STUDIJSKI PROGRAM MEHATRONIKA

K TET

Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

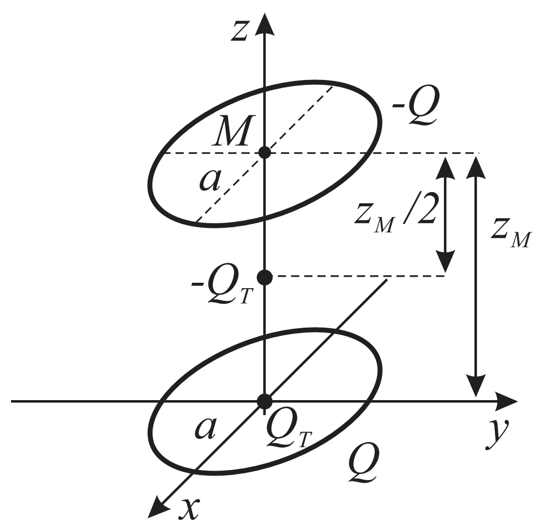
23.01.2023.

ZADACI

Zadatak 1. Na slici 1 su prikazana dva tanka prstena poluprečnika *a,* naelektrisana ravnomemerno jednakim količinama naelektrisanja *Q*, ali suprotnog znaka. Pozitivno naelektrisan prsten leži u *x-y* ravni Dekartovog koordinatnog sistema, dok je negativno naelektrisan prsten postavljen u ravan koja je paralalena sa *x-y* ravni i translirana duž *z* ose za rastojanje z_M (kooridantni početak ove ravni odgovara tački *M*). Duž *z* ose, na rastojanjima z_M i z_M /2 od tačke *M*, postavljena su dva tačkasta naelektrisanja, Q_T i $-Q_T$. Sistem se nalazi u vazduhu.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački *M* stvaraju oba prstena i oba tačkasta naelektrisanja.
b) (**bonus – 5 poena**) Izračunati, brojno, potencijal tačke *M* u odnosu na referentnu tačku u beskonačnosti, koji potiče samo od **negativno** naelektrisanih struktura.

Brojni podaci su: a = 1 cm, z_M = 2 cm, Q = 1 nC, Q_T = 1 pC, $\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m.

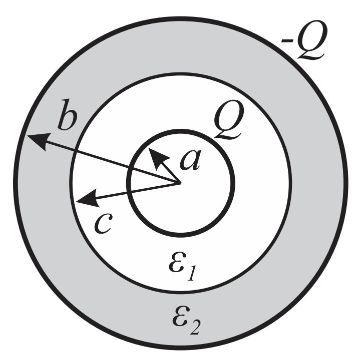


Slika 1.

Zadatak 2. Na slici 2 je prikazan **sferni** kondenzator, ispunjen sa dva sloja dielektrika relativnih permitivnosti ε_{r1} = 8 i ε_{r2} = 6. Poluprečnici elektroda kondenzatora su a = 1 mm i b = 3 mm. Poluprečnik razdvojne površi dva dielektrika je c = 2 mm. Kabl je priključen na izvor napona *U.*

a) Razmotriti granične uslove i odrediti kako se u zavisnosti od rastojanja tačke od centra kondenzatora menjaju intenziteti vektora električnog pomeraja i vektora jačine električnog polja.
b) Odrediti izraz za kapacitivnost kondenzatora, te potom izračunati njenu brojnu vrednost.
c) Da li će doći do proboja kondenzatora, ukoliko je poznato da maksimalni napon na koji kondenzator sme da se priključi iznosi 10 kV?
d) Izračunati količinu vezanog naelektrisanja uz unutrašnju elektrodu kondenzatora.

Ostali brojni podaci su: Q = 10 nC.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MEHATRONIKA

K **TET** Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **23.01.2023.**

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

a) Deo mreže između tačaka 1 i 2 predstaviti ekvivalentnim Tevenenovim generatorom E_{T1}, R_{T1}.

b) Transformacijom generatora uprostiti deo mreže između tačaka 2 i 3 i predstaviti ga ekvivalentnim realnim naponskim generetorom E_{T2}, R_{T2}.

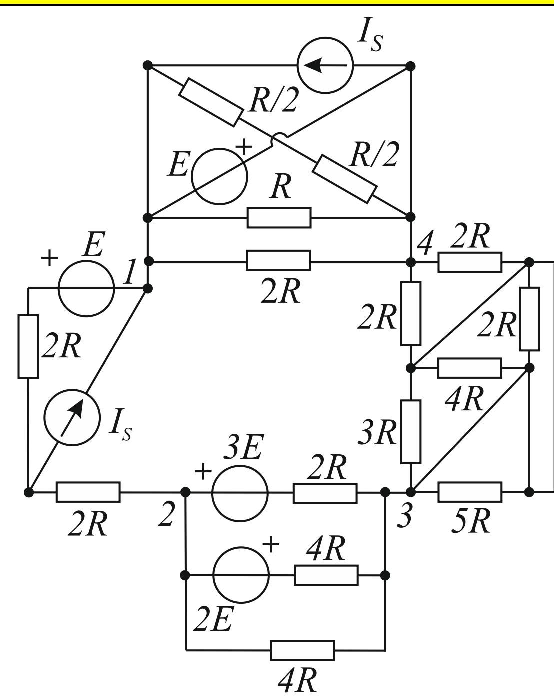
c) Deo mreže između tačaka 3 i 4 predstaviti ekvivalentnim otpornikom R_e.

d) Deo mreže između tačaka 4 i 1 predstaviti Tevenenovim generatorom E_{T3}, R_{T3}.

e) U uprošćenom kolu, izračunati snage generatora E_{T1}, E_{T2} i E_{T3}.

f) U uprošćenom kolu, izračunati snage otpornika.

Brojni podaci su: $R = 5\ \mathrm{k\Omega}$, $E = 5\ \mathrm{V}$, $I_S = 1\ \mathrm{mA}$.



Slika 1.

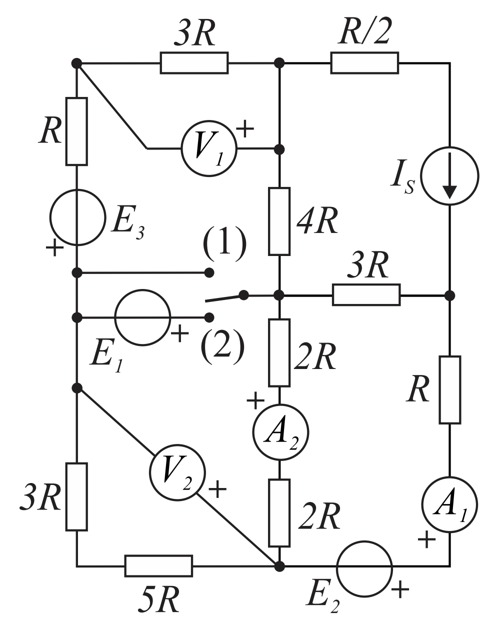
Zadatak 2. U kolu vremenski konstantne struje sa slike 2, primenom teoreme superpozicije:

a) Odrediti za koliko će se promeniti pokazivanje idealnih mernih instrumenata, nakon što se preklopnik prebaci iz položaja (1) u položaj (2). **Napomena**: kolo rešavati primenom metode potencijala čvorova.

b) Odrediti promenu snage strujnog generatora i promenu snage otpornika otpornosti $R/2$.

c) **(bonus – 5 poena)** Odrediti promenu snage naponskih generatora E_2 i E_3.

Brojni podaci su: $R = 10\ \mathrm{k\Omega}$, $E_1 = 3\ \mathrm{V}$, $E_2 = 10\ \mathrm{V}$, $E_3 = 8\ \mathrm{V}$, $I_S = 1\ \mathrm{mA}$.



Slika 2.

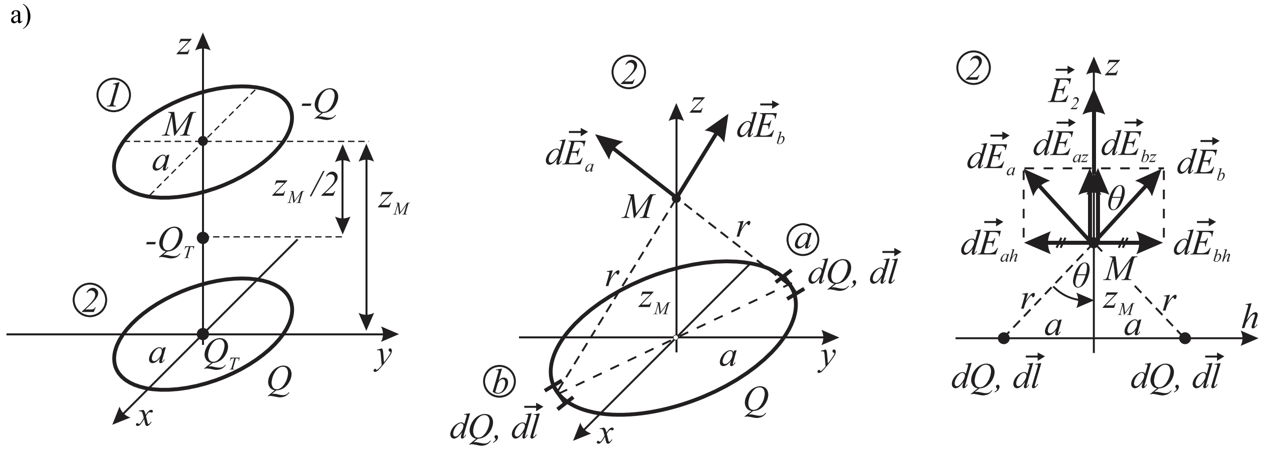
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MEH ISPIT 23.01.2023.

K1 Z1

a)



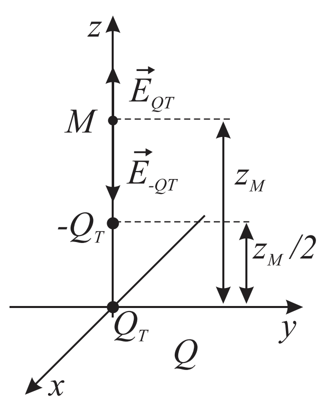
Prsten 1: na osnovu simetrije, s obzirom da se tačka M nalazi u centru prstena 1, zaključujemo da je $\boxed{\vec{E}_1 = 0}$

Prsten 2:

$$dE_{az} = dE_{bz} = dE_a\cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 r^2}\,\frac{z_M}{r} = \frac{Q'\,z_M}{4\pi\varepsilon_0 r^3}dl \qquad \left(Q' = \frac{Q}{2a\pi}\right)$$

$$E_2 = \int\limits_{\substack{po\\ prstenu\,2}} dE_{az} = \frac{Q'\,z_M}{4\pi\varepsilon_0 r^3}\int\limits_0^{2a\pi} dl = \frac{\frac{Q}{2a\pi}z_M}{4\pi\varepsilon_0 r^3}2a\pi = \frac{Q\,z_M}{4\pi\varepsilon_0\left(a^2+z_M^2\right)^{\frac{3}{2}}} \qquad \boxed{\vec{E}_2 = \frac{Q\,z_M}{4\pi\varepsilon_0\left(a^2+z_M^2\right)^{\frac{3}{2}}}\cdot\vec{i}_z}$$

Tačkasta naelektrisanja:



$$E_{QT} = \frac{Q_T}{4\pi\varepsilon_0 {z_M}^2} \qquad \boxed{\vec{E}_{QT} = \frac{Q_T}{4\pi\varepsilon_0 {z_M}^2}\cdot\vec{i}_z}$$

$$E_{-QT} = \frac{Q_T}{4\pi\varepsilon_0\left(z_M/2\right)^2} \qquad \boxed{\vec{E}_{-QT} = \frac{Q_T}{\pi\varepsilon_0 {z_M}^2}\cdot\left(-\vec{i}_z\right)}$$

$$\vec{E}_M = \vec{E}_1 + \vec{E}_2 + \vec{E}_{QT} + \vec{E}_{-QT}$$

$$\boxed{\vec{E}_M = \left(\frac{Q\,z_M}{4\pi\varepsilon_0\left(a^2+z_M^2\right)^{\frac{3}{2}}} - \frac{3Q_T}{4\pi\varepsilon_0 {z_M}^2}\right)\cdot\left(\vec{i}_z\right)}$$

b) **bonus (5-poena)**

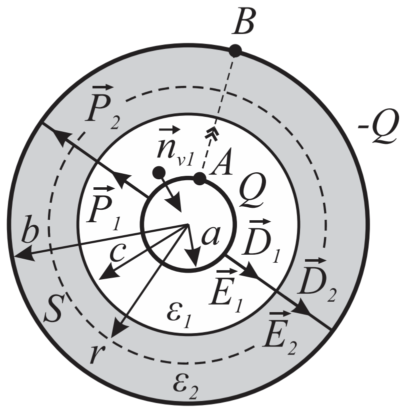
$$dV = \frac{dQ}{4\pi\varepsilon_0 r} = \frac{Q'dr}{4\pi\varepsilon_0 r}$$

$$V_{M(-Q)} = \int\limits_{po\ \ prstenu} dV = \int\limits_{po\ \ prstenu}\frac{-Q'dl}{4\pi\varepsilon_0 a} = \frac{-Q'}{4\pi\varepsilon_0 a}\int_0^{2a\pi} dl = \frac{\frac{-Q}{2a\pi}}{4\pi\varepsilon_0 a}2a\pi = \frac{-Q}{4\pi\varepsilon_0 a}$$

$$V_{M(-Q_T)} = \frac{-Q_T}{4\pi\varepsilon_0\frac{z_M}{2}} = \frac{-Q_T}{2\pi\varepsilon_0 z_M} \qquad V_M = V_{M(-Q)} + V_{M(-Q_T)} = -\frac{Q}{4\pi\varepsilon_0 a} - \frac{Q_T}{2\pi\varepsilon_0 z_M} \qquad \boxed{V_M = -900{,}08\,\mathrm{V}}$$

K1 Z2

a)



Granični uslov:

$D_{n1} = D_{n2}$ $\qquad D_1 = D_2 = D$

$E_{t1} = E_{t2} = 0$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_S D\, ds = Q$$

$$D\, 4r^2\pi = Q \qquad \boxed{D = \frac{Q}{4\pi r^2}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_1 r^2}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_2 r^2}}, \qquad c \le r \le b$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr + \int_c^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$U_{AB} = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_1}\frac{c-a}{a\,c} + \frac{1}{\varepsilon_2}\frac{b-c}{b\,c}\right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi}{\frac{1}{\varepsilon_1}\frac{c-a}{a\,c} + \frac{1}{\varepsilon_2}\frac{b-c}{b\,c}}} \qquad \boxed{C = 1{,}23\ \text{pF}}$$

c)

$Q = 10\ \text{nC}$

$$U = \frac{Q}{C} = \frac{10 \cdot 10^{-9}}{1{,}23 \cdot 10^{-12}} = 8{,}13\ \text{kV}$$

$U_{\max} = 10\ \text{kV}$

$\boxed{U < U_{\max} \;\Rightarrow\; \text{neće doći do proboja kondenzatora}}$

d)

$$P_1 = D - \varepsilon_0 E_1 = D - \varepsilon_0 \frac{D}{\varepsilon_1} = \left(1 - \frac{1}{\varepsilon_{r1}}\right) D = \left(1 - \frac{1}{8}\right)\frac{Q}{4\pi r^2} = \frac{7}{8}\frac{Q}{4\pi r^2}$$

$$\sigma_{v1} = \vec{P}_1 \cdot \vec{n}_{v1} = -P_1(r = a) = -\frac{7}{8}\frac{Q}{4\pi a^2}$$

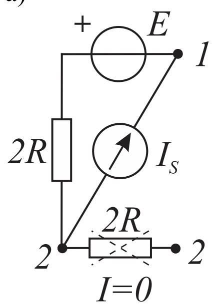
$$Q_{v1} = \sigma_{v1}\, 4\pi a^2 = -\frac{7}{8}\frac{Q}{4\pi a^2}\, 4\pi a^2 = -\frac{7}{8} Q = -\frac{7}{8} \cdot 10\ nC$$

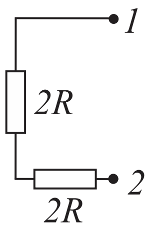
$\boxed{Q_{v1} = -8{,}75\ \text{nC}}$

MH ISPIT, K2, 23.01.2023.

K2 Z1

a)



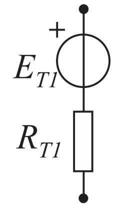


$$U_{12} = -E + 2RI_S = -5 + 10\text{k}\cdot 1\text{m} = 5\text{V}$$

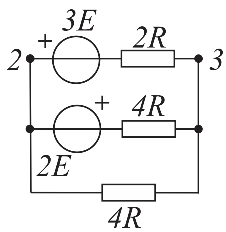
$$\boxed{E_{T1} = U_{12} = 5\text{V}}$$

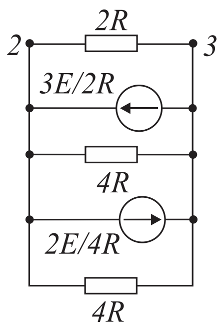
$$R_{T1} = R_{12} = 2R + 2R = 4R$$

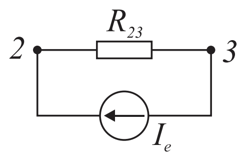
$$\boxed{R_{T1} = 20\,\text{k}\Omega}$$

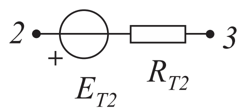


b)









$$R_{23} = R_{T2} = 2R \,||\, 4R \,||\, 4R = 2R \,||\, 2R = R$$

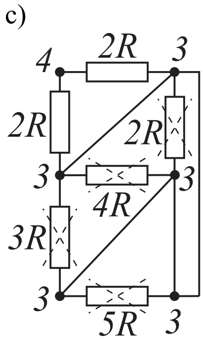
$$\boxed{R_{T2} = 5\,\text{k}\Omega}$$

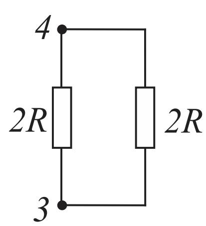
$$I_e = \frac{3E}{2R} - \frac{2E}{4R} = 1{,}5\frac{E}{R} - 0{,}5\frac{E}{R} = \frac{E}{R} = \frac{5}{5\,\text{k}} = 1\,\text{mA}$$

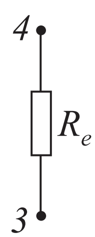
$$E_{T2} = U_{23} = R_{23}\cdot I_e = 5\text{k}\cdot 1\text{m}$$

$$\boxed{E_{T2} = 5\,\text{V}}$$

c)



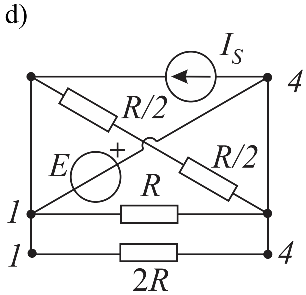


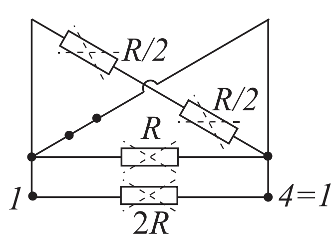


$$R_e = 2R \,||\, 2R = R$$

$$\boxed{R_e = 5\,\text{k}\Omega}$$

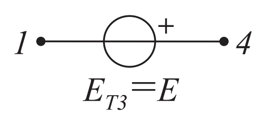
d)



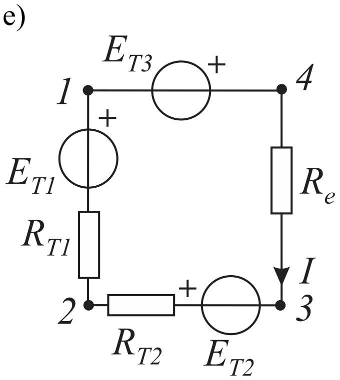


$$\boxed{R_{T3} = 0}$$

$$\boxed{E_{T3} = E = 5\,\text{V}}$$



e)



$$I = \frac{E_{T1}+E_{T2}+E_{T3}}{R_{T1}+R_{T2}+R_e} = \frac{5+5+5}{20\text{k}+5\text{k}+5\text{k}} = \frac{15}{30\,\text{k}} = 0{,}5\,\text{mA}$$

$$P_{E_{T1}} = E_{T1}I = 5\cdot 0{,}5\text{m} = 2{,}5\,\text{mW}$$

$$E_{T1} = E_{T2} = E_{T3} \quad \Rightarrow \quad \boxed{P_{E_{T1}} = P_{E_{T2}} = P_{E_{T3}} = 2{,}5\,\text{mW}}$$

f)

$$P_{R_{T1}} = R_{T1}I^2 = 20\,\text{k}\cdot(0{,}5\text{m})^2 \qquad \boxed{P_{R_{T1}} = 5\,\text{mW}}$$

$$P_{R_{T2}} = R_{T2}I^2 = 5\,\text{k}\cdot(0{,}5\text{m})^2 \qquad \boxed{P_{R_{T1}} = 1{,}25\,\text{mW}}$$

$$P_{R_e} = R_eI^2 = 5\,\text{k}\cdot(0{,}5\text{m})^2 \qquad \boxed{P_{R_e} = 1{,}25\,\text{mW}}$$

K2 Z2

1. Na osnovu uslova teksta zadatka: $U_V^{(2)} = U_V^{(1)} + \Delta U_V$, $\quad I_A^{(2)} = I_A^{(1)} + \Delta I_A$

2. Na osnovu teoreme superpozicije:

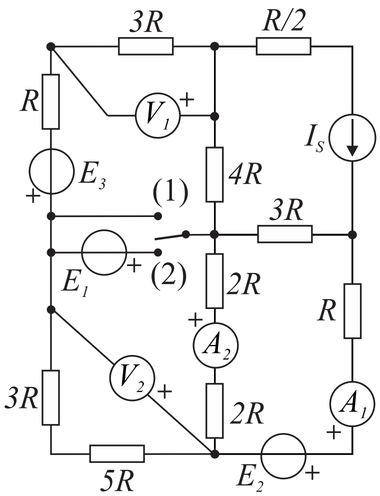
$$\underset{(2)}{\boxed{\begin{matrix}\text{Svi}\\ \text{generatori}\end{matrix}}} = \underset{(1)}{\boxed{\begin{matrix}\text{Svi}\\ \text{sem } E_1\end{matrix}}} + \boxed{\begin{matrix}\text{Samo}\\ E_1\end{matrix}}$$

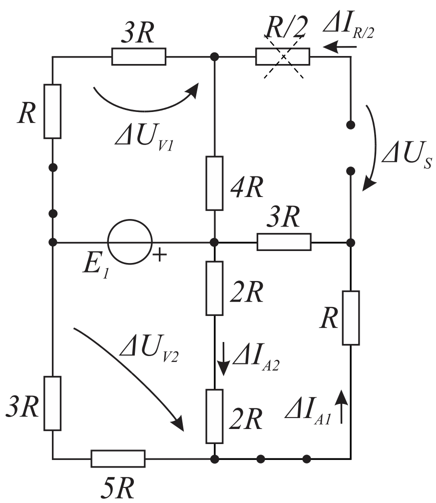
Preklopnik u položaju (2)
(deluju svi generatori)
$U_V^{(2)}$
$I_A^{(2)}$

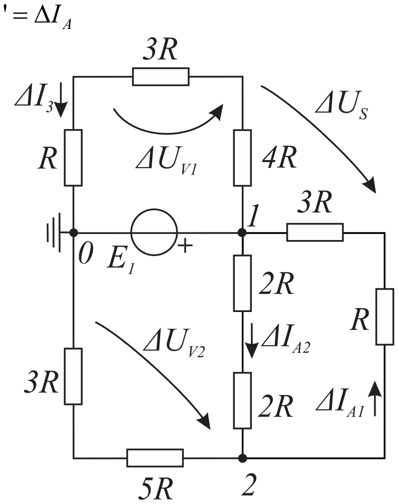
Preklopnik u položaju (1)
(svi generatori osim naponskog generotora ems E_1)
$U_V^{(1)}$
$I_A^{(1)}$

Deluje samo naponski generator ems E_1
U_V'
I_A'

Na osnovu (1) i (2) sledi da je $U_V' = \Delta U_V$, $\quad I_A' = \Delta I_A$







$V_0 = 0\,V, \qquad V_1 = E_1 = 3\,V$

$$V_2\left(\frac{1}{3R+R}+\frac{1}{2R+2R}+\frac{1}{3R+5R}\right) - V_1\left(\frac{1}{3R+R}+\frac{1}{2R+2R}\right) = 0$$

$$V_2\left(\frac{1}{4R}+\frac{1}{4R}+\frac{1}{8R}\right) - 3\cdot\left(\frac{1}{4R}+\frac{1}{4R}\right) = 0 \qquad /\cdot 8R$$

$$5V_2 = 12 \qquad \Rightarrow \qquad V_2 = 2{,}4\ \text{V}$$

a)

$$\Delta I_{A1} = \frac{V_2 - V_1}{R+3R} = \frac{2{,}4-3}{40\,\text{k}} \qquad \boxed{\Delta I_{A1} = -0{,}015\ \text{mA}}$$

$$\Delta I_{A2} = \frac{V_1 - V_2}{2R+2R} = \frac{3-2{,}4}{40\,\text{k}} \qquad \boxed{\Delta I_{A2} = 0{,}015\ \text{mA}}$$

$$\Delta I_3 = \frac{V_1 - V_0}{R+3R+4R} = \frac{3-0}{80\,k} = 0{,}0375\ \text{mA}$$

$$\Delta U_{V1} = 3R\,\Delta I_3 = 30\,\text{k}\cdot 0{,}0375\ \text{m} \qquad \boxed{\Delta U_{V1} = 1{,}125\ \text{V}}$$

$$\Delta U_{V2} = V_2 - V_0 = V_2 \qquad \boxed{\Delta U_{V2} = 2{,}4\ \text{V}}$$

b)

$$\Delta U_S = 3R\Delta I_{A1} + 4R\Delta I_3 = 30\text{k}\cdot(-0{,}015\text{m}) + 40\text{k}\cdot 0{,}0375\text{m} = 1{,}05\ \text{V}$$

$$\Delta P_{IS} = I_S \cdot \Delta U_S = 1\text{m}\cdot 1{,}05 \qquad \boxed{\Delta P_{IS} = 1{,}05\ \text{mW}}$$

$$\Delta I_{R/2} = 0 \qquad \Delta P_{R/2} = \frac{R}{2}\cdot(\Delta I_{R/2})^2 \qquad \boxed{\Delta P_{R/2} = 0\ \text{W}}$$

c) (bonus – 5 poena)

$$\Delta P_{E2} = E_2\,\Delta I_{A1} = 10\cdot(-0{,}0375\ \text{m}) \qquad \boxed{\Delta P_{E2} = -0{,}375\ \text{mW}}$$

$$\Delta P_{E3} = E_3\,\Delta I_3 = 8\cdot 0{,}0375\ \text{m} \qquad \boxed{\Delta P_{E3} = 0{,}3\ \text{mW}}$$