STUDIJSKI PROGRAM ZA PRIMENJENO SOFTVERSKO INŽENJERSTVO

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike (I kolokvijum)

K1

11.02.2023.

ZADACI

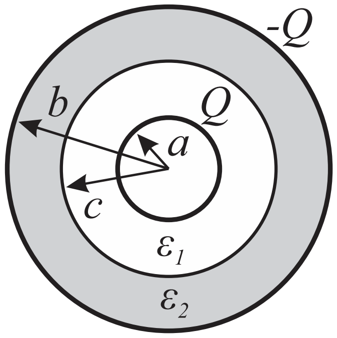
Zadatak 1. Na slici 1 je prikazan poprečni presek vazdušnog koaksijalnog kabla, poluprečnika elektroda a = 1 mm i b = 7,29 mm, dužine L = 1 m, sa dva sloja dielektrika, permitivnosti $\varepsilon_1 = 2\varepsilon_0$ i $\varepsilon_2 = 4\varepsilon_0$. Poluprečnik granične površine između dielektrika je c = 2,7 mm. Kabl je priključen na izvor napona U.

a) Odrediti kako se u zavisnosti od rastojanja r, menjaju intenziteti vektora jačine električnog polja, vektora električnog pomeraja i vektora polarizacije.

b) Odrediti izraz za kapacitivnost koaksijalnog kabla C, i izračunati njenu brojnu vrednost.

c) Proveriti da li će doći do proboja ako se kabl priključi na napon U_1 = 20 kV.

Brojni podaci su:
$E_{č1}$ = 100 kV/cm, $E_{č2}$ = 40 kV/cm, $\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m.



Slika 1.

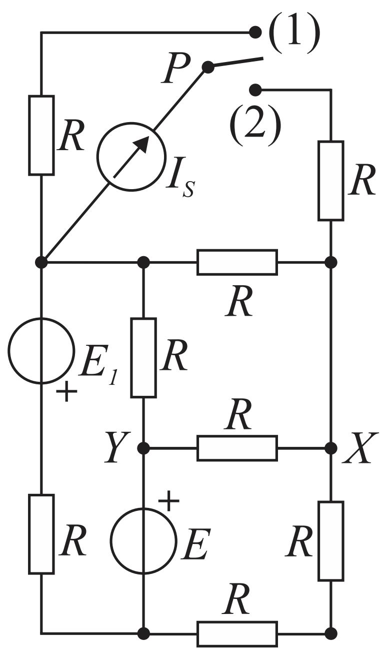
Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, posle prebacivanja preklopnika P iz položaja (1) u položaj (2), napon između tačaka X i Y se promeni za ΔU_{XY} = 2 V.

a) Odrediti jačinu struje strujnog generatora I_S.

b) Izračunati snagu koju razvija strujni generator u stacionarnom stanju mreže, koje nastane kada je preklopnik u položaju (2).

c) Izračunati snagu idealnog naponskog generatora E kada je preklopnik u položaju (2).

Brojni podaci su: $E = E_1$ = 6,5 V, R = 6,5 Ω.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum je neophodno sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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ZADACI

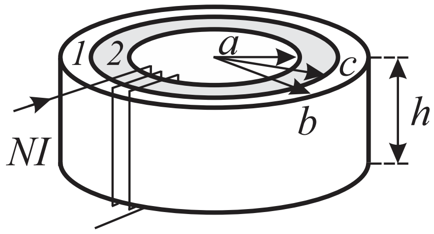
Zadatak 1. Ne debelom torusnom jezgru načinjenom od dva feromagnetska materijala, prikazanom na slici 1a, nalazi se namotaj sa $N = 314$ zavojaka tanke, provodne žice, namotanih ravnomerno i gusto po celom torusu. Krive prvobitnog magnetisanja oba materijala su prikazane na slici 1b.

a) Izračunati maksimalnu vrednost jačine struje u namotajima, I, tako da oba sloja jezgra budu u linearnom režimu rada.

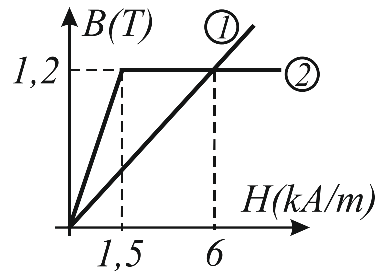
b) Odrediti režime rada, oba sloja jezgra, nakon povećanja struje određene pod *a*) na duplo veću vrednost, $I_1 = 2 \cdot I$.

c) Odrediti energiju sadržanu u jezgru nakon povećanja struje određene pod *a*) na duplo veću vrednost, $I_1 = 2 \cdot I$.

Dimenzije jezgra su $a = 4\ cm$, $c = 6\ cm$, $b = 8\ cm$, $h = 3\ cm$.



Slika 1a.



Slika 1b.

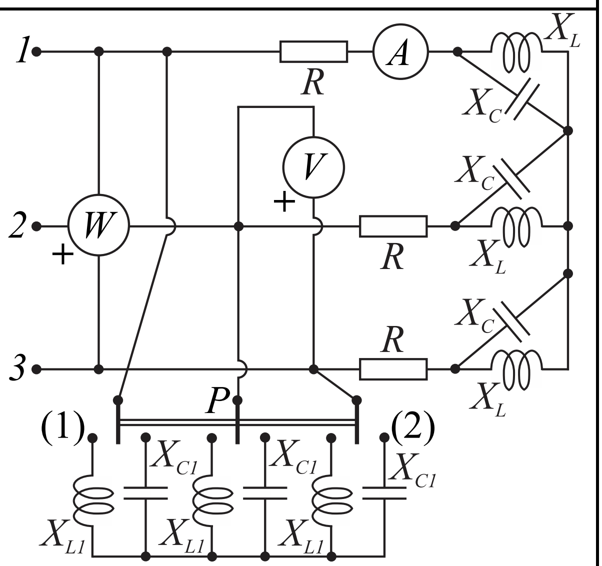
Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik, sastavljen od otpornika otpornosti $R = 20\ \Omega$, kalemova reaktanse $X_L = 20\ \Omega$ i kondenzatora reaktanse $X_C = 10\ \Omega$. Prijemnik je priključen na mrežu faznog napona $\underline{U}_1 = 230\ V$.

a) Odrediti položaj prekidača P, koji omogućava da se faktor snage prijemnika popravi na jedinicu.

b) Odrediti reaktanse elemenata čijim dodavanjem se vrši popravak faktora snage.

c) Odrediti pokazivanja idealnih mernih instrumenata pre i posle popravka faktora snage.

d) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



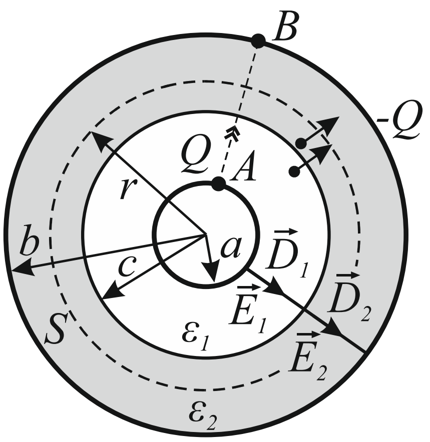
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

PSI ISPIT 11.02.2023.

K1 Z1



a)

Granični uslov:

$D_{n1} = D_{n2}$ $\qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{OM} D\,ds = Q$$

$$D2\pi rL = Q \qquad \boxed{D = \frac{Q}{2\pi rL}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 rL}}, \quad a \le r \le c \qquad \boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 rL}}, \quad c \le r \le b$$

$$P_1 = D - \varepsilon_0 E_1 = D - \varepsilon_0\frac{D}{\varepsilon_1} = D\left(1 - \varepsilon_0\frac{1}{\varepsilon_{r1}\varepsilon_0}\right) = D\left(1 - \frac{1}{\varepsilon_{r1}}\right) = \frac{Q}{2\pi rL}\cdot\left(1 - \frac{1}{2}\right) = \frac{1}{2}\cdot\frac{Q}{2\pi rL} = \frac{Q}{4\pi rL} \qquad \boxed{P_1 = \frac{Q}{4\pi rL}},\ a \le r \le c$$

$$P_2 = D - \varepsilon_0 E_2 = D - \varepsilon_0\frac{D}{\varepsilon_2} = D\left(1 - \varepsilon_0\frac{1}{\varepsilon_{r2}\varepsilon_0}\right) = D\left(1 - \frac{1}{\varepsilon_{r2}}\right) = \frac{Q}{2\pi rL}\cdot\left(1 - \frac{1}{4}\right) = \frac{3}{4}\cdot\frac{Q}{2\pi rL} = \frac{3Q}{8\pi rL} \qquad \boxed{P_2 = \frac{3Q}{8\pi rL}},\ c \le r \le b$$

b) $C = \dfrac{Q}{U}$

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E dr = \int_a^c E_1 dr + \int_c^b E_2 dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 rL}dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 rL}dr = \frac{Q}{2\pi L}\left(\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}\right)$$

$$C = \frac{2\pi L}{\frac{1}{\varepsilon_1}\ln\frac{c}{a} + \frac{1}{\varepsilon_2}\ln\frac{b}{c}} = \frac{2\pi\cdot 1}{\frac{1}{2\varepsilon_0}\ln 2{,}7 + \frac{1}{4\varepsilon_0}\ln 2{,}7} = \frac{8}{3}\pi\varepsilon_0$$

$$\boxed{C = 74{,}14\ \text{pF}}$$

c)

$$E_{1\max}(r = a) = \frac{Q_{1\max}}{2\pi\varepsilon_1 aL} \le E_{\check{c}1} \quad\rightarrow\quad Q_{1\max} = E_{\check{c}1}2\pi\varepsilon_1 aL = 1{,}11\ \mu\text{C}$$

$$E_{2\max}(r = c) = \frac{Q_{2\max}}{2\pi\varepsilon_2 cL} \le E_{\check{c}2} \quad\rightarrow\quad Q_{2\max} = E_{\check{c}2}2\pi\varepsilon_2 cL = 2{,}4\ \mu\text{C}$$

$$Q_{\max} = \min\{Q_{1\max}, Q_{2\max}\} = Q_{1\max} = 1{,}11\ \mu\text{C}$$

$$U_{\max} = \frac{Q_{\max}}{C} = 15\ \text{kV}$$

$U_1 = 20\ kV$

$U_1 > U_{\max} = 15\ \text{kV}$

Doći će do proboja.

K1 Z2

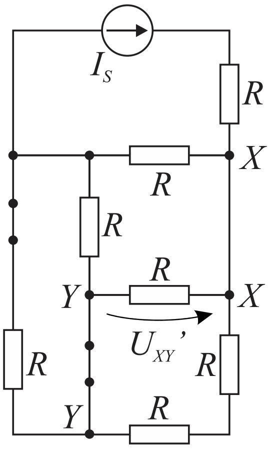
a)

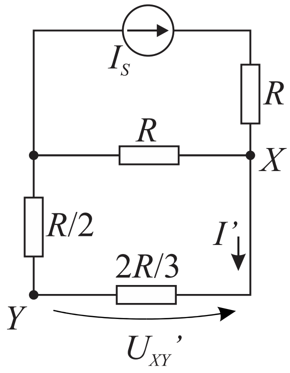
$$\boxed{\begin{matrix} Svi \\ generatori \end{matrix}} = \boxed{\begin{matrix} Svi \\ sem\ I_S \end{matrix}} + \boxed{\begin{matrix} Samo \\ I_S \end{matrix}}$$

(2) (1)

$$U_{XY}^{(2)} = U_{XY}^{(1)} + U_{XY}'$$

$$\Delta U_{XY} = U_{XY}^{(2)} - U_{XY}^{(1)} = U_{XY}' = 2\ \text{V}$$





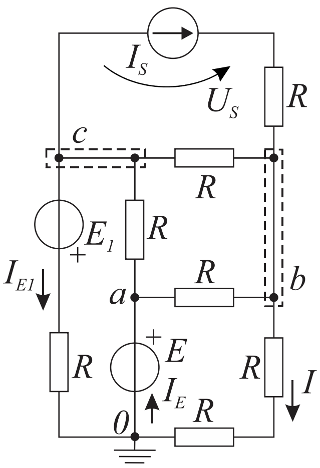
$$I' = \frac{U_{XY}'}{2R/3} = \frac{2}{13/3} = 0,462\ \text{A}$$

$$I' = \frac{R}{R + R/2 + 2R/3} I_S$$

$$I_S = \frac{R + R/2 + 2R/3}{R} I' = \frac{13}{6} I'$$

$$\boxed{I_S = 1\ \text{A}}$$

b)



$$n_č = 4, \quad n_{i.n.g.} = 1$$

$$MPČ: \quad n_č - 1 - n_{i.n.g.} = 4 - 1 - 1 = 2$$

$$V_0 = 0\ \text{V}, \qquad V_a = E = 6,5\ \text{V}$$

$$V_b\left(\frac{1}{R+\infty} + \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}\right) - V_a\left(\frac{1}{R}\right) - V_c\left(\frac{1}{R+\infty} + \frac{1}{R}\right) = I_S \qquad / \cdot 2R$$

$$V_c\left(\frac{1}{R+\infty} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R+0}\right) - V_a\left(\frac{1}{R}\right) - V_b\left(\frac{1}{R+\infty} + \frac{1}{R}\right) = -I_S - \frac{E_1}{R} \qquad / \cdot R$$

$$5V_b - 2V_a - 2V_c = 2R\, I_S$$

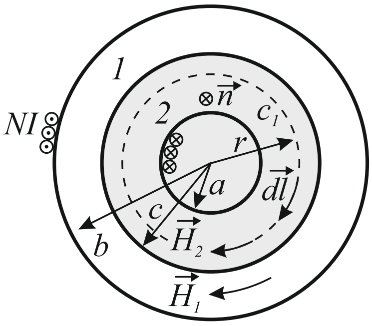
$$3V_c - V_a - V_b = -R\, I_S - E_1$$

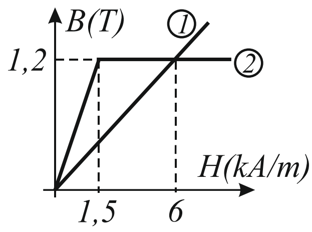
$$\left.\begin{aligned} 5V_b - 2V_c &= 26 \\ 3V_c - V_b &= -6,5 \end{aligned}\right\} \quad \Rightarrow \quad V_b = 5\ \text{V} \qquad V_c = -0,5\ \text{V}$$

$$U_S = RI_S + (V_b - V_c) = 6,5 \cdot 1 + (5 - (-0,5)) = 12\ \text{V} \qquad \boxed{P_S = U_S\, I_S = 12\ \text{W}}$$

$$P_E = E \cdot I_E = E \cdot (I_{E1} + I) = E \cdot \left(\frac{V_c - V_0 + E_1}{R} + \frac{V_b - V_0}{2R}\right) = 8,5\,\text{W}$$

K2 Z1





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_{c1} \vec{H} \cdot \vec{dl} = NI$$

$$H\, 2r\pi = NI$$

$$H = \frac{NI}{2r\pi} \qquad a < r < b$$

a)

Materijal 1 je stalno u linearnom režimu rada.

$$H_{2\max}(r = a) = \frac{NI}{2\pi a} \le H_{K2} = 1500\ A/m$$

$$I = \frac{H_{K2}\, 2\pi a}{N} = \frac{1500 \cdot 2\pi \cdot 0,04}{314}$$

$$\boxed{I = 1,2\ A}$$

b)

$$I_1 = 2I = 2,4\ A$$

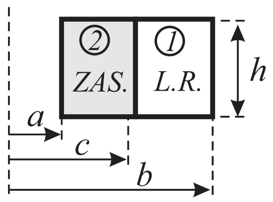
Materijal 1 je stalno u linearnom režimu rada.

$$H_{2\max}(r = a) = \frac{NI_1}{2\pi a} = 3000\ A/m \ge H_{K2}$$

$$H_{2\min}(r = c) = \frac{NI_1}{2\pi c} = 2000\ A/m \ge H_{K2}$$

$\Rightarrow$ Materijal 2 u zasićenju.

c)



$$W_{m2} = \frac{1}{2} B_{K2} H_{K2} V_2 = \frac{1}{2} B_{K2} H_{K2} \left(c^2 - a^2\right) \pi\, h$$

$$W_{m2} = \frac{1}{2} \cdot 1,2 \cdot 1500 \cdot \left(0,06^2 - 0,04^2\right) \cdot 3,14 \cdot 0,03$$

$$\boxed{W_{m2} = 169,6\ mJ}$$

$$W_{m1} = \int_{V_1} \omega_{m1}\, dV = \int_{V_1} \frac{1}{2} B_1 H_1\, dV = \int_{V_1} \frac{1}{2} \mu_1 \left(\frac{NI_1}{2\pi r}\right)^2 2\pi r\, dr\, h = \int_{V_1} \frac{1}{2} \mu_1 \frac{(NI_1)^2}{2\pi r}\, dr\, h$$

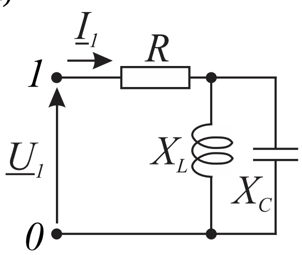
$$W_{m1} = \frac{1}{2} \mu_1 \frac{(NI_1)^2}{2\pi} h \int_c^b \frac{dr}{r} = \frac{1}{2} \mu_1 \frac{(NI_1)^2}{2\pi} h \ln\frac{b}{c} = \frac{1}{2} \frac{1,2}{6000} \frac{(314 \cdot 2,4)^2}{6,28} \cdot 0,03 \cdot \ln\frac{0,08}{0,06}$$

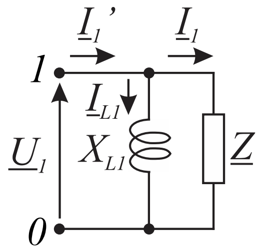
$$\boxed{W_{m1} = 78\ mJ}$$

$$\boxed{W_m = W_{m1} + W_{m2} = 247,6\ mJ}$$

K2 Z2

a)



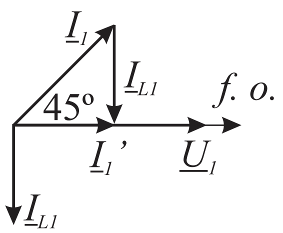


$$\underline{Z} = R + \frac{jX_L\left(-jX_C\right)}{jX_L + \left(-jX_C\right)} = 20 + \frac{j20\cdot\left(-j10\right)}{j20 + \left(-j10\right)}$$

$$\underline{Z} = \left(20 - j20\right)\Omega = 20\sqrt{2}\, e^{-j45^\circ}\ \Omega$$

$$\boxed{\text{Treba dodati } X_{\text{L1}}\text{. Prekidač u položaj }(1).}$$

b)



I KZ: $\underline{I}_1{}' = \underline{I}_{L1} + \underline{I}_1$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\, e^{j0^\circ}}{Z\, e^{j\varphi}} = \frac{230}{20\sqrt{2}\, e^{-j45^\circ}} = 8{,}13\, e^{j45^\circ}\ A$$

$$\underline{I}_{L1} = \frac{U_1\, e^{j0}}{jX_{L1}} = \frac{U_1\, e^{j0}}{X_{L1}\, e^{j90^\circ}} = \frac{U_1}{X_{L1}}\, e^{-j90^\circ}\ A$$

$$I_{L1} = I_1 \sin 45^\circ$$

$$\frac{U_1}{X_{L1}} = \frac{U_1}{Z}\sin 45^\circ \quad\Rightarrow\quad \frac{1}{X_{L1}} = \frac{\sin 45^\circ}{Z} \quad\Rightarrow\quad X_{L1} = \frac{20\sqrt{2}}{\sin 45^\circ}$$

$$\boxed{X_{L1} = 40\ \Omega}$$

$$I_1{}' = I_1 \cos 45^\circ = 8{,}13\cdot\cos 45^\circ = 5{,}75\ A \qquad\qquad \underline{I}_1{}' = 5{,}75\, e^{j0^\circ}\ A$$

c)

$$\boxed{I_A{}^{PRE} = I_A{}^{POSLE} = \left|\underline{I}_1\right| = 8{,}13\ A}$$

$$\boxed{U_V{}^{PRE} = U_V{}^{POSLE} = \left|\underline{U}_{32}\right| = \sqrt{3}\left|\underline{U}_1\right| = 398{,}4\ V}$$

$$P_W{}^{PRE} = \operatorname{Re}\left\{\underline{U}_{31}\, \underline{I}_2{}^{*}\right\} = U_{31}\, I_2 \cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_2\right)$$

$$P_W{}^{PRE} = \sqrt{3}U_1\, I_1 \cos\left(210^\circ - 75^\circ\right) = \sqrt{3}\cdot 230\cdot 8{,}13\cdot\cos 135^\circ$$

$$\boxed{P_W{}^{PRE} = -2290{,}15\ W}$$

$$P_W{}^{POSLE} = \operatorname{Re}\left\{\underline{U}_{31}\, \underline{I}_2{}'^{*}\right\} = U_{31}\, I_2{}'\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_2{}'\right)$$

$$P_W{}^{POSLE} = \sqrt{3}U_1\, I_1{}'\cos\left(210^\circ - 120^\circ\right) = \sqrt{3}\cdot 230\cdot 5{,}75\cdot\cos 90^\circ$$

$$\boxed{P_W{}^{POSLE} = 0\ W}$$

d)