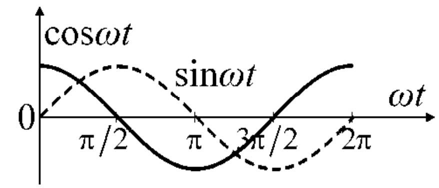
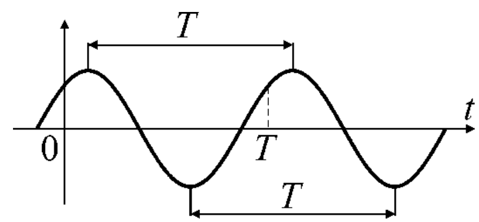
Prostoperiodične struje (PPS)

Prostoperiodične funkcije

- Prostoperiodične funkcije su $\sin\omega t,\ \cos\omega t$ i njihova linearna kombinacija $\cos(\omega t+\theta)$.
- $\sin\omega t=\cos(\omega t-\pi/2),\ -\sin\omega t=\cos(\omega t+\pi/2)$.



- Osnovna perioda T [s]
  - najlakše se vidi kao vremenski interval između dva uzastopna maksimuma (minimuma).



- ω $[\mathrm{rad/s}]$ kružna frekvencija
  f $[\mathrm{Hz}]$ frekvencija

$$\boxed{f=\frac{\omega}{2\pi}=\frac{1}{T}}$$

primer : $f=50$ Hz
$T=1/f=1/50=0{,}02\,\mathrm{s}=20\,\mathrm{ms}$
$\omega=2\pi f=2\pi\cdot 50=314\,\mathrm{rad/s}$

-

$\varphi[\mathrm{rad}]$	2π	π	$\pi/2$	$\pi/3$	$\pi/4$
$\varphi[^\circ]$	360°	180°	90°	60°	45°

$$\frac{\varphi[\mathrm{rad}]}{\pi}=\frac{\varphi[^\circ]}{180^\circ}$$

$$1\,\mathrm{rad}:\quad \varphi[^\circ]=180^\circ\cdot\frac{1}{\pi}=57{,}3^\circ$$

Standardni oblik pp. veličine

$$\boxed{u(t) = U_{\mathrm{m}} \cos(\omega t + \theta)}$$

$u(t)$ $[\mathrm{V}]$ trenutna vrednost napona

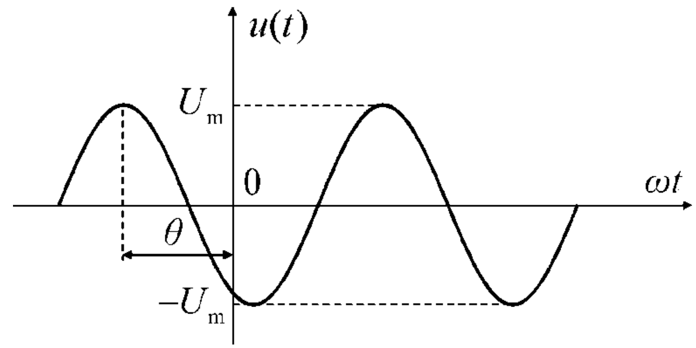
U_{m} $[\mathrm{V}]$ amplituda (maksimalna vrednost)

ω $[\mathrm{rad/s}]$ kružna učestanost

t $[\mathrm{s}]$ vreme

$(\omega t + \theta)$ $[\mathrm{rad}]$ faza

θ $[\mathrm{rad}]$ početna faza (za $t = 0$ faza je θ)



– napon je potpuno opisan sa amplitudom, kružnom frekvencijom i početnom fazom.

– definicije su primenljive i na druge pp. veličine:

– početne faze napona i struje:

θ, odnosno ψ.

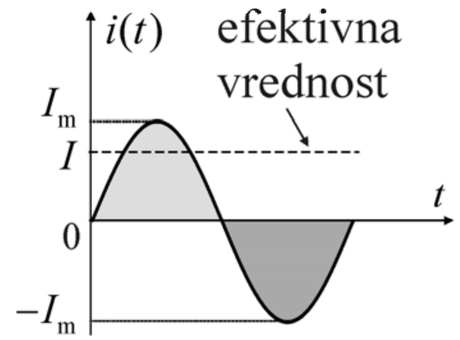
kružna frekvencija

$$i(t) = \underbrace{I_{\mathrm{m}}}_{\text{amplituda}} \cos\,(\omega t + \underbrace{\psi}_{\text{početna faza}})$$

Efektivna vrednost

- Srednja vrednost pp. veličine je nula
- Veza između efektivne vrednosti I i amplitude I_{m}

$$I = \frac{I_{\mathrm{m}}}{\sqrt{2}} \approx 0,707\, I_{\mathrm{m}} \qquad I_{\mathrm{m}} = \sqrt{2}\, I \approx 1,41 I$$



  - važi za sve pp. veličine: napon, ems, fluks, itd. (ne važi za periodične signali drugih oblika)
  - instrumenti pokazuju efektivnu vrednost
  - kada se govori o pp. veličini koristi se efektivna vrednost, npr. u gradskoj mreži napon je 230 V (efektivna vrednost), maksimalna vrednost je 1,41· 230 V = 325 V
  - pp. veličine se često pišu kao:

$$u(t) = \sqrt{2}\, U \cos(\omega t + \theta),$$
$$i(t) = \sqrt{2}\, I \cos(\omega t + \psi).$$

Oznake:
i, u i e su trenutne vrednosti,
I, U i E su efektivne vrednosti,
I_{m}, U_{m} i E_{m} su amplitude.

Primer 1

Dat je prostoperiodičan napon

$$u(t) = 35\cos(314t)\,\mathrm{V}.$$

Odrediti njegovu

a) maksimalnu vrednost,

b) efektivnu vrednost,

c) početnu fazu,

d) kružnu frekvenciju,

e) frekvenciju,

f) osnovnu periodu.

Rešenje. Poređenjem

$u(t) = 35\cos(314t)\,\mathrm{V}$ sa

standardnim oblikom

$u(t) = U_\mathrm{m}\cos(\omega t + \theta)$, dobija se

a) $U_\mathrm{m} = 35\,\mathrm{V}$,

b) $U = U_\mathrm{m}/\sqrt{2} = 0,707 \cdot 35\,\mathrm{V} = 24,75\,\mathrm{V}$

c) $\theta = 0$,

d) $\omega = 314\,\mathrm{rad/s}$,

e) $f = \omega/(2\pi) = 314/(2\pi) = 50\,\mathrm{Hz}$,

f) $T = 1/f = 1/50 = 0,02\,\mathrm{s} = 20\,\mathrm{ms}$.

Primer 2

Data je prostoperiodična struja

$$i(t) = 75\cos(200\pi t - \pi/4)\,\mathrm{mA}.$$

Odrediti njenu

a) maksimalnu vrednost,

b) efektivnu vrednost,

c) početnu fazu,

d) kružnu frekvenciju,

e) frekvenciju,

f) osnovnu periodu.

Rešenje. Poređenjem

$i(t) = 75\cos(200\pi t - \pi/4)\,\mathrm{mA}$ sa standardnim oblikom

$i(t) = I_\mathrm{m}\cos(\omega t + \psi)$, dobija se

a) $I_\mathrm{m} = 75\,\mathrm{mA}$,

b) $I = I_\mathrm{m}/\sqrt{2} = 0,707 \cdot 75\,\mathrm{mA} = 53\,\mathrm{mA}$,

c) $\psi = -\pi/4$,

d) $\omega = 200\pi\ \mathrm{rad/s}$,

e) $f = \omega/(2\pi) = 200\pi/(2\pi) = 100\,\mathrm{Hz}$,

f) $T = 1/f = 1/100 = 0,01\,\mathrm{s} = 10\,\mathrm{ms}$.

Fazna razlika

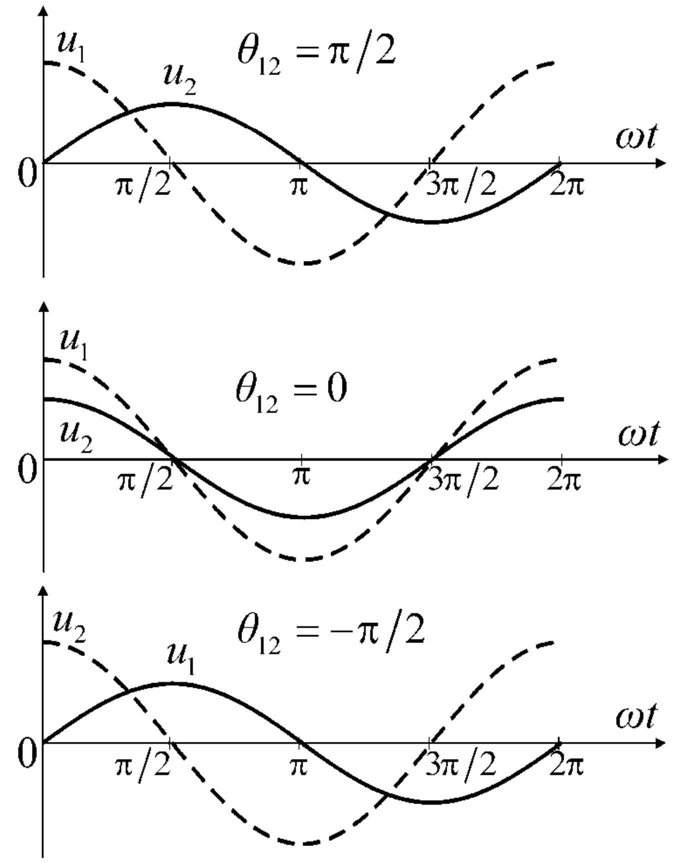
- Fazna razlika dva napona istih učestanosti $u_1(t) = U_{m1}\cos(\omega t + \theta_1)$ i $u_2(t) = U_{m2}\cos(\omega t + \theta_2)$ je
  $\theta_{12} = (\omega t + \theta_1) - (\omega t + \theta_2)$

  $\boxed{\theta_{12} = \theta_1 - \theta_2}$ θ_{12} ne zavisi od vremena,

  $\theta_{12} > 0$: u_1 prednjači u_2 (ili u_2 kasni za u_1)
  $\theta_{12} = 0$: u_1 i u_2 su u fazi (sinhronizovani su)
  $\theta_{12} < 0$: u_2 prednjači u_1 (ili u_1 kasni za u_2)

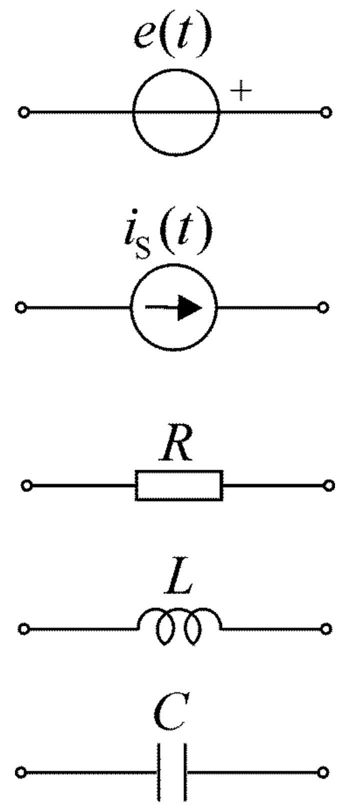
- Fazna razlika između napona i struje:
  $u(t) = U_m\cos(\omega t + \theta)$ i $i(t) = I_m\cos(\omega t + \psi)$ je
  $\boxed{\varphi = \theta - \psi}$

  $\varphi > 0$: napon prednjači struji
  $\varphi = 0$: napon i struja su u fazi
  $\varphi < 0$: napon kasni za strujom



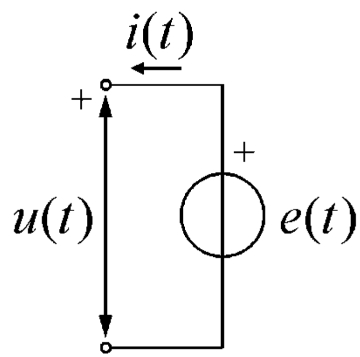
Osnovni elementi u kolima PPS

- Osnovni elementi
  - generatori
    - idealan naponski generator
    - idealan strujni generator
  - prijemnici
    - otpornik
    - kalem
    - kondenzator



Idealni generatori

- <u>Naponski</u>:



$$\boxed{u(t) = e(t)}$$

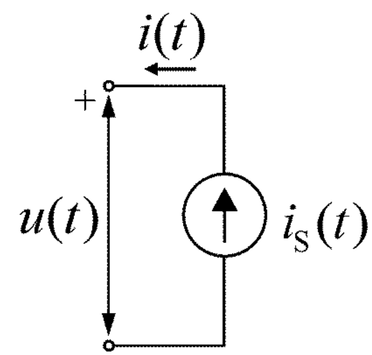
– bez obzira na jačinu struje.

- Trenutna snaga (za usaglašene referentne smerove napona i struje)

$$p(t) = u(t)i(t) = e(t)i(t)$$

– jedinica je [W].

- <u>Strujni</u>:



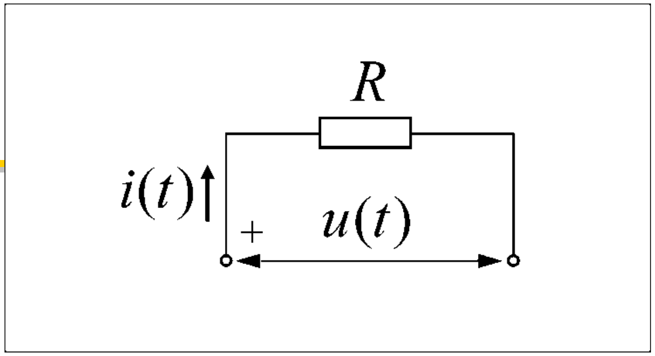
$$\boxed{i(t) = i_S(t)}$$

– bez obzira na napon.

- Trenutna snaga

$$p(t) = u(t)i(t) = u(t)i_S(t).$$

Otpornik



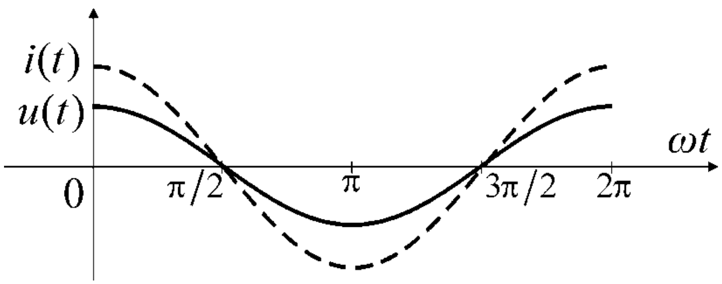
- Omov zakon: $\boxed{u(t) = Ri(t)}$

- $i(t) = \sqrt{2}I\cos(\omega t + \psi)$

  $u(t) = \sqrt{2}RI\cos(\omega t + \psi)$

  $\boxed{U = RI}$ (Omov zakon za efektivne vrednosti)

  $\boxed{\theta = \psi}$ (napon i struja su u fazi, $\varphi = 0$)



- Trenutna snaga [W]:

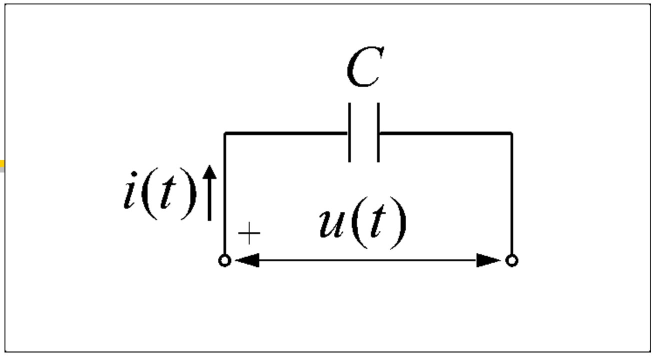
  $p(t) = Ri^2(t) = 2RI^2\cos^2(\omega t + \psi)$

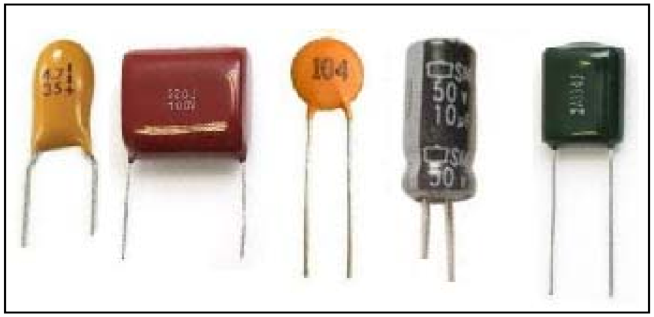
  $= RI^2 + RI^2\cos(2\omega t + 2\psi)$

  Srednja snaga [W]:

  $\boxed{P = RI^2}$

Kondenzator





- $q(t) = Cu(t)$

  $$\frac{dq(t)}{dt} = C\frac{du(t)}{dt}$$

  $$\boxed{i(t) = C\frac{du(t)}{dt}}$$

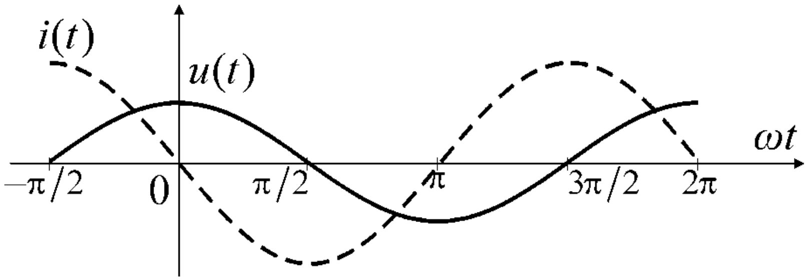
- $u(t) = \sqrt{2}U\cos(\omega t + \theta),$

  $i(t) = -\sqrt{2}U\omega C\sin(\omega t + \theta)$

  $= \sqrt{2}U\omega C\cos(\omega t + \theta + \pi/2)$

  $\boxed{I = U\omega C}$

  $\boxed{\varphi = \theta - \psi = -\pi/2}$ (struja prednjači)

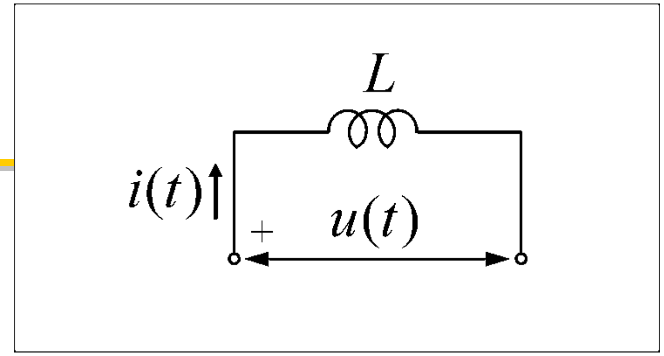


- Trenutna snaga:

  $p(t) = u(t)i(t) = -\omega CU^2\sin(2\omega t + 2\theta)$

  Srednja snaga: $\boxed{P = 0}$

Kalem

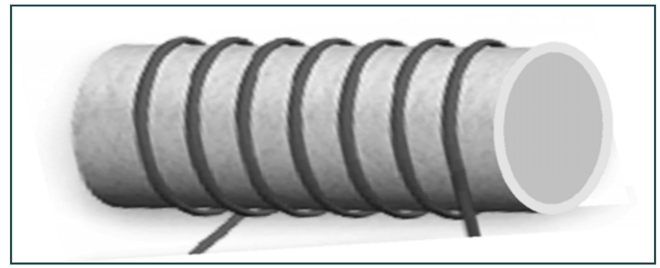


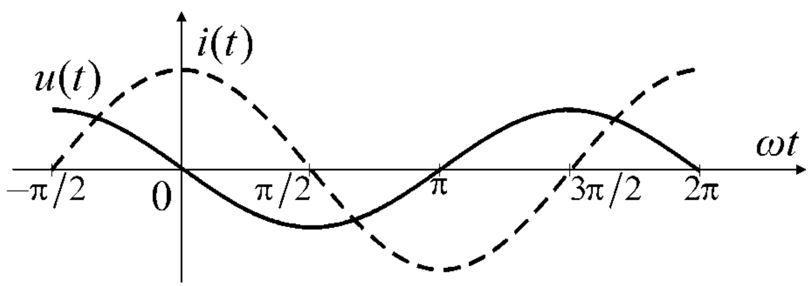
- $\boxed{u(t) = L\dfrac{di(t)}{dt}}$

  Analogno izvođenje kao kod kondenzatora.

  $\boxed{U = \omega LI}$ (relacija između efektivnih vrednosti)

  $\boxed{\theta = \psi + \pi/2}$ (napon prednjači struji $\varphi = \pi/2$)





Prijemnik	Osnovna relacija	U/I	φ	P
i + R	$u = Ri$	R	0	RI^2
i + L	$u = L\dfrac{di}{dt}$	ωL	$\dfrac{\pi}{2}$	0
i + C	$i = C\dfrac{du}{dt}$	$\dfrac{1}{\omega C}$	$-\dfrac{\pi}{2}$	0

- Trenutna snaga: $p(t) = u(t)i(t)$

  Srednja snaga: $P = 0$

Kirhofovi zakoni

- ista formulacija kao i za kola VKS,
- pravila o predznacima: ista kao i za kola VKS,
- I KZ: $\boxed{\sum i(t) = 0}$ II KZ: $\boxed{\sum u(t) = 0}$

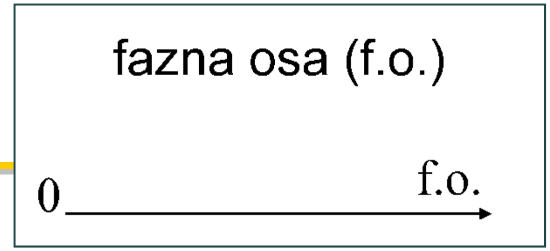
- Analiza kola PPS
  - broj nezavisnih jednačina po I KZ: $(n_{č} - 1)$,
  - broj nezavisnih jednačina po II KZ: $n_g - (n_{č} - 1)$,
  - ukupan broj nezavisnih jednačina: n_g.

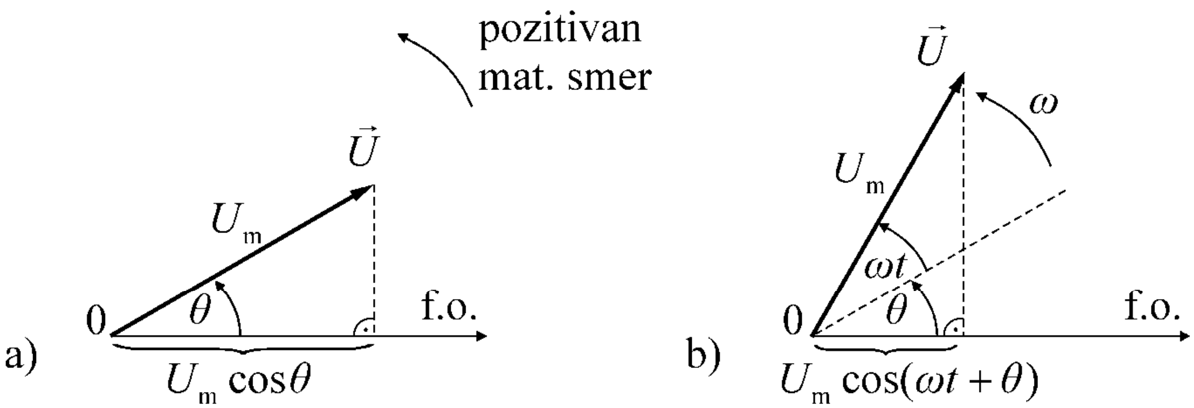
- Relacije između napona i struje
  - na kalemu i kondenzatoru sadrže izvode po vremenu
  - integro-diferencijalne jednačine
  - zato se koriste drugi alati za rešavanje kola PPS…

$$\boxed{u(t) = L\frac{di(t)}{dt}}$$

$$\boxed{i(t) = C\frac{du(t)}{dt}}$$

Fazori / 1

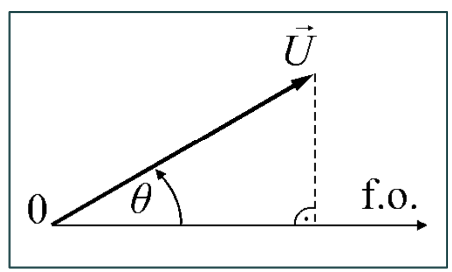




– Vektor rotira ugaonom brzinom ω oko 0.

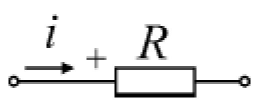
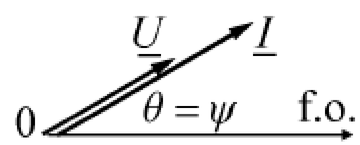
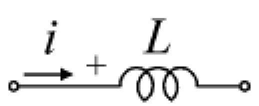
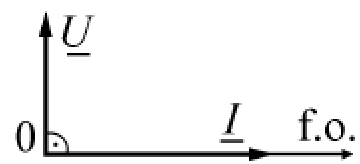
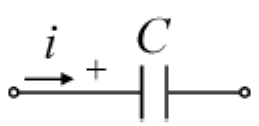
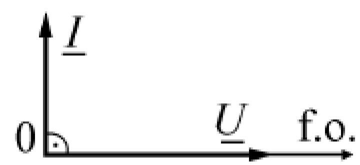
- a) U $t = 0$: $\vec{U}$ zaklapa ugao θ sa f. o.
- b) $\vec{U}$ zaklapa ugao $(\omega t + \theta)$ sa f. o.

  - projekcija $U_m \cos(\omega t + \theta)$ može da predstavlja napon $u(t)$.
- Vektor $\vec{U}$ se naziva <u>fazor.</u>
- Dovoljno je nacrtati fazor u $t = 0$.

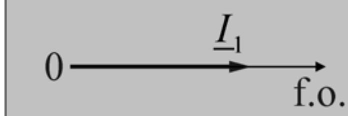


Fazori / 2

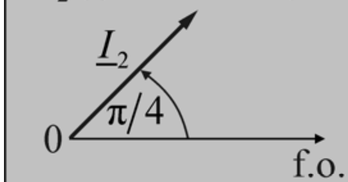
- dužina fazora odgovara efektivnoj vrednosti (a ne amplitudi kao da sada),
- ugao između fazora i f.o. jednak je početnoj fazi,
- oznake za fazore <u>U</u>, <u>I</u>.

Prijemnik	φ	Fazorski dijagram
	0	
	$\frac{\pi}{2}$	
	$-\frac{\pi}{2}$	

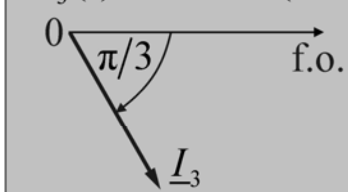
$$i_1(t)=\sqrt{2}I\cos(\omega t)$$

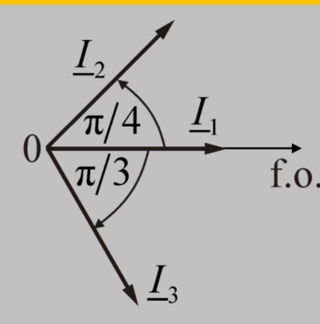


$$i_2(t)=\sqrt{2}I\cos(\omega t+\pi/4)$$



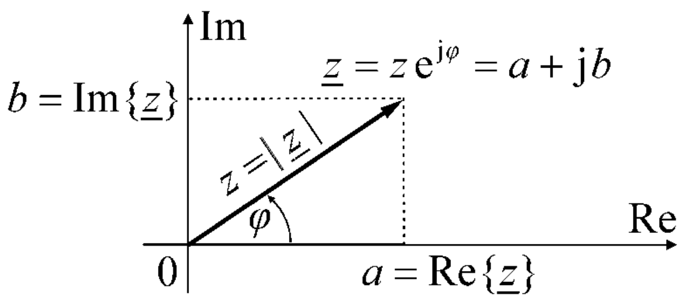
$$i_3(t)=\sqrt{2}I\cos(\omega t-\pi/3)$$





Kompleksni brojevi / 1

- U elektrotehnici $\boxed{\mathrm{j}=\sqrt{-1}}$ $\quad \mathrm{j}^2=-1,\ \ \mathrm{j}(-\mathrm{j})=1,\ \ 1/\mathrm{j}=-\mathrm{j}$
- Eksponencijalni oblik $\boxed{\underline{z}=z\,\mathrm{e}^{\mathrm{j}\varphi}}$ $\quad z=|\,\underline{z}\,|$ je moduo, φ je argument
- Algebarski oblik $\boxed{\underline{z}=a+\mathrm{j}b}$ realni deo je a, imaginarni b

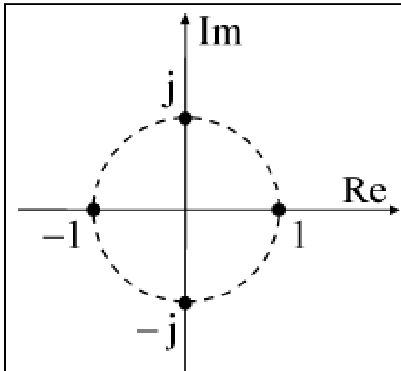


$$z=|\,\underline{z}\,|=\sqrt{a^2+b^2}$$

$$\mathrm{tg}\,\varphi=\frac{b}{a}$$

- Ojlerov obrazac: $\boxed{\mathrm{e}^{\mathrm{j}\varphi}=\cos\varphi+\mathrm{j}\sin\varphi}$
- transformacija iz eksponencijalnog u algebarski oblik:

$$\underline{z}=z\,\mathrm{e}^{\mathrm{j}\varphi}=\underbrace{z\cos\varphi}_{a}+\mathrm{j}\underbrace{z\sin\varphi}_{b}=a+\mathrm{j}b.$$



$$1=\mathrm{e}^{\mathrm{j}0}$$

$$\mathrm{j}=\mathrm{e}^{\mathrm{j}\pi/2}$$

$$-1=\mathrm{e}^{\mathrm{j}\pi}$$

$$-\mathrm{j}=\mathrm{e}^{-\mathrm{j}\pi/2}$$

Kompleksni brojevi / 2

Neka su $\underline{z}_1 = a_1 + \mathrm{j}b_1$ i $\underline{z}_2 = a_2 + \mathrm{j}b_2$.

- Zbir: $\underline{z}_1 + \underline{z}_2 = (a_1 + a_2) + \mathrm{j}(b_1 + b_2)$
- Razlika: $\underline{z}_1 - \underline{z}_2 = (a_1 - a_2) + \mathrm{j}(b_1 - b_2)$
- Proizvod: $\underline{z}_1\underline{z}_2 = (a_1 + \mathrm{j}b_1)(a_2 + \mathrm{j}b_2) = (a_1a_2 - b_1b_2) + \mathrm{j}(a_1b_2 + b_1a_2)$
- Konjugovano kompleksni broj:

  $\underline{z}^* = (a + \mathrm{j}b)^* = a - \mathrm{j}b$

  $\underline{z}^* = (z\mathrm{e}^{\mathrm{j}\varphi})^* = z\mathrm{e}^{-\mathrm{j}\varphi}$

  $\underline{z}\,\underline{z}^* = a^2 + b^2 = z^2$ (kvadrat modula kompleksnog broja)
- Količnik: $\dfrac{\underline{z}_1}{\underline{z}_2} = \dfrac{\underline{z}_1\underline{z}_2^*}{\underline{z}_2\underline{z}_2^*} = \dfrac{(a_1 + \mathrm{j}b_1)(a_2 - \mathrm{j}b_2)}{a_2^2 + b_2^2}$

Kompleksni predstavnik / 1

– kompleksni broj koji predstavlja pp. veličinu.

- Vremenski oblik: $u(t)=\sqrt{2}U\cos(\omega t+\theta)$

  (U je efektivna vrednost, a θ početna faza)

- Kompleksni predstavnik: $\boxed{\underline{U}=U\,\mathrm{e}^{\mathrm{j}\theta}}$

  - moduo U (efektivna vrednost)

  - argument θ (početna faza)

  - izražava se u istim jedinicama kao i pp. veličina koju predstavlja.

- Primer:

  Vremenski oblik: $u(t)=28{,}2\cos(\omega t-\pi/3)\ \mathrm{V}$

  $U_\mathrm{m}=28{,}2\mathrm{V}$

  $U=U_\mathrm{m}/\sqrt{2}=28{,}2\mathrm{V}/1{,}41=20\,\mathrm{V}$

  $\theta=-\pi/3$

  Kompleksni predstavnik: $\underline{U}=20\,\mathrm{e}^{-\mathrm{j}\pi/3}\mathrm{V}$

Kompleksni predstavnik / 2

- Isto pravilo važi i za druge pp. veličine,

  vremenski oblik: $i(t)=\sqrt{2}I\cos(\omega t+\psi)$, kompleksni predstavnik: $\underline{I}=I\,\mathrm{e}^{\mathrm{j}\psi}$

  vremenski oblik: $e(t)=\sqrt{2}E\cos(\omega t+\theta)$, kompleksni predstavnik: $\underline{E}=E\,\mathrm{e}^{\mathrm{j}\theta}$
- Primer: $i(t)=10\sqrt{2}\cos(\omega t+\pi/4)$ A.

  $I_\mathrm{m}=10\sqrt{2}$ A, $I=10$ A, $\psi=\pi/4$, $\underline{I}=10\mathrm{e}^{\mathrm{j}\pi/4}$ A.

- <u>Ojlerov obrazac</u>: $\mathrm{e}^{\mathrm{j}\varphi}=\cos\varphi+\mathrm{j}\sin\varphi$.

  Kompleksni predstavnici u algebarskom obliku:

  $\underline{U}=U\,\mathrm{e}^{\mathrm{j}\theta}=U\cos\theta+\mathrm{j}U\sin\theta$

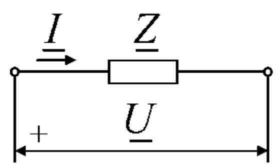
  $\underline{I}=I\,\mathrm{e}^{\mathrm{j}\psi}=I\cos\psi+\mathrm{j}I\sin\psi$
- Primeri:

  $\underline{U}=20\,\mathrm{e}^{-\mathrm{j}\pi/3}\,\mathrm{V}=20\cos(-\pi/3)+\mathrm{j}20\sin(-\pi/3)=(10-\mathrm{j}10\sqrt{3})\,\mathrm{V}$

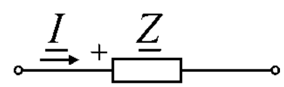
  $\underline{I}=10\,\mathrm{e}^{\mathrm{j}\pi/4}\mathrm{A}=10\cos(\pi/4)+\mathrm{j}10\sin(\pi/4)=(5\sqrt{2}+\mathrm{j}5\sqrt{2})\,\mathrm{A}$

(Kompleksna) impedansa

$$\boxed{\underline{Z} = \frac{\underline{U}}{\underline{I}} \qquad [\Omega]}$$



ili

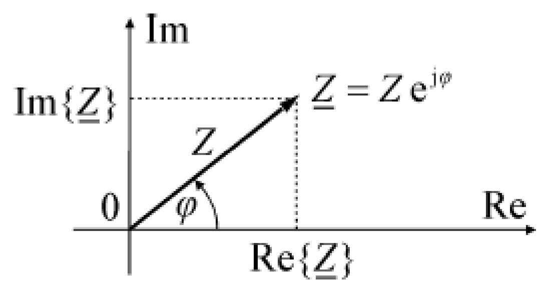


$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U\,\mathrm{e}^{\mathrm{j}\theta}}{I\,\mathrm{e}^{\mathrm{j}\psi}} = \frac{U}{I}\mathrm{e}^{\mathrm{j}(\theta-\psi)}$$

moduo: $Z = \dfrac{U}{I}$

argument: $\varphi = \theta - \psi$

$$\boxed{\underline{Z} = Z\,\mathrm{e}^{\mathrm{j}\varphi}}$$

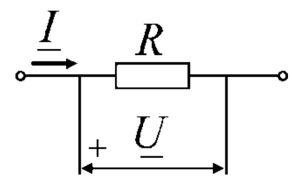


- (Kompleksna) admitansa
  - jedinica je S (simens)

$$\boxed{\underline{Y} = \frac{1}{\underline{Z}}}$$

Impedanse osnovnih prijemnika

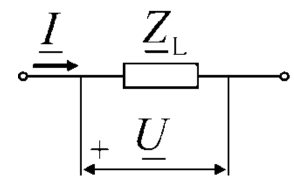
- Otpornik $\underline{U} = R\underline{I}$, <u>impedansa otpornika</u> $\underline{Z} = R$



- Kalem $\underline{U} = \mathrm{j}\omega L\underline{I}$

  <u>impedansa kalema</u> $\boxed{\underline{Z} = \mathrm{j}\omega L}$ ili $\underline{Z} = \omega L\,\mathrm{e}^{\mathrm{j}\pi/2}$

  primer $L = 1{,}5\,\mathrm{H}$, $\omega = 314\,\mathrm{rad/s}$, $\underline{Z}_\mathrm{L} = \mathrm{j}\omega L = \mathrm{j}314\cdot 1{,}5 = \mathrm{j}471\,\Omega$

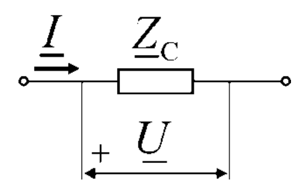


- Kondenzator $\underline{I} = \mathrm{j}\omega C\underline{U}$

  <u>impedansa kondenzatora</u> $\boxed{\underline{Z} = \frac{1}{\mathrm{j}\omega C} = \frac{-\mathrm{j}}{\omega C}}$ ili $\underline{Z} = \frac{1}{\omega C}\mathrm{e}^{-\mathrm{j}\pi/2}$

  primer $C = 10\,\mu\mathrm{F} = 10\cdot 10^{-6}\mathrm{F}$, $\omega = 314\,\mathrm{rad/s}$

$$\underline{Z}_\mathrm{C} = -\frac{\mathrm{j}}{\omega C} = -\mathrm{j}\frac{1}{314\cdot 10\cdot 10^{-6}} = -\mathrm{j}\frac{10^6}{3140} = -\mathrm{j}318{,}5\,\Omega$$



Kola PPS u kompleksnom domenu

- Kirhofovi zakoni

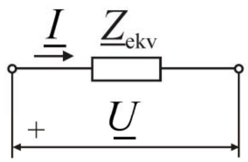
I KZ: $\boxed{\sum \underline{I} = 0}$

II KZ: $\boxed{\sum \underline{U} = 0}$

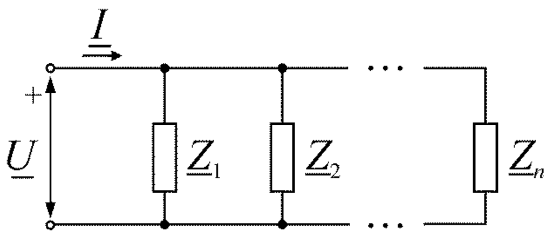
  – pravila o predznacima su ista kao u kolima VKS.

- Za kolo sa $n_č$ čvorova i n_g grana:
  – broj linearno nezavisnih jednačina po I KZ: $(n_č - 1)$,
  – broj linearno nezavisnih jednačina po II KZ: $n_g - (n_č - 1)$,
  – ukupan broj lin. nezavisnih jednačina po I KZ i II KZ: n_g .

Veze impedansi

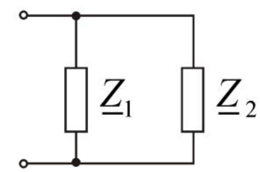


- Paralelna veza



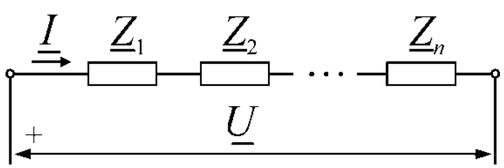
$$\frac{\underline{U}}{\underline{Z}_{\text{ekv}}} = \frac{\underline{U}}{\underline{Z}_1} + \frac{\underline{U}}{\underline{Z}_2} + \ldots + \frac{\underline{U}}{\underline{Z}_n}$$

$$\boxed{\frac{1}{\underline{Z}_{\text{ekv}}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \ldots + \frac{1}{\underline{Z}_n}}$$



$$\underline{Z}_{\text{ekv}} = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

- Redna veza



$$\underline{Z}_{\text{ekv}} \underline{I} = \underline{Z}_1 \underline{I} + \underline{Z}_2 \underline{I} + \ldots + \underline{Z}_n \underline{I}$$

$$\boxed{\underline{Z}_{\text{ekv}} = \underline{Z}_1 + \underline{Z}_2 + \ldots + \underline{Z}_n}$$

- Mešovita veza
  - postepeno zamenjivanje rednih i paralelnih veza ekvivalentnim impedansama

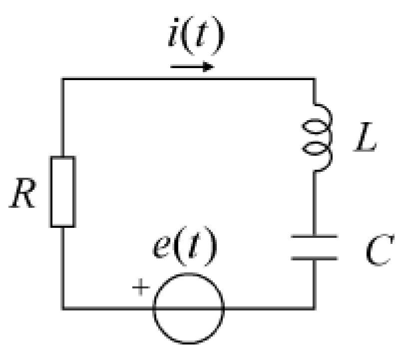
Primer rešavanja kola PPS

- <u>Vremenski domen</u>:

  Zadato:

  $R, L, C,$

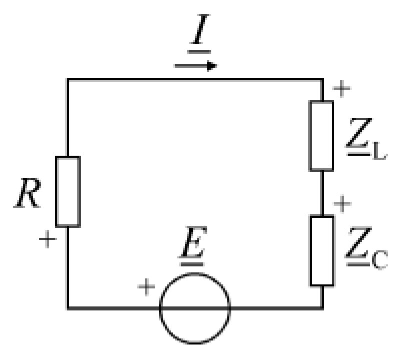
  $e(t) = \sqrt{2}E\cos(\omega t + \theta).$



- <u>Kompleksni domen</u>:

  Računa se:

  $\underline{Z}_L = j\omega L,\ \ \underline{Z}_C = \dfrac{-j}{\omega C},\ \ \underline{E} = E\,e^{j\theta}.$

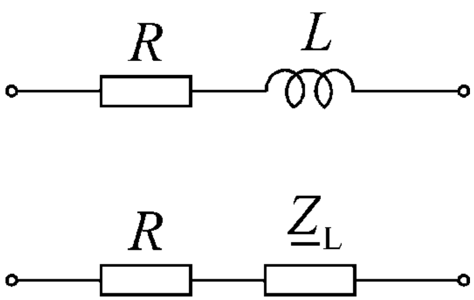


II KZ: $-\underline{E} + R\underline{I} + \underline{Z}_L\underline{I} + \underline{Z}_C\underline{I} = 0,$

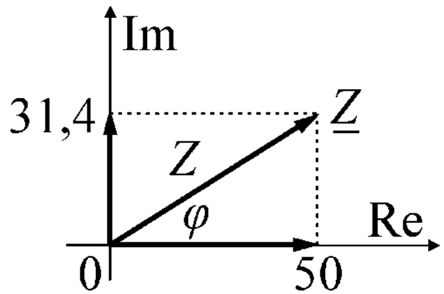
sledi: $\underline{I} = \dfrac{\underline{E}}{R + \underline{Z}_L + \underline{Z}_C}.$

Primer redne veze impedansi

- Potrošač modelovan kao redna veza otpornika otpornosti $50\,\Omega$ i kalema induktivnosti $100\,\text{mH}$ priključen je na napon efektivne vrednosti $230\,\text{V}$ i frekvencije $50\,\text{Hz}$.
  Izračunati:
  a) kompleksnu impedansu potrošača,
  b) moduo impedanse potrošača i
  c) argument ove impedanse.



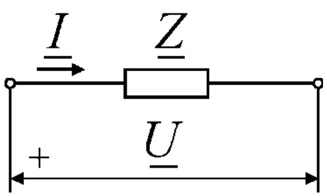
- Rešenje.
  a) Kružna frekvencija: $\omega = 2\pi f = 2\pi \cdot 50 = 314\,\text{rad/s}$.
  Impedansa kalema: $\underline{Z}_\text{L} = \text{j}\omega L = \text{j}314 \cdot 0{,}1 = \text{j}31{,}4\,\Omega$.
  Impedansa potrošača: $\underline{Z} = R + \underline{Z}_\text{L} = (50 + \text{j}31{,}4)\,\Omega$.
  b) Moduo impedance: $Z = \sqrt{Z_\text{Re}^2 + Z_\text{Im}^2} = \sqrt{50^2 + 31{,}4^2} = 59\,\Omega$.
  c) Argument impedance: $\varphi = \text{arctg}\dfrac{Z_\text{Im}}{Z_\text{Re}} = \text{arctg}\dfrac{31{,}4}{50} = 0{,}56$.



Primer

Kroz prijemnik impedanse $\underline{Z} = (3 + \mathrm{j}4)\,\Omega$ postoji kompleksna struja $\underline{I} = (10 - \mathrm{j}8)\,\mathrm{A}$. Kružna frekvencija je $\omega = 120\pi\,\mathrm{rad/s}$. Odrediti:

a) kompleksni napon na prijemniku,
b) efektivnu i maksimalnu vrednost ovog napona,
c) argument napona,
d) napisati vremenski oblik napona na prijemniku.

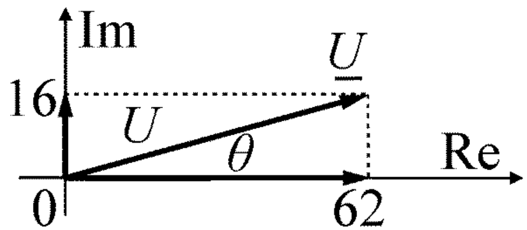


----

a) Kompleksni napon: $\underline{U} = \underline{Z}\,\underline{I} = (3 + \mathrm{j}4)(10 - \mathrm{j}8) = (62 + \mathrm{j}16)\,\mathrm{V}$.

b) Efektivna vrednost: $U = \sqrt{U_{\mathrm{Re}}^2 + U_{\mathrm{Im}}^2} = \sqrt{62^2 + 16^2} = 64\,\mathrm{V}$.

Maksimalna vrednost: $U_{\mathrm{m}} = \sqrt{2}\,U = 64\sqrt{2} = 90,5\,\mathrm{V}$.



c) Argument: $\theta = \mathrm{arctg}\dfrac{U_{\mathrm{Im}}}{U_{\mathrm{Re}}} = \mathrm{arctg}\dfrac{16}{62} = 0,25$.

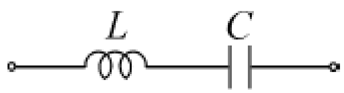
d) Standardni oblik (vremenski domen): $u(t) = U_{\mathrm{m}}\cos(\omega t + \theta)$.

Zamenom $U_{\mathrm{m}} = 90,5\,\mathrm{V}$, $\omega = 120\pi\,\mathrm{rad/s}$ i $\theta = 0,25$:

$u(t) = 90,5\cos(120\pi t + 0,25)\,\mathrm{V}$.

Rezonancija Antirezonancija

- ekvivalentna impedansa može biti jednaka nuli
- najjednostavniji primer: redna LC veza

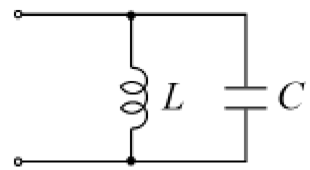


$$\underline{Z}_{\mathrm{ekv}} = \underline{Z}_{\mathrm{L}} + \underline{Z}_{\mathrm{C}} = \mathrm{j}\left(\omega L - \frac{1}{\omega C}\right) = \mathrm{j}\frac{\omega^2 LC - 1}{\omega C}$$

$\underline{Z}_{\mathrm{ekv}} = 0$ za $\underline{Z}_{\mathrm{L}} = -\underline{Z}_{\mathrm{C}}$ tada je $\omega^2 LC = 1$

$\omega = \dfrac{1}{\sqrt{LC}}$ rezonantna kružna učestanost

- ekvivalentna impedansa može biti beskonačna
- najjednostavniji primer: paralelna LC veza



$$\underline{Z}_{\mathrm{ekv}} = \frac{\underline{Z}_{\mathrm{L}}\underline{Z}_{\mathrm{C}}}{\underline{Z}_{\mathrm{L}} + \underline{Z}_{\mathrm{C}}} = \frac{\frac{1}{\mathrm{j}\omega C}\cdot \mathrm{j}\omega L}{\frac{1}{\mathrm{j}\omega C} + \mathrm{j}\omega L} = \frac{\mathrm{j}\omega L}{1 - \omega^2 LC}.$$

$\underline{Z}_{\mathrm{ekv}} = \infty$ za $\underline{Z}_{\mathrm{L}} = -\underline{Z}_{\mathrm{C}}$ tada je $\omega^2 LC = 1$

$\omega = \dfrac{1}{\sqrt{LC}}$ antirezonantna kružna učestanost

Rezistansa i reaktansa

- Rezistansa: $R = \mathrm{Re}\{\underline{Z}\}$.

  Reaktansa: $X = \mathrm{Im}\{\underline{Z}\}$.

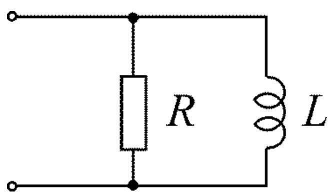
  Jedinica Ω.

  – Primer: ekvivalentna impedansa paralelne RL veze:

$$\underline{Z}_{\mathrm{ekv}} = \frac{R \cdot \mathrm{j}\omega L}{R + \mathrm{j}\omega L} = \frac{R \cdot \mathrm{j}\omega L \cdot (R - \mathrm{j}\omega L)}{(R + \mathrm{j}\omega L)(R - \mathrm{j}\omega L)} = \left(\frac{\omega^2 L^2 R}{R^2 + \omega^2 L^2} \right) + \mathrm{j} \left(\frac{\omega L R^2}{R^2 + \omega^2 L^2} \right),$$

  rezistansa i reaktansa su

$$\mathrm{Re}\{\underline{Z}_{\mathrm{ekv}}\} = \frac{\omega^2 L^2 R}{R^2 + \omega^2 L^2}, \qquad \mathrm{Im}\{\underline{Z}_{\mathrm{ekv}}\} = \frac{\omega L R^2}{R^2 + \omega^2 L^2}.$$



- Reaktansa kalema $X_{\mathrm{L}} = \omega L$ je pozitivna.

  Reaktansa kondenzatora $X_{\mathrm{C}} = -1/(\omega C)$ je negativna.

  Prijemnik impedanse $\underline{Z} = R + \mathrm{j}X$ ima

  - induktivni karakter ako je $X > 0$,
  - kapacitivni karakter ako je $X < 0$.

Snage prijemnika

$u(t) = \sqrt{2}U\cos(\omega t + \theta)$, $i(t) = \sqrt{2}I\cos(\omega t + \psi)$, $\varphi = \theta - \psi$

Trenutna snaga $p(t) = UI\cos\varphi + UI\cos(2\omega t + \theta + \psi)$ [W]

Srednja (aktivna) snaga $P = UI\cos\varphi$ [W]

Prividna snaga: $S = UI$ [VA]

$P = S\cos\varphi$ faktor snage $\cos\varphi = \dfrac{P}{S}$

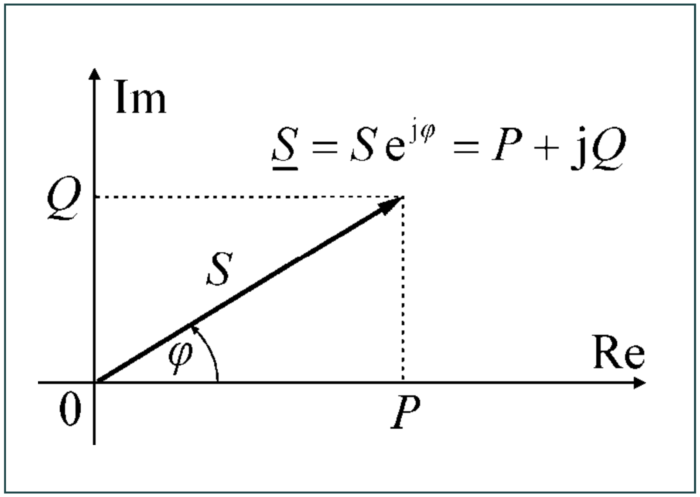
Reaktivna snaga: $Q = S\sin\varphi$ [var]

Kompleksna snaga: $\boxed{\underline{S} = P + \mathrm{j}Q}$ [VA]

$\underline{S} = S\cos\varphi + \mathrm{j}S\sin\varphi = S\,\mathrm{e}^{\mathrm{j}\varphi}$

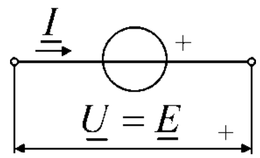
$\underline{U}\,\underline{I}^* = U\,\mathrm{e}^{\mathrm{j}\theta}\cdot(I\,\mathrm{e}^{\mathrm{j}\psi})^* = UI\,\mathrm{e}^{\mathrm{j}(\theta-\psi)} = UI\,\mathrm{e}^{\mathrm{j}\varphi} = S\,\mathrm{e}^{\mathrm{j}\varphi}$

$\boxed{\underline{S} = \underline{U}\,\underline{I}^*}$

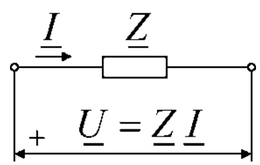


Kompleksna snaga

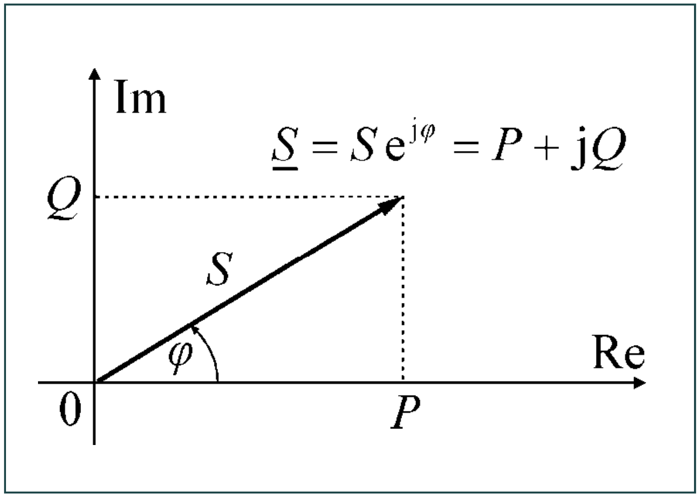
- Na generatoru $\boxed{\underline{U} = \underline{E}, \qquad \underline{S}_{\mathrm{E}} = \underline{E}\,\underline{I}^*}$



- Na impedansi $\boxed{\underline{U} = \underline{Z}\,\underline{I}, \qquad \underline{S} = \underline{U}\,\underline{I}^*}$



snaga	izraz	jedinica
trenutna	$p(t) = u(t)i(t)$	W
kompleksna	$\underline{S} = \underline{U}\,\underline{I}^*$	VA
srednja (aktivna)	$P = \mathrm{Re}\{\underline{S}\}$	W
reaktivna (jalova)	$Q = \mathrm{Im}\{\underline{S}\}$	var
prividna (nominalna)	$S = \sqrt{P^2 + Q^2}$	VA



Snage – primer 1

- Kroz prijemnik impedanse $\underline{Z} = (10 - \mathrm{j}5)\,\Omega$ postoji struja $\underline{I} = (4 + \mathrm{j}3)\,\mathrm{A}$.

  Odrediti njegovu
  a) kompleksnu snagu,
  b) aktivnu snagu,
  c) reaktivnu snagu,
  d) prividnu snagu,
  e) faktor snage.

Rešenje.

$\underline{U} = \underline{Z}\,\underline{I} = (10 - \mathrm{j}5)(4 + \mathrm{j}3)$

$\underline{U} = (55 + \mathrm{j}10)\,\mathrm{V}.$

a) $\underline{S} = \underline{U}\,\underline{I}^* = (55 + \mathrm{j}10)(4 + \mathrm{j}3)^*$

$\underline{S} = (55 + \mathrm{j}10)(4 - \mathrm{j}3)$

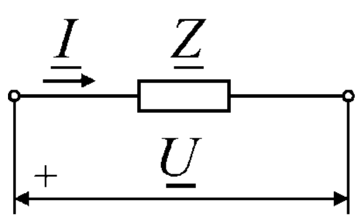
$\underline{S} = (250 - \mathrm{j}125)\,\mathrm{VA},$

b) $P = \mathrm{Re}\{\underline{S}\} = 250\ \mathrm{W},$

c) $Q = \mathrm{Im}\{\underline{S}\} = -125\ \mathrm{var},$

d) $S = \sqrt{P^2 + Q^2} = \sqrt{250^2 + 125^2} = 279{,}5\ \mathrm{VA},$

e) $\cos\varphi = P/S = 250/279{,}5 = 0{,}89.$



Snage – primer 2

- Kroz prijemnik impedanse $\underline{Z} = (20 + \mathrm{j}10)\,\Omega$ postoji struja $\underline{I} = (4 + \mathrm{j}3)\,\mathrm{A}$.

  Odrediti njegovu

  a) kompleksnu snagu,

  b) aktivnu snagu,

  c) reaktivnu snagu,

  d) prividnu snagu,

  e) faktor snage.

Rešenje.

$\underline{U} = \underline{Z}\,\underline{I} = (20 + \mathrm{j}10)(4 + \mathrm{j}3)$

$\underline{U} = (50 + \mathrm{j}100)\,\mathrm{V}.$

a) $\underline{S} = \underline{U}\,\underline{I}^* = (50 + \mathrm{j}100)(4 + \mathrm{j}3)^*$

$\underline{S} = (50 + \mathrm{j}100)(4 - \mathrm{j}3)$

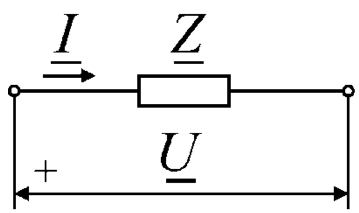
$\underline{S} = (500 + \mathrm{j}250)\,\mathrm{VA},$

b) $P = \mathrm{Re}\{\underline{S}\} = 500\ \mathrm{W},$

c) $Q = \mathrm{Im}\{\underline{S}\} = 250\ \mathrm{var},$

d) $S = \sqrt{P^2 + Q^2} = \sqrt{500^2 + 250^2} = 559\ \mathrm{VA},$

e) $\cos\varphi = P/S = 500/559 = 0{,}89.$



Teoreme

- Teorema održanja snage (TOS)

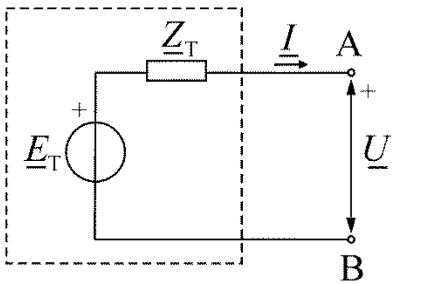
  $\boxed{\sum p_p(t) = \sum p_g(t)}$ TOS za trenutne snage

  $\boxed{\sum \underline{S}_p = \sum \underline{S}_g}$ TOS za kompleksne snage

  Iz $\sum (P_p + jQ_p) = \sum (P_g + jQ_g)$ sledi da TOS važi i za aktivnu i za reaktivnu snagu

  TOS <u>ne</u> važi za prividne snage

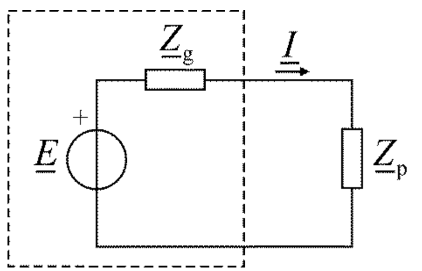
- Tevenenova teorema
  – kolo se u odnosu na bilo koje svoje dve tačke ponaša kao realan naponski generator
  – ekvivalentan (Tevenenov) generator: $\underline{E}_T$ i $\underline{Z}_T$



- Uslov prilagođenja po snazi: $\boxed{\underline{Z}_p = \underline{Z}_g^*}$

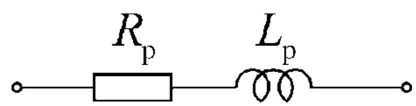
  Primer: $\underline{Z}_g = (100 - j50)\,\Omega$.

  Impedansa prijemnika prilagođenog po snazi:
  $\underline{Z}_p = \underline{Z}_g^* = (100 + j50)\,\Omega$.

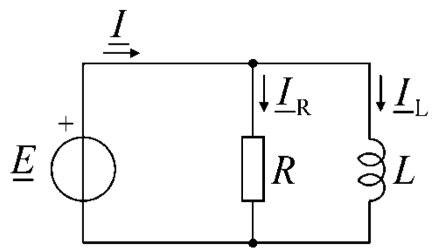


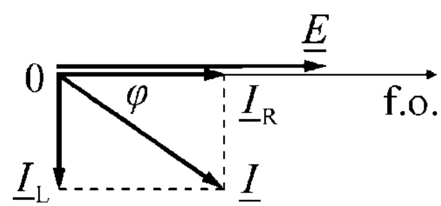
Popravka faktora snage

- Motori su induktivni prijemnici



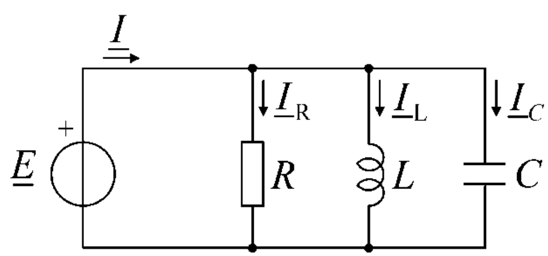
  - može se transformisati u paralelnu vezu

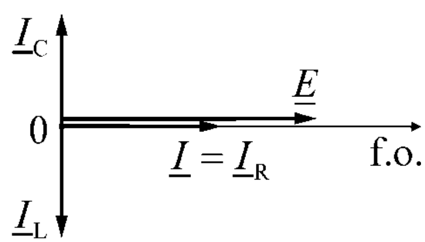




- $P = UI\cos\varphi$, idealno $\cos\varphi = 1 \quad (\varphi = 0)$

  povećati $\cos\varphi$ (smanjiti φ)





$$\underline{Z}_L = -\underline{Z}_C$$

$$\boxed{C = \frac{1}{\omega^2 L}}$$