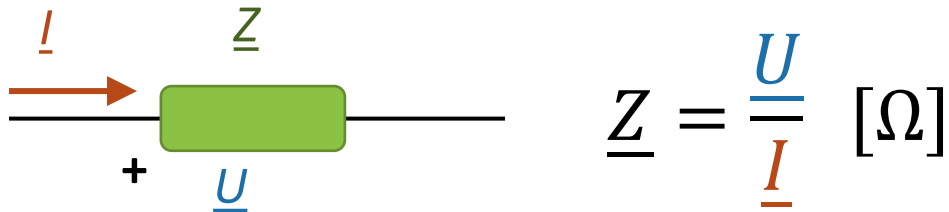
$$\underline{I} = I e^{j\psi}$$

$$\underline{U} = U e^{j\theta}$$

$$\varphi = \theta - \psi$$



Impedansa prijemnika



$$\underline{Z} = \frac{U e^{j\theta}}{I e^{j\psi}} = \frac{U}{I} e^{j(\theta - \psi)} = Z e^{j\varphi}$$

Moduo

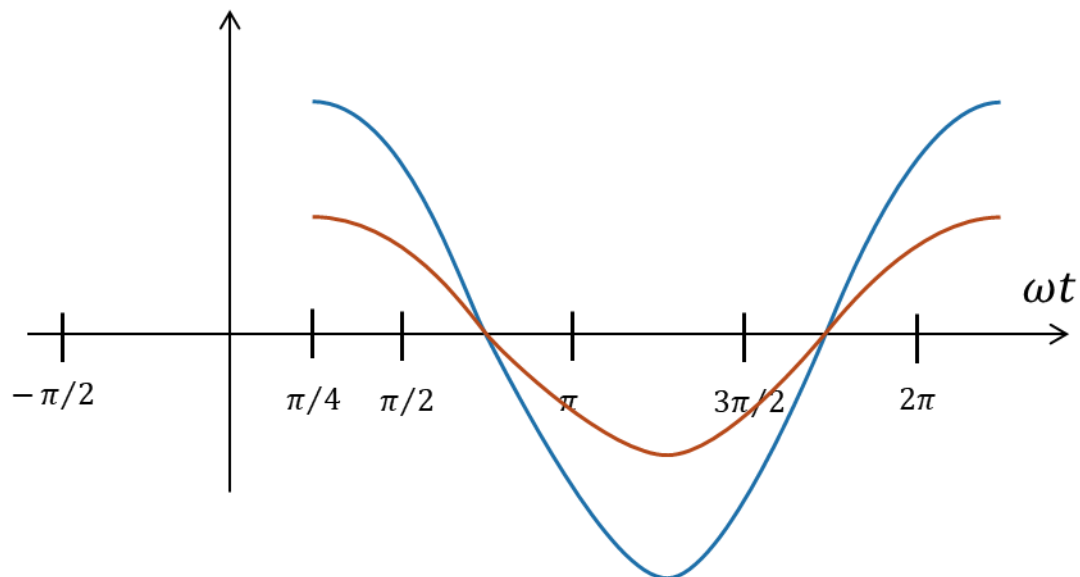
$$Z = \frac{U}{I}$$

Argument

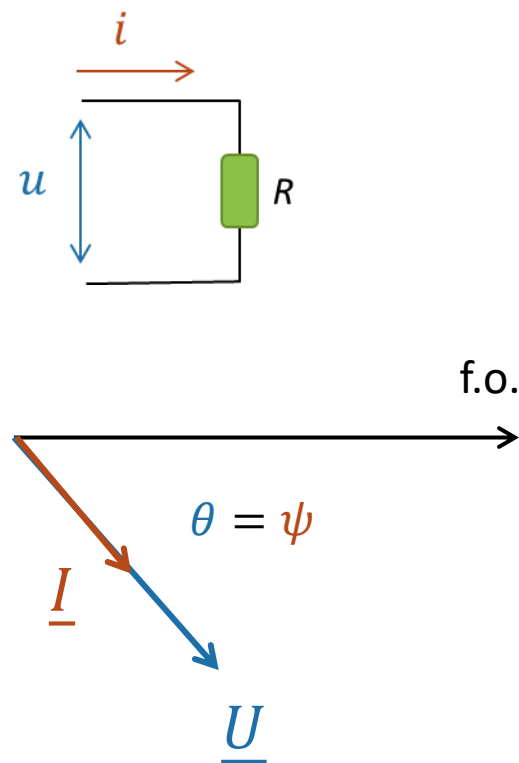
$$\varphi = \theta - \psi$$

Impedansa prijemnika

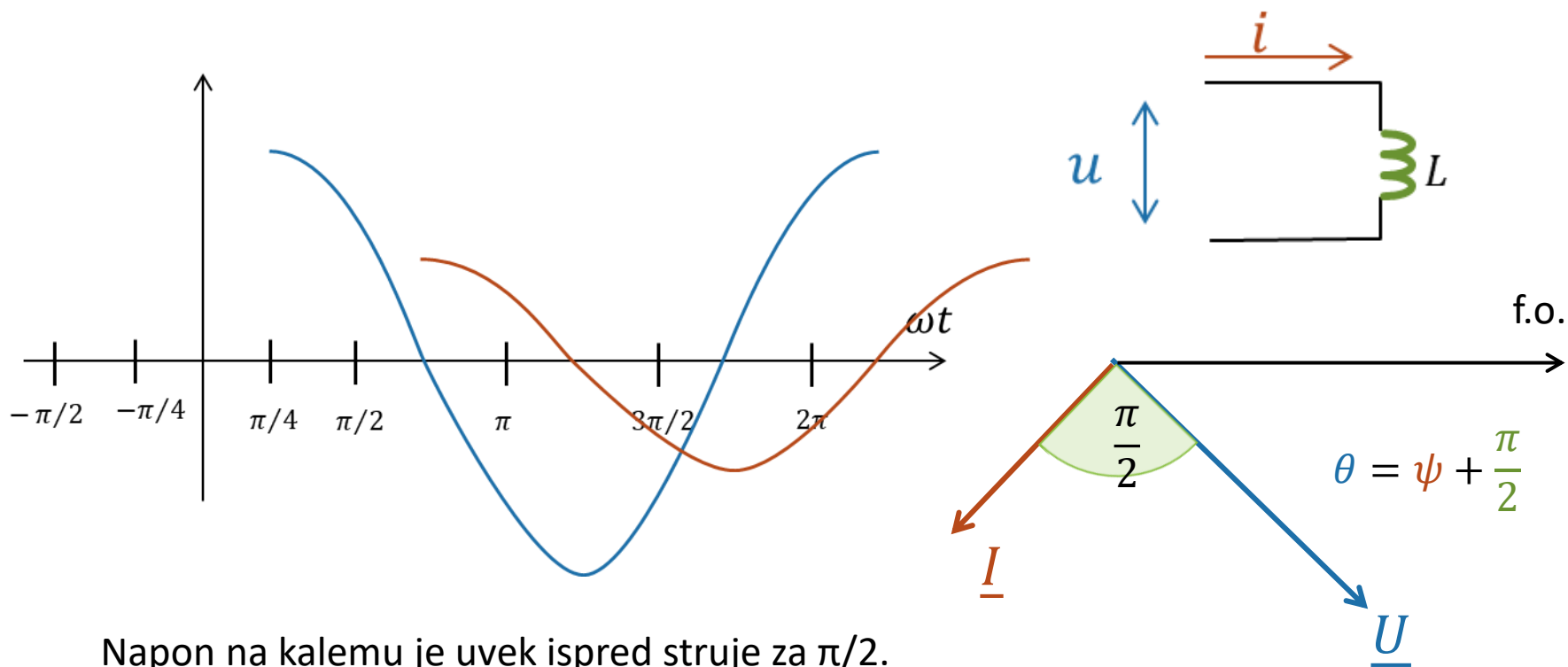
Na otporniku su napon i struja u fazi.



$$\underline{Z} = \frac{Ue^{j\theta}}{Ie^{j\psi}} = \frac{RI}{I} e^{j(\theta-\psi)} = Re^{j0}$$



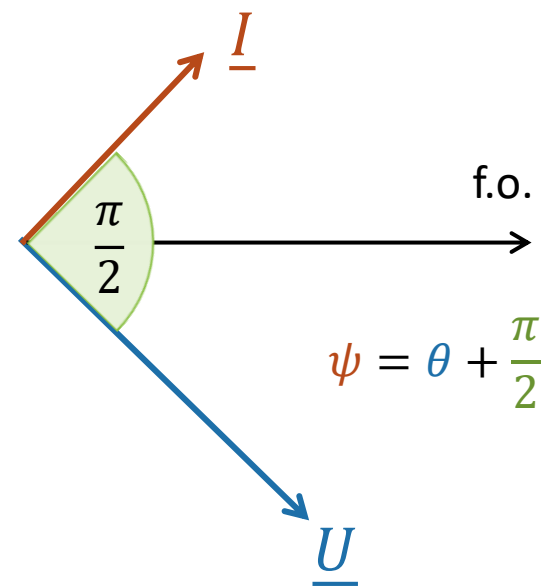
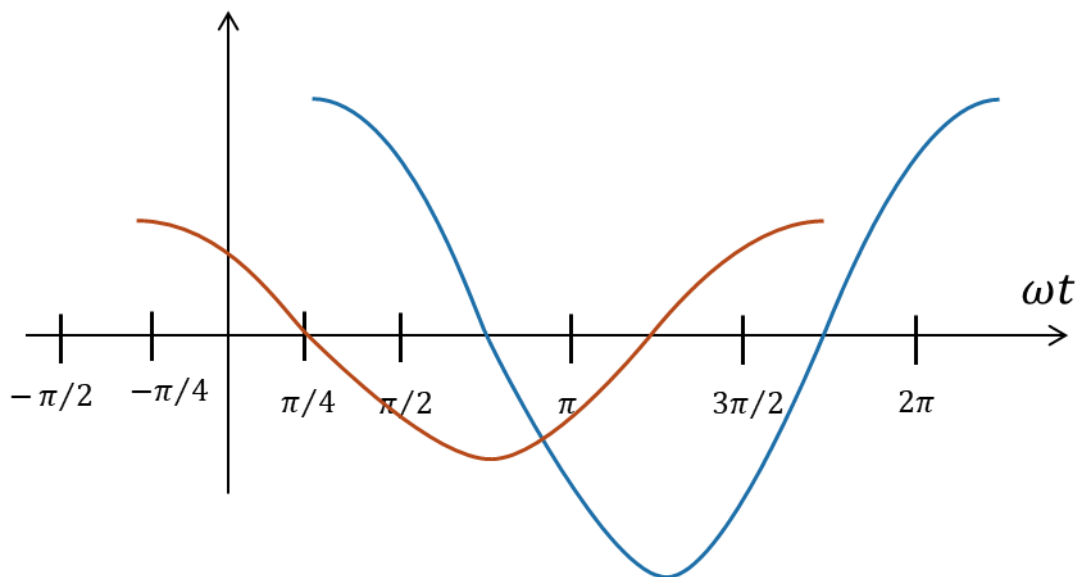
Impedansa prijemnika



Napon na kalemu je uvek ispred struje za $\pi/2$.

$$\underline{Z} = \frac{U e^{j\theta}}{I e^{j\psi}} = \frac{\omega L I}{I} e^{j(\theta - \psi)} = \omega L e^{j\frac{\pi}{2}}$$

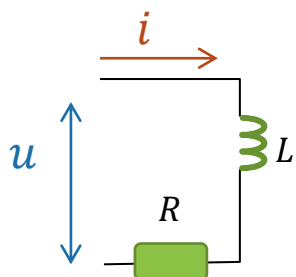
Impedansa prijemnika



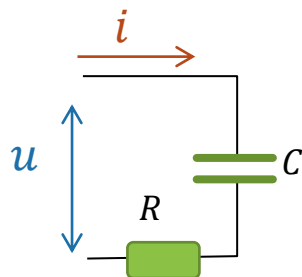
Na kondenzatoru struja je uvek ispred napona za $\pi/2$.

$$\underline{Z} = \frac{U e^{j\theta}}{I e^{j\psi}} = \frac{U}{\omega C U} e^{j(\theta - \psi)} = \frac{1}{\omega C} e^{-j\frac{\pi}{2}}$$

Impedansa prijemnika



$$0 < \varphi < \frac{\pi}{2}$$



$$-\frac{\pi}{2} < \varphi < 0$$

$$\underline{Z} = Z \cos(\varphi) + jZ \sin(\varphi)$$

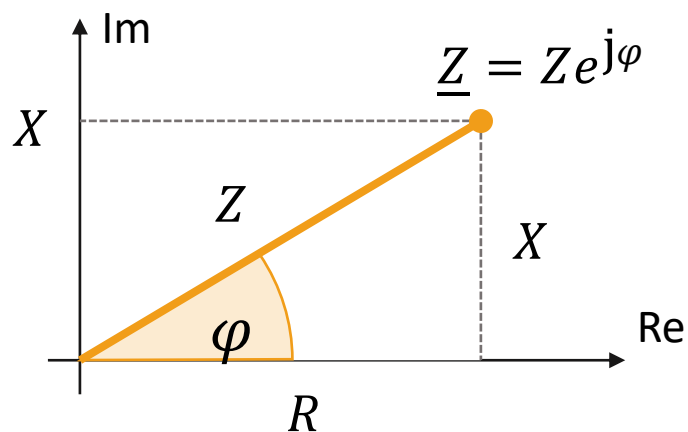
$$= R + jX$$

R - Rezistansa

$$R \geq 0$$

X - Reaktansa

$$X \in \mathbb{R}$$

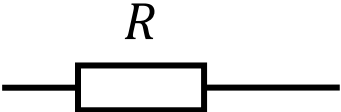
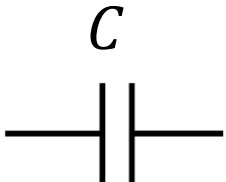


$X > 0$ – induktivni prijemnik

$X < 0$ – kapacitivni prijemnik

$X = 0$ – rezistivni prijemnik

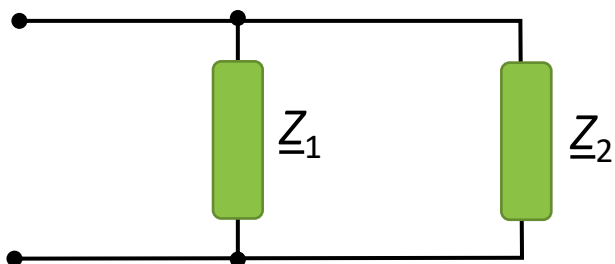
Impedanse osnovnih elemenata

Otpornost		$\underline{Z}_R = R + j0$	$X = 0$
Induktivnost		$\underline{Z}_L = 0 + j\omega L$	$X > 0$
Kapacitivnost		$\underline{Z}_C = 0 - j\frac{1}{\omega C}$	$X < 0$

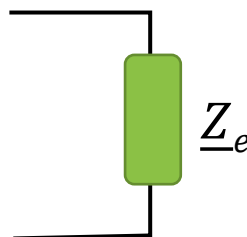
Modelovanje prijemnika

Prijemnici se u kolima prostoperiodične struje modeluju rednom, paralelenom i mešovitom vezom osnovnih elemenata.

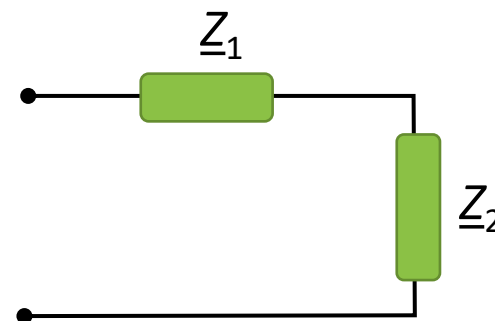
Paralelna veza



$$\underline{Z}_e = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$



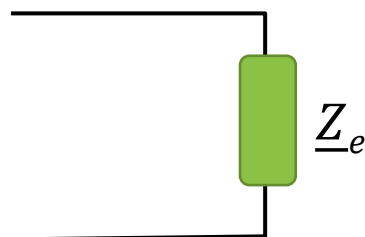
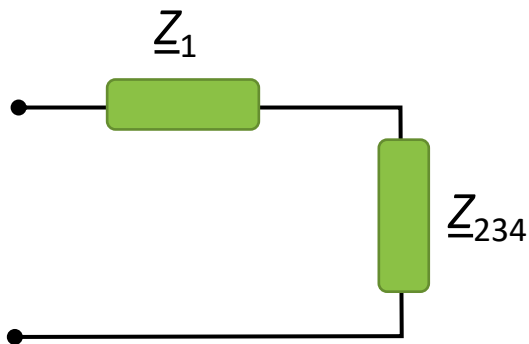
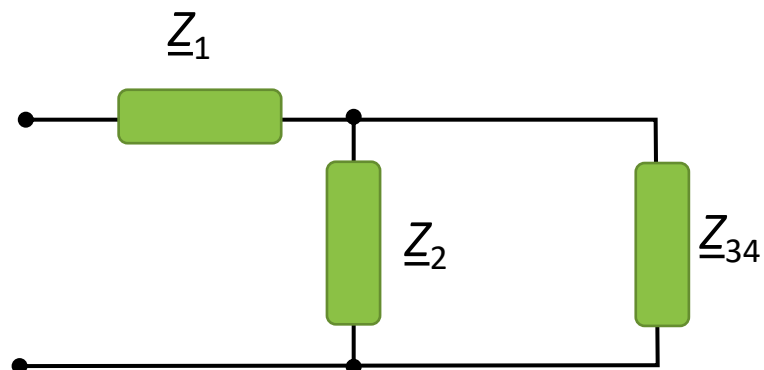
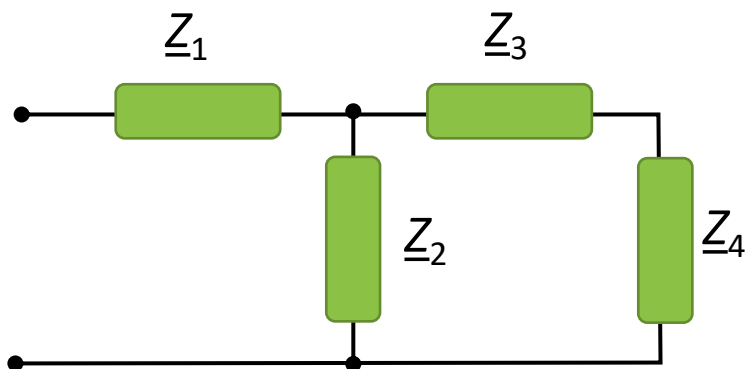
Redna veza



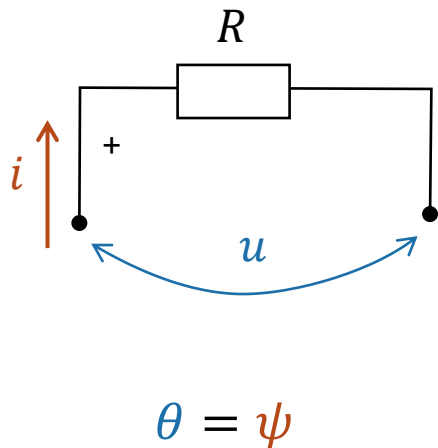
$$\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2$$

Modelovanje prijemnika

Mešovita veza



Srednja snaga prijemnika



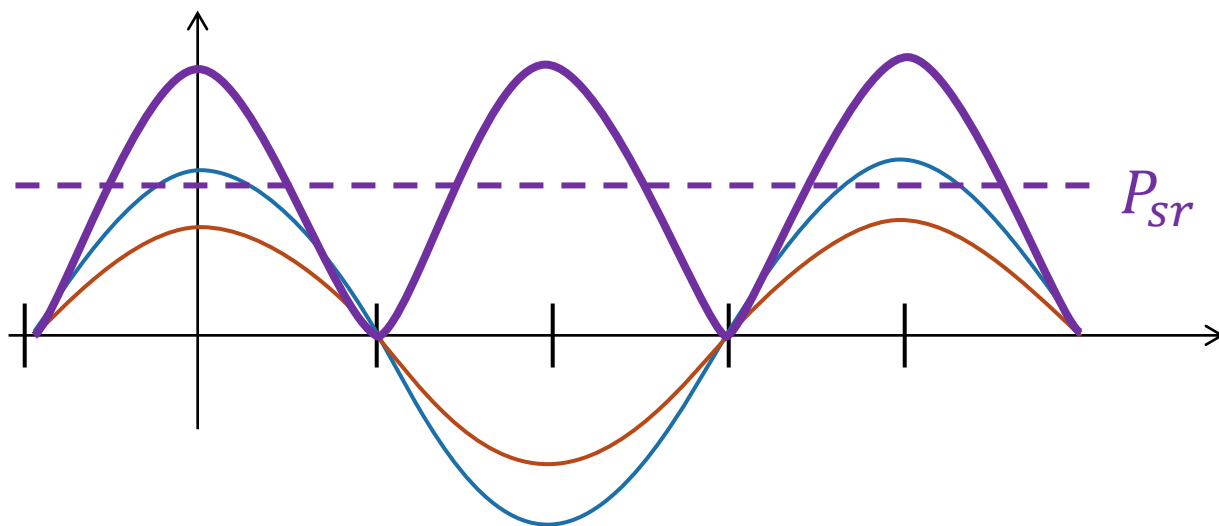
$$u = U_m \cos(\omega t + \theta)$$

$$i = I_m \cos(\omega t + \psi)$$

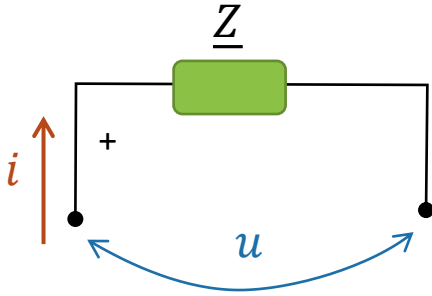
$$P_{sr} = \frac{1}{T} \int_{t_0}^{t_0+T} p dt = \frac{U_m I_m}{2}$$

Džulov zakon

$$p = u \times i$$



Srednja snaga prijemnika

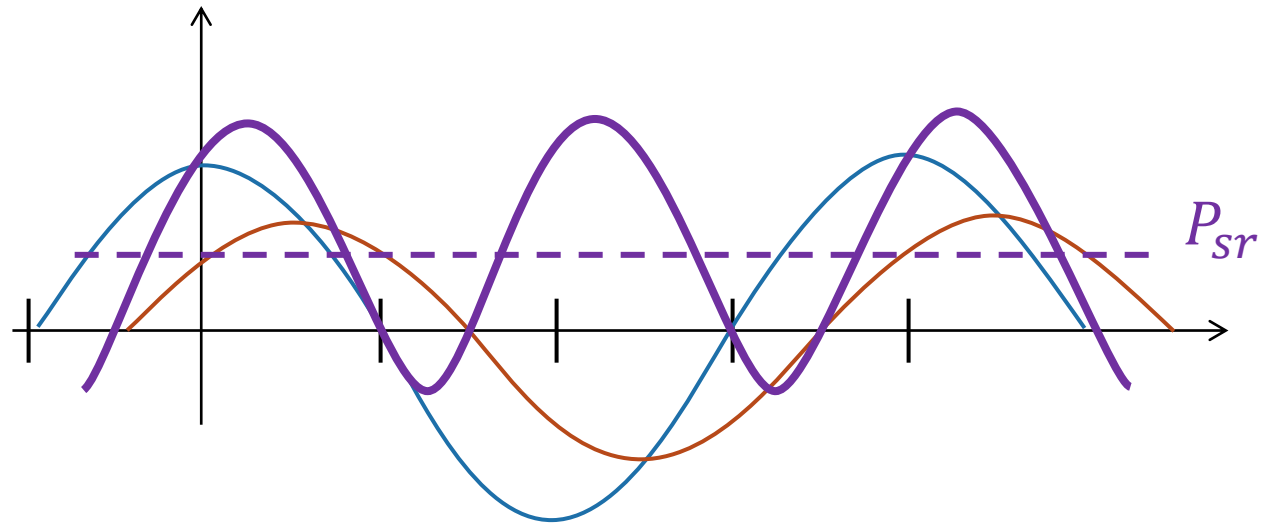


$$u = U_m \cos(\omega t + \theta)$$

$$i = I_m \cos(\omega t + \psi)$$

$$\underline{Z} = Z e^{j\varphi}$$

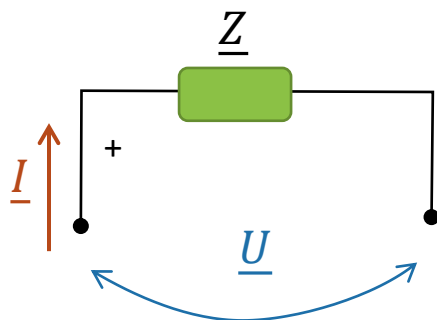
$$\theta = \psi + \varphi$$



$$P_{sr} < \frac{U_m I_m}{2}$$

$$P_{sr} = \frac{U_m I_m}{2} \cos \varphi = \frac{U_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \varphi = UI \cos \varphi$$

Kompleksna snaga prijemnika



Kompleksna **snaga** prijemnika je proizvod kompleksnog **napona** i **konjugovano** kompleksne **struje** prijemnika

$$\underline{S} = \underline{U} \underline{I}^*$$

jedinica [VA]

$$\underline{I} = I e^{j\psi} \quad \underline{U} = U e^{j\theta} \quad \underline{S} = UI e^{j(\theta - \psi)}$$

Eksponencijalni oblik

$$\underline{S} = S e^{j\varphi}$$

S - Prividna snaga [VA]

$$S = UI$$

Algebarski oblik $\underline{S} = P + jQ$

$$P - \text{Aktivna snaga [W]} \quad P = S \cos \varphi = UI \cos \varphi$$

$$Q - \text{Reaktivna snaga [var]} \quad Q = S \sin \varphi = UI \sin \varphi$$

Kompleksna snaga prijemnika

Primer:

Odrediti kompleksnu, prividnu, aktivnu i reaktivnu snagu prijemnika ako je poznato:

$$\underline{U} = 40e^{j\pi/2}V$$

$$\underline{I} = 0,2e^{j\pi/3}A$$

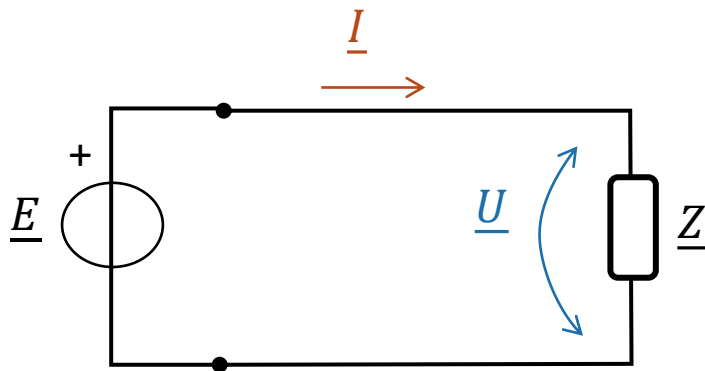
Primer:

Odrediti kompleksnu, prividnu, aktivnu i reaktivnu snagu prijemnika ako je poznato:

$$\underline{U} = (40 + j20)V$$

$$\underline{I} = (1 - j2)A$$

Snaga generatora



$$\underline{I} = I e^{j\psi} \quad \underline{E} = E e^{j\theta}$$

$$\underline{S}_g = \underline{E} \underline{I}^* = (P_g + jQ_g)$$

jedinica [VA]

P_g - Aktivna snaga [W]

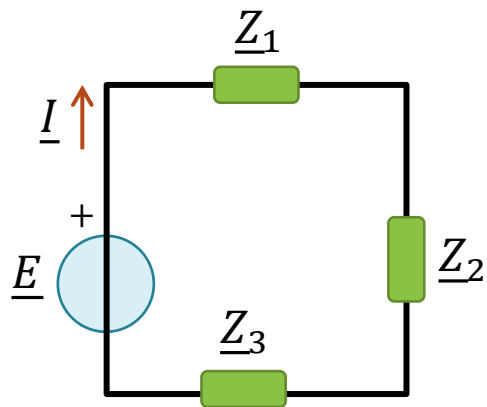
Q_g - Reaktivna snaga [var]

Primer: Odrediti kompleksnu snagu generatora.

$$\underline{E} = 50 e^{j\pi/3} \text{ V}$$

$$\underline{I} = 0,1 e^{j\pi/6} \text{ A}$$

Teorema održanja snage



Ukupna kompleksna snaga svih generatora mora biti jednaka ukupnoj kompleksnoj snazi svih prijemnika (potrošača).

$$\sum \underline{S}_g = \sum \underline{S}_p$$

$$\underline{S}_g = \underline{E} \underline{I}^*$$

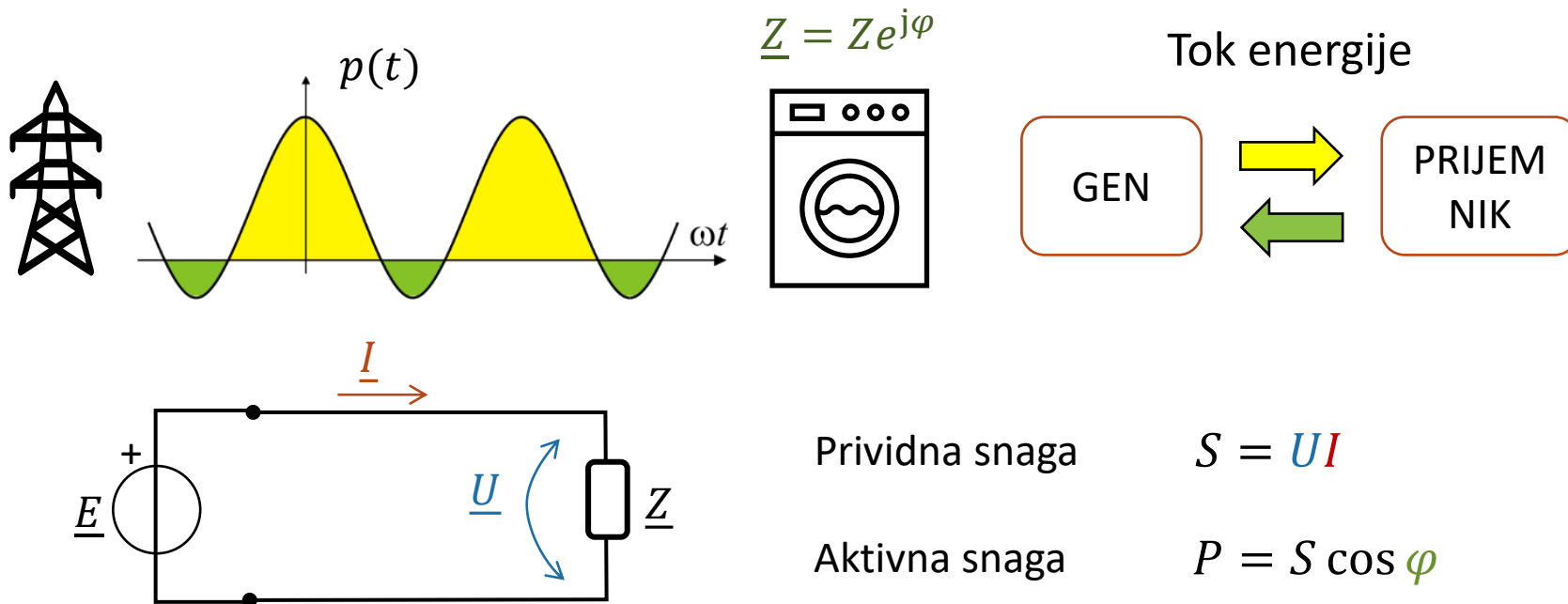
$$\underline{S}_p = \underline{U}_p \underline{I}^*$$

$$\underline{U}_p = \underline{Z}_p \underline{I}$$

Primer:

U kolu sa generatorom i dva prijemnika kompleksna snaga generatora je $(500+j50)$ VA a kompleksna snaga jednog prijemnika $(200+j100)$ VA. Koliko iznosi kompleksna snaga drugog prijemnika?

Faktor snage prijemnika



Prividna snaga

$$S = UI$$

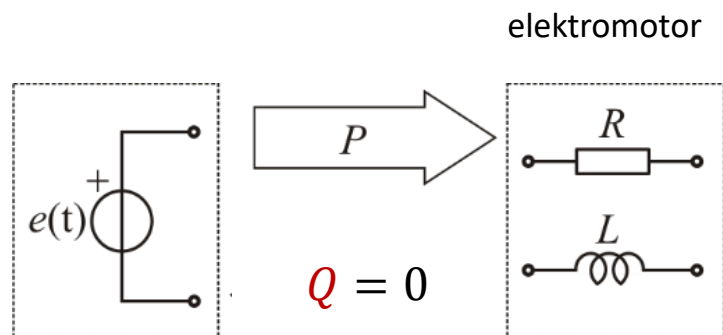
Aktivna snaga

$$P = S \cos \varphi$$

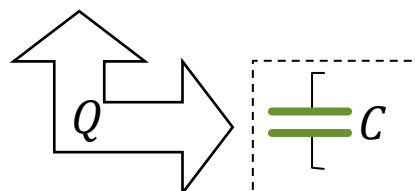
Faktor snage

$$\text{f. s.} = \frac{P}{S} = \cos \varphi \leq 1$$

Popravlak faktora snage



- Prijemnik dobija od generatora kompleksnu snagu (aktivnu i reaktivnu)
- Može da iskoristi samo aktivnu snagu.
- Reaktivna snaga opterećuje mrežu.



Dodavanjem kondenzatora obezbeđuje se reaktivna snaga za elektromotor. Generator ne mora više da obezbeđuje i reaktivnu snagu.

$$\cos \varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{P}{\sqrt{P^2 + 0}} = 1$$

Pitanja

1. Kako se definiše impedansa prijemnika?
2. Šta predstavlja moduo a šta argument impedanse?
3. Kako se zovu realni i imaginarni delovi impedanse?
4. Koja je jedinica za impedansu, rezistansu i raktansu?
5. Kada je impedansa prijemnika pretežno induktivna?
6. Kako se može moelovati prijamnik ako napon prednjači struji za ugao od $\varphi > 0$?
7. Šta je kompleksna snaga? Šta je aktivna, a šta reaktivna snaga?
8. Ako je aktivna snaga 80 W a reaktivna 60 var, koliko iznosi prividna snaga?
9. Kako glasi teorema održanja snage u kolima prostoperiodične struje?
10. U kolu sa generatorom i dva prijemnika kompleksna snaga generatora je $(500+j50)$ VA a kompleksna snaga jednog prijemnika $(200+j100)$ VA. Koliko iznosi kompleksna snaga drugog prijemika?
11. Ako je prividna snaga 100 VA a reaktivna -60 VAR, kolika je aktivna snaga?
12. U kolu sa generatorom i dva prijemnika kompleksna snaga generatora je $(800+j20)$ VA. Ako je aktivna snaga jednog prijemnika 200W, koliko iznosi aktivna snaga drugog prijemika?
13. Šta je to faktor snage? Ako je kompleksna snaga prijemnika $(80 + j 60)$ VA koliki je faktor snage.
14. Kako se popravljja faktor snage kod elektromotora?