STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **23.01.2023.**

ZADACI

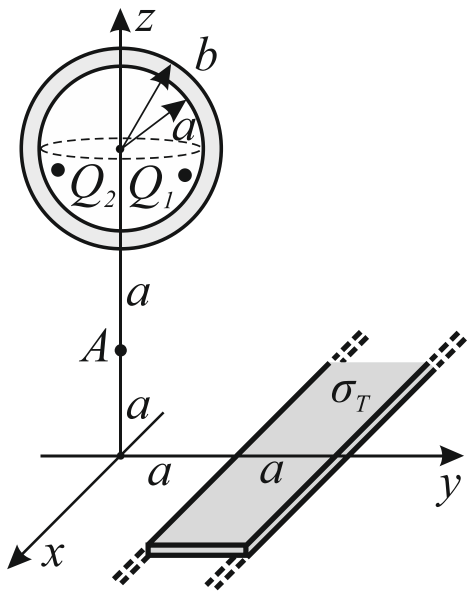
Zadatak 1. Nenaelektrisana provodna sferna ljuska, unutrašnjeg poluprečnika a i spoljašnjeg poluprečnika b, postavljena je kao što je prikazano na slici 1. Centar ljuske se nalazi na z osi, na udaljenosti $2a+b$ od centra Dekartovog koordinatnog sistema i unutar nje se nalaze dva tačkasta naelektrisanja, Q_1 i Q_2. Tanka, veoma dugačka, traka od dielektrika, širine a, ravnomerno je naelektrisana površinskim naelektrisanjem σ_T i postavljena u x-y ravni, paralelno sa x osom. Sferna ljuska i traka se nalaze na dovoljno velikom međusobnom rastojanju, tako da je njihov međusobni uticaj na raspodelu naelektrisanja zanemarljiv. Sistem se nalazi u vazduhu.

a) Odrediti, u opštim brojevima, ukupne količine naelektrisanja na unutrašnjoj i spoljašnjoj površi provodne ljuske.

b) Odrediti, u opštim brojevima, ukupni vektor jačine električnog polja koji u tački A stvaraju sferna ljuska i traka. Tačka A se nalazi na z osi, na visini a.

c) Odrediti površinsko naelektrisanje trake, σ_T, tako da ukupni vektor jačine električnog polja u tački A nema z komponentu.

Brojni podaci su: $a = 1\ cm$, $b = 1{,}1\ cm$, $Q_1 = 2\ nC$, $Q_2 = 5\ nC$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



Slika 1.

$$\int tg\alpha \, d\alpha = -\ln|\cos\alpha|$$

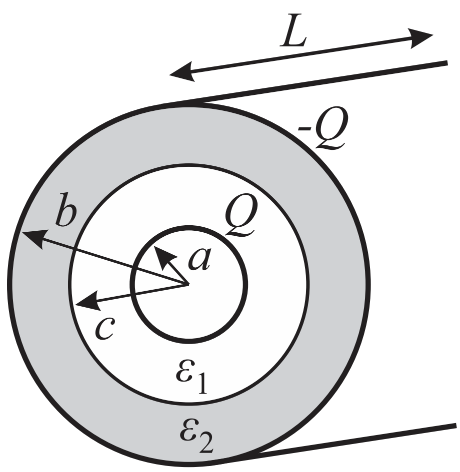
Zadatak 2. Na slici 2 je prikazan koaksijalni kabl dužine $L = 1\ m$, ispunjen sa dva sloja dielektrika: čvrsti – relativne permitivnosti $\varepsilon_{r1} = 5$ i tečni – relativne permitivnosti ε_{r2}. Poluprečnici elektroda kabla su a i b, dok je poluprečnik razdvojne površi dva dielektrika c.

a) Odrediti poluprečnik razdvojne površi dva dielektrika, c, ako je najveći intenzitet vektora jačine električnog polja u čvrstom dielektriku, permitivnosti ε_1, duplo veći od najmanjeg intenziteta vektora jačine električnog polja u tom sloju.

b) Odrediti relativnu permitivnost tečnog dielektrika, ε_{r2}, ako se nakon njegovog potpunog ispuštanja iz kabla, ukupna količina vezanog naelektrisanja uz razdvojnu površ dva dielektrika poveća četiri puta.

c) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora, pre ispuštanja tečnog dielektrika.

Ostali brojni podaci: $a = 1\ cm$, $b = 5\ cm$, $Q = 1\ nC$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

23.01.2023.

ZADACI

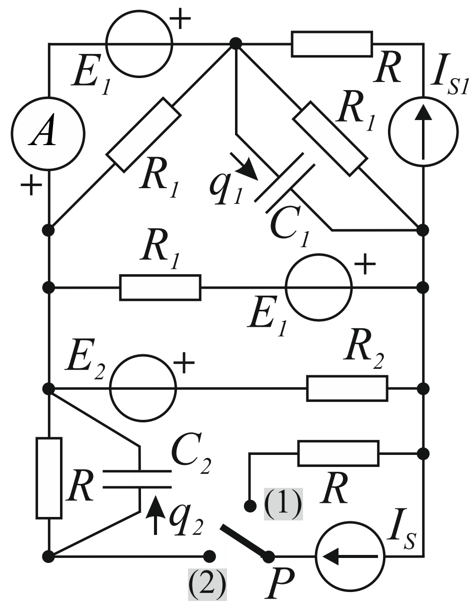
Zadatak 1. Kada se u kolu vremenski konstantnih struja sa slike 1 preklopnik prebaci iz položaja (1) u položaj (2), kroz kondenzator kapacitivnosti $C_1 = 4\ nF$ protekne količina naelektrisanja $q_1 = 6\ nC$, u naznačenom referentnom smeru.

a) Primenjujući teoremu superpozicije, odrediti jačinu struje strujnog generatora, I_S, kao i količinu naelektrisanja q_2, koja protekne kroz kondenzator kapacitivnosti $C_2 = 1\ nF$, u naznačenom referentnom smeru, nakon prebacivanja preklopnika iz položaja (1) u položaj (2).

b) Izračunati snagu strujnog generatora I_{S1} u stacionarnom stanju mreže, koje nastane kada je preklopnik u položaju (2). Kolo rešavati primenom metode potencijala čvorova.

c) Izračunati pokazivanje idealnog ampermetra, kada je preklopnik u položaju (2).

Brojni podaci su: $E_1 = 3\ V$, $E_2 = 5\ V$, $I_{S1} = 0{,}5\ A$, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, $R = 20\ \Omega$.



Slika 1.

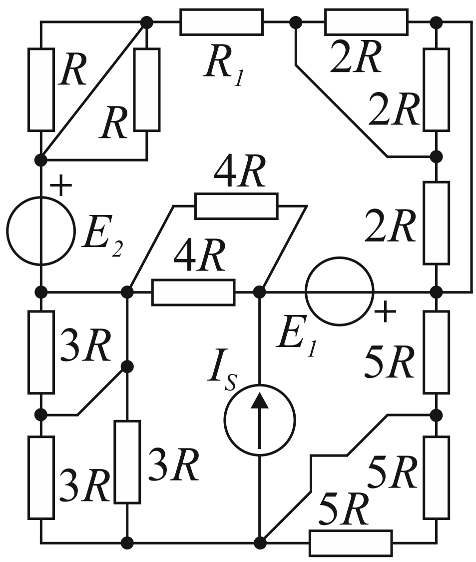
Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, otpornost otpornika R_1 je takva da se na njemu razvija maksimalno moguća snaga.

a) Nacrtati pojednostavljenu šemu kola, zamenom grupa otpornika otpornosti R, $2R$, $3R$, $4R$ i $5R$ sa po jednim otpornikom.

b) Primenom Tevenenove teoreme, izračunati vrednost otpornosti otpornika R_1.

c) Primenom metode konturnih struja, izračunati snage svih generatora u kolu.

Brojni podaci su: $R = 51\ \Omega$, $E_1 = E_2 = 51\ V$, $I_S = 1\ A$.



Slika 2.

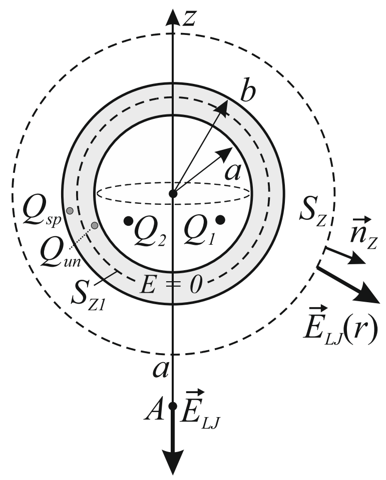
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 23.01.2023.

I-1

a)



$$\oint_{S_{Z1}} \vec{E}\cdot d\vec{S} = \frac{Q_{unutar\,S_{Z1}}}{\varepsilon_0} \qquad E = 0,\ \text{u provodniku}$$

$$0 = \frac{(Q_1+Q_2)+Q_{un}}{\varepsilon_0} \qquad \boxed{Q_{un} = -Q_1 - Q_2}$$

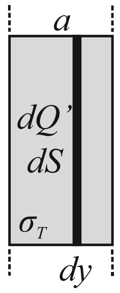
$$Q_{LJ} = Q_{un} + Q_{sp} = 0 \qquad \boxed{Q_{sp} = -Q_{un} = Q_1 + Q_2}$$

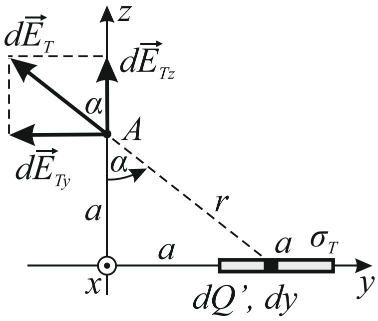
b)

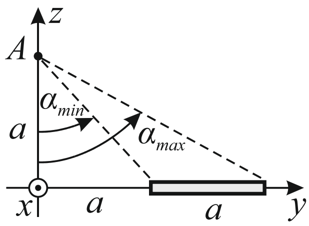
$$\oint_{S_Z} \vec{E}\cdot d\vec{S} = \frac{Q_{unutar\,S_Z}}{\varepsilon_0} \qquad \sphericalangle\left(\vec{E}_{LJ}, \vec{n}_Z\right) = 0$$

$$E_{LJ}(r)\,4\pi r^2 = \frac{Q_1+Q_2+Q_{un}+Q_{sp}}{\varepsilon_0} = \frac{Q_1+Q_2}{\varepsilon_0} \qquad E_{LJ}(r) = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}, \quad r > b$$

$$E_{LJ} = E_{LJ\,u\,A}(r = a+b) = \frac{Q_1+Q_2}{4\pi\varepsilon_0 (a+b)^2} \qquad \boxed{\vec{E}_{LJ} = \frac{Q_1+Q_2}{4\pi\varepsilon_0 (a+b)^2}\cdot\left(-\vec{i}_z\right)}$$







$$dQ = \sigma_T\, dS = \sigma\, l\, dy$$

$$(l \to \infty)$$

$$\frac{dQ}{l} = \sigma_T\, dy$$

$$dQ' = \sigma_T\, dy$$

$$dE_{Ty} = dE_T \sin\alpha = \frac{dQ'}{2\pi\varepsilon_0 r}\sin\alpha = \frac{\sigma_T\, dy}{2\pi\varepsilon_0 r}\sin\alpha = \frac{\sigma_T \frac{r\,d\alpha}{\cos\alpha}}{2\pi\varepsilon_0 r}\sin\alpha = \frac{\sigma_T}{2\pi\varepsilon_0} tg\alpha\, d\alpha$$

$$E_{Ty} = \int dE_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\int_{\alpha_{\min}}^{\alpha_{\max}} tg\alpha\, d\alpha = \frac{\sigma_T}{2\pi\varepsilon_0}\left(-\ln|\cos\alpha|\right)\Big|_{\alpha_{\min}}^{\alpha_{\max}} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\ln|\cos\alpha_{\min}| - \ln|\cos\alpha_{\max}|\right) = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{|\cos\alpha_{\min}|}{|\cos\alpha_{\max}|}$$

$$E_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\frac{a}{\sqrt{a^2+a^2}}}{\frac{a}{\sqrt{a^2+(2a)^2}}} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\sqrt{5}}{\sqrt{2}} \qquad \boxed{\vec{E}_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\sqrt{5}}{\sqrt{2}}\cdot\left(-\vec{i}_y\right)}$$

$$dE_{Tz} = dE_T \cos\alpha = \frac{dQ'}{2\pi\varepsilon_0 r}\cos\alpha = \frac{\sigma_T\, dy}{2\pi\varepsilon_0 r}\cos\alpha = \frac{\sigma_T \frac{r\,d\alpha}{\cos\alpha}}{2\pi\varepsilon_0 r}\cos\alpha = \frac{\sigma_T}{2\pi\varepsilon_0}\, d\alpha$$

$$E_{Tz} = \int dE_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\alpha_{\max} - \alpha_{\min}\right) = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\text{arctg}\frac{2a}{a} - \text{arctg}\frac{a}{a}\right) \qquad \boxed{\vec{E}_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\text{arctg}\,2 - \text{arctg}\,1\right)\cdot\vec{i}_z}$$

c)

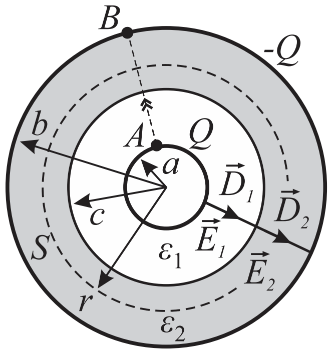
$$\vec{E}_A = \vec{E}_{Ty} + \vec{E}_{Tz} + \vec{E}_{LJ} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\sqrt{5}}{\sqrt{2}}\cdot\left(-\vec{i}_y\right) + \frac{\sigma_T}{2\pi\varepsilon_0}\left(\text{arctg}\,2 - \text{arctg}\,1\right)\cdot\vec{i}_z + \frac{Q_1+Q_2}{4\pi\varepsilon_0(a+b)^2}\cdot\left(-\vec{i}_z\right)$$

$$\vec{E}_{Az} = 0 \quad \Rightarrow \quad \frac{\sigma_T}{2\pi\varepsilon_0}\left(\text{arctg}\,2 - \text{arctg}\,1\right) = \frac{Q_1+Q_2}{4\pi\varepsilon_0(a+b)^2} \qquad \boxed{\sigma_T = 24{,}7\ \mu C/m^2}$$

Ispit iz OET1, 23.01.2023.

I-2

a)



Granični uslov:

$$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{OM}} D\, ds = Q$$

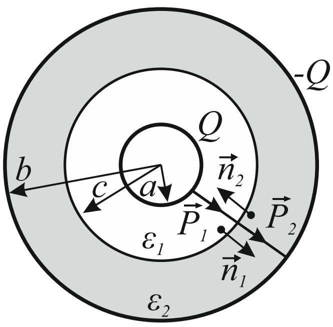
$$D\, 2r\pi\, L = Q \qquad \boxed{D = \frac{Q}{2\pi r\, L}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{2\pi\varepsilon_1 r\, L}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{2\pi\varepsilon_2 r\, L}}, \qquad c \le r \le b$$

$$\left.\begin{aligned} E_{1\max}(r=a) &= \frac{Q}{2\pi\varepsilon_1 a\, L} \\ E_{1\min}(r=c) &= \frac{Q}{2\pi\varepsilon_1 c\, L} \end{aligned}\right\} \Rightarrow \quad E_{1\max} = 2\, E_{1\min}, \qquad \frac{Q}{2\pi\varepsilon_1 a\, L} = 2 \cdot \frac{Q}{2\pi\varepsilon_1 c\, L} \qquad c = 2a \qquad \boxed{c = 2\, cm}$$

b)



Pre ispuštanja ε_2:

$$P = D - \varepsilon_0 E = D - \varepsilon_0 \frac{D}{\varepsilon} = \left(1 - \frac{1}{\varepsilon_r}\right) D = \left(1 - \frac{1}{\varepsilon_r}\right) \frac{Q}{2\pi r\, L}$$

$$\sigma_{V1}{}^{PRE} = \vec{P}_1 \cdot \vec{n}_1 = P_1(c) = \left(1 - \frac{1}{5}\right) \frac{Q}{2\pi c\, L} = \frac{4}{5} \frac{Q}{2\pi c\, L}$$

$$\sigma_{V2}{}^{PRE} = \vec{P}_2 \cdot \vec{n}_2 = -P_2(c) = -\left(1 - \frac{1}{\varepsilon_{r2}}\right) \frac{Q}{2\pi c\, L}$$

$$Q_V{}^{PRE} = \left(\sigma_{V1}{}^{PRE} + \sigma_{V2}{}^{PRE}\right) 2\pi c\, L \qquad \boxed{Q_V{}^{PRE} = \left[\frac{4}{5} - \left(1 - \frac{1}{\varepsilon_{r2}}\right)\right] Q}$$

Posle ispuštanja ε_2:

$$\sigma_{V1}{}^{POSLE} = \sigma_{V1}{}^{PRE} = \vec{P}_1 \cdot \vec{n}_1 = P_1(c) = \left(1 - \frac{1}{5}\right) \frac{Q}{2\pi c\, L} = \frac{4}{5} \frac{Q}{2\pi c\, L} \qquad \sigma_{V2}{}^{POSLE} = 0$$

$$Q_V{}^{POSLE} = \sigma_{V1}{}^{POSLE}\, 2\pi c\, L \qquad \boxed{Q_V{}^{POSLE} = \frac{4}{5} Q}$$

$$Q_V{}^{POSLE} = 4 Q_V{}^{PRE}, \qquad \frac{4}{5} Q = 4 \cdot \left[\frac{4}{5} - \left(1 - \frac{1}{\varepsilon_{r2}}\right)\right] Q \qquad \boxed{\varepsilon_{r2} = 2{,}5}$$

c)

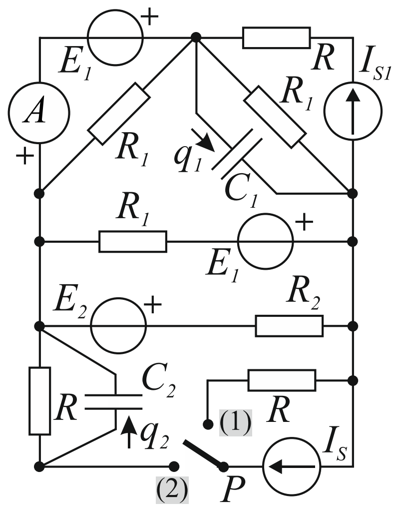
$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{2\pi\varepsilon_1 r\, L} dr + \int_c^b \frac{Q}{2\pi\varepsilon_2 r\, L} dr = \frac{Q}{2\pi\varepsilon_1\, L} \ln\frac{c}{a} + \frac{Q}{2\pi\varepsilon_2\, L} \ln\frac{b}{c}$$

$$C = \frac{Q}{U_{AB}} \qquad \boxed{C = \frac{2\pi\varepsilon_0\, L}{\frac{1}{\varepsilon_{r1}} \ln\frac{c}{a} + \frac{1}{\varepsilon_{r2}} \ln\frac{b}{c}}}$$

Ispit iz OET1, 23.01.2023.

II-1

a)



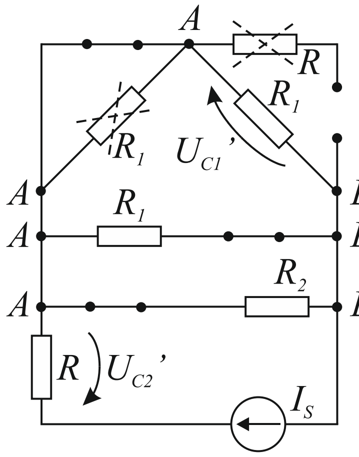
$$\boxed{\begin{matrix}\text{Svi}\\ \text{generatori}\end{matrix}} = \boxed{\begin{matrix}\text{Svi sem}\\ I_S\end{matrix}} + \boxed{\begin{matrix}\text{Samo}\\ I_S\end{matrix}}$$

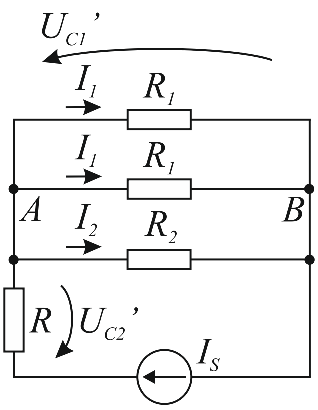
(2) (1)

$$U_{C1}^{(2)} = U_{C1}^{(1)} + U_{C1}'$$

$$\Delta U_{C1} = U_{C1}^{(2)} - U_{C1}^{(1)} = U_{C1}'$$

$$U_{C1}' = \Delta U_{C1} = \frac{q_1}{C_1} = \frac{6\,nC}{4\,nF} = 1{,}5\,V$$





$$U_{AB} = U_{C1}' = 1{,}5\,V$$

$$I_1 = \frac{U_{AB}}{R_1} = \frac{1{,}5}{5} = 0{,}3\,A$$

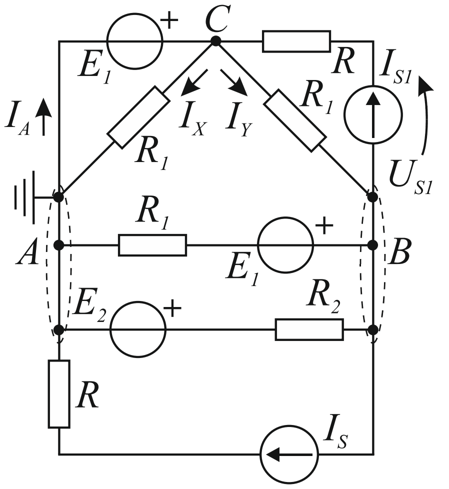
$$I_2 = \frac{U_{AB}}{R_2} = \frac{1{,}5}{10} = 0{,}15\,A$$

$$I_S = I_1 + I_1 + I_2 \qquad \boxed{I_S = 0{,}75\,A}$$

$$U_{C2}' = R\,I_S = 20 \cdot 0{,}75 = 15\,V$$

$$q_2 = C_2 U_{C2}' = 1 \cdot 10^{-9} \cdot 15 \qquad \boxed{q_2 = 15\,nC}$$

b)



$n_č = 3, \quad n_{i.n.g.} = 1$

$MPČ: \quad n_č - 1 - n_{i.n.g.} = 3 - 1 - 1 = 1$

$V_A = 0\,V, \qquad V_C = E_1 = 3\,V$

$$V_B\left(\frac{1}{\infty} + \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{\infty}\right) - V_C\left(\frac{1}{R_1} + \frac{1}{\infty}\right) = -I_S + \frac{E_2}{R_2} + \frac{E_1}{R_1} - I_{S1}$$

$$V_B\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{5}\right) - 3 \cdot \left(\frac{1}{5}\right) = -0{,}75 + \frac{5}{10} + \frac{3}{5} - 0{,}5 \qquad / \cdot 10$$

$$5\,V_B = 4{,}5 \qquad \Rightarrow \qquad V_B = 0{,}9\,V$$

$$U_{S1} = R\,I_{S1} + (V_C - V_B) = 20 \cdot 0{,}5 + (3 - 0{,}9) = 12{,}1\,V$$

$$P_{S1} = U_{S1}\,I_{S1} = 12{,}1 \cdot 0{,}5 \qquad \boxed{P_{S1} = 6{,}05\,W}$$

c)

$$I_X = \frac{V_C - V_A}{R_1} = \frac{3 - 0}{5} = 0{,}6\,A$$

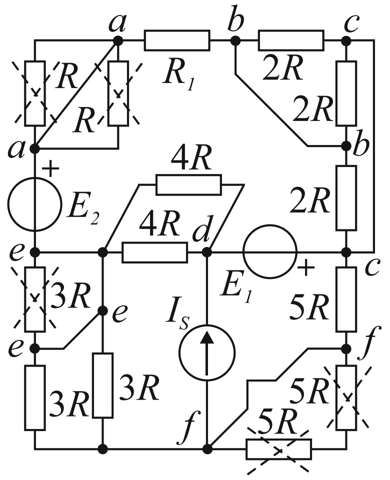
$$I_Y = \frac{V_C - V_B}{R_1} = \frac{3 - 0{,}9}{5} = 0{,}42\,A$$

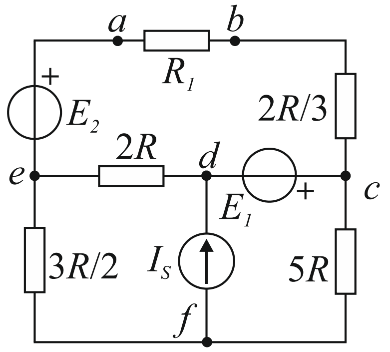
$$I_A = I_X + I_Y - I_{S1} = 0{,}6 + 0{,}42 - 0{,}5 \qquad \boxed{I_A = 0{,}52\,A}$$

Ispit iz OET1, 23.01.2023.

II-2

a)





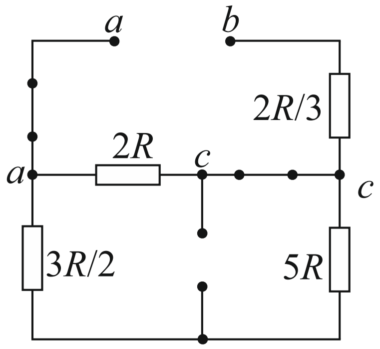
$$R_{bc} = 2R \parallel 2R \parallel 2R = \frac{2R}{3}$$

$$R_{cf} = 5R$$

$$R_{de} = 4R \parallel 4R = 2R$$

$$R_{ef} = 3R \parallel 3R = \frac{3R}{2}$$

b)

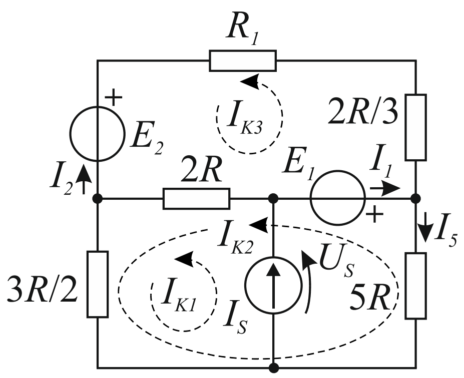


$$R_T = R_{ab} = \left[2R \parallel \left(\frac{3R}{2} + 5R\right)\right] + \frac{2R}{3} = \left(2R \parallel \frac{13R}{2}\right) + \frac{2R}{3}$$

$$R_T = \frac{26R}{17} + \frac{2R}{3} = \frac{78R}{51} + \frac{34R}{51} = \frac{112R}{51} = 112\,\Omega$$

$$\boxed{R_1 = R_T = 112\,\Omega}$$

c)



$n_g = 6, \quad n_č = 4, \quad n_{s.g.} = 1$

$MKS: \quad n_g - (n_č - 1) - n_{s.g.} = 6 - (4-1) - 1 = 3 - 1 = 2$

$K1: \quad I_{K1} = I_S = 1\,A$

$K2: \quad \left(2R + 5R + \frac{3R}{2}\right) I_{K2} + \left(2R + \frac{3R}{2}\right) I_{K1} - 2R\, I_{K3} = -E_1$

$K3: \quad \left(R_1 + 2R + \frac{2R}{3}\right) I_{K3} - 2R\, I_{K1} - 2R\, I_{K2} = E_1 - E_2$

$K2: \quad \frac{17R}{2} I_{K2} + \frac{7R}{2} \cdot 1 - 2R\, I_{K3} = -51 \qquad / \cdot \frac{2}{R}$

$K3: \quad \frac{248R}{51} I_{K3} - 2R \cdot 1 - 2R\, I_{K2} = 51 - 51 \qquad / \cdot \frac{51}{R}$

$$\left.\begin{array}{l} 17 I_{K2} - 4 I_{K3} = -9 \\ -102 I_{K2} + 248 I_{K3} = 102 \end{array}\right\} \Rightarrow \quad I_{K2} = -0{,}479\,A, \quad I_{K3} = 0{,}214\,A$$

$P_{E1} = E_1 I_1 = E_1 (I_{K3} - I_{K2}) = 51 \cdot (0{,}214 + 0{,}479)$ $\qquad \boxed{P_{E1} = 35{,}34\,W}$

$P_{E2} = E_2 I_2 = E_2 (-I_{K3}) = 51 \cdot (-0{,}214)$ $\qquad \boxed{P_{E2} = -10{,}91\,W}$

$U_S = -E_1 + 5R\, I_5 = -E_1 + 5R(-I_{K2}) = -51 + 5 \cdot 51 \cdot 0{,}479 = 71{,}15\,V$

$P_S = U_S I_S = 71{,}15 \cdot 1$ $\qquad \boxed{P_S = 71{,}15\,W}$