STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

03.12.2022.

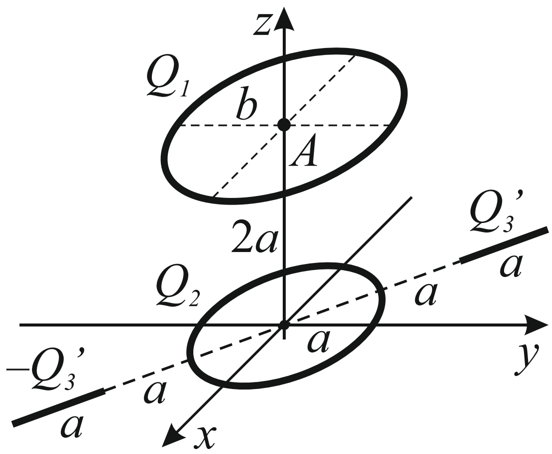
ZADACI

Zadatak 1. Jedan tanak prsten, poluprečnika a, naelektrisan je ravnomerno količinom naelektrisanja Q_2, i postavljen je u x-y ravni Dekartovog koordinatnog sistema, kao što je prikazano na slici 1. Drugi tanak prsten, poluprečnika b, naelektrisan je ravnomerno količinom naelektrisanja Q_1, i postavljen je u ravni koja je paralelna sa x-y ravni, pri čemu je njegov centar na z osi, na visini $2a$ od koordinatnog početka (tačka A). Dva tanka štapa, dužine a, naelektrisana su istim podužnim gustinama naelektrisanja suprotnog znaka, Q_3' i $-Q_3'$. Štapovi su postavljeni na pravcu simetrale drugog i četvrtog kvadranta x-y ravni. Početak oba štapa se nalazi na rastojanju $2a$ od koordinatnog početka. Sistem se nalazi u vazduhu.

a) Odrediti, u opštim brojevima, vektor jačine električnog polja koji u tački A stvaraju prsteni i štapovi.

b) Odrediti količinu naelektrisanja Q_2, prstena poluprečnika a, tako da sve komponente vektora jačine električnog polja u tački A budu jednake.

Brojni podaci su: $a = 1\ cm$, $b = 1{,}5\ cm$, $Q_1 = 1\ nC$, $Q_3' = 1\ nC/m$, $\varepsilon_0 = 8{,}85\cdot 10^{-12}\ F/m$.



Slika 1.

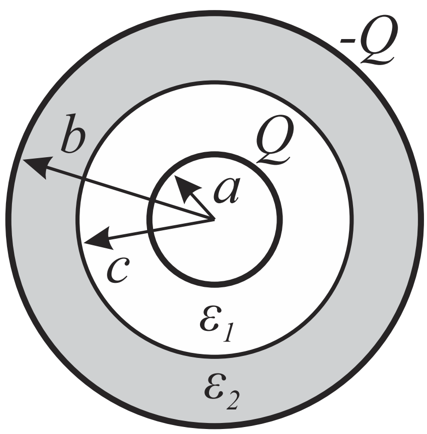
Zadatak 2. Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika relativnih permitivnosti $\varepsilon_{r1} = 2\cdot\varepsilon_{r2}$ i ε_{r2}. Poluprečnici elektroda kondenzatora su $a = 1\ mm$ i $b = 3\ mm$, dok je poluprečnik razdvojne površi dva dielektrika $c = 2\ mm$.

a) Odrediti, u opštim brojevima, izraz za kapacitivnost kondenzatora.

b) Odrediti relativne permitivnosti oba dielektrika, ε_{r1} i ε_{r2}, ako se zna da je ukupna količina vezanog naelektrisanja uz razdvojnu površ dva dielektrika, jednaka jednoj šestini količine slobodnog naelektrisanja na unutrašnjoj elektrodi kondenzatora.

c) Izračunati maksimalni napon na koji sme da se priključi kondenzator.

Brojni podaci su: $E_{Č1} = 52\ kV/cm$, $E_{Č2} = 78\ kV/cm$.



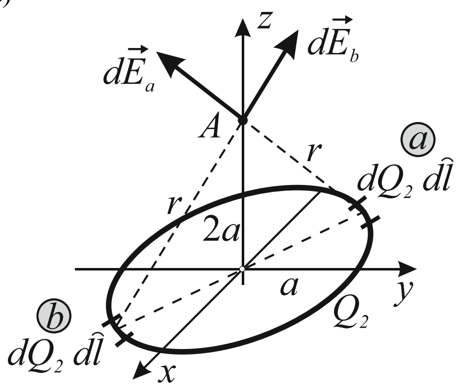
Slika 2.

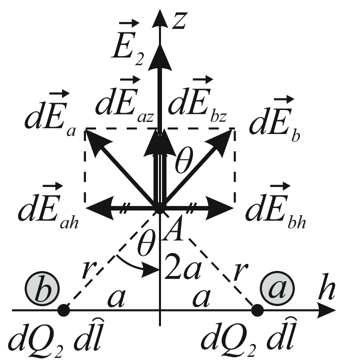
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

I-1

a)





$\boxed{\vec{E}_1 = 0}$ Tačka A u centru prstena.

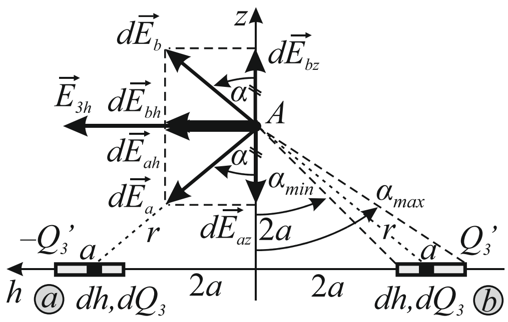
Zbog simetrije je:

$d\vec{E}_{ah} + d\vec{E}_{bh} = 0 \quad \Rightarrow \quad \boxed{E_h = 0}$

$\boxed{E_x = 0} \qquad \boxed{E_y = 0}$

$$dE_{az} = dE_{bz} = dE_a \cos\theta = \frac{dQ_2}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q_2'\,dl}{4\pi\varepsilon_0 r^2}\,\frac{2a}{r} = \frac{Q_2'\,a}{2\pi\varepsilon_0 r^3}dl \qquad \left(Q_2' = \frac{Q_2}{2a\pi}\right)$$

$$E_2 = \int_{\substack{po\\luku}} dE_{az} = \frac{Q_2'\,a}{2\pi\varepsilon_0 r^3}\int_0^{2a\pi} dl = \frac{Q_2'\,a}{2\pi\varepsilon_0 r^3}2a\pi = \frac{Q_2\,a}{2\pi\varepsilon_0\left(a^2+4a^2\right)^{\frac{3}{2}}} = \frac{Q_2}{10\sqrt{5}\,\pi\varepsilon_0 a^2} \qquad \boxed{\vec{E}_2 = \frac{Q_2}{10\sqrt{5}\,\pi\varepsilon_0 a^2}\cdot\vec{i}_z}$$

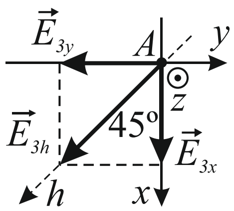


$d\vec{E}_{az} + d\vec{E}_{bz} = 0 \quad \Rightarrow \quad \boxed{E_{3z} = 0}$

$$dE_{ah} = dE_a\sin\alpha = \frac{dQ_3}{4\pi\varepsilon_0 r^2}\sin\alpha = \frac{Q_3'\,dh}{4\pi\varepsilon_0 r^2}\sin\alpha = \frac{Q_3'\,\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\sin\alpha$$

$$dE_{ah} = \frac{Q_3'\,\dfrac{d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r}\sin\alpha = \frac{Q_3'\,\dfrac{d\alpha}{\cos\alpha}}{4\pi\varepsilon_0\dfrac{2a}{\cos\alpha}}\sin\alpha = \frac{Q_3'}{8\pi\varepsilon_0 a}\sin\alpha\,d\alpha$$

$$dh = \frac{r\,d\alpha}{\cos\alpha}, \qquad r = \frac{2a}{\cos\alpha} \qquad E_{3h} = 2\int dE_{ah} = 2\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{Q_3'}{8\pi\varepsilon_0 a}\sin\alpha\,d\alpha = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\cos\alpha_{\min} - \cos\alpha_{\max}\right)$$



$$E_{3h} = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{2a}{\sqrt{(2a)^2+(2a)^2}} - \frac{2a}{\sqrt{(2a)^2+(3a)^2}}\right) = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{2\sqrt{13}}{13}\right)$$

$$E_{3x} = E_{3h}\cos 45^\circ = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{2\sqrt{13}}{13}\right)\cdot\frac{\sqrt{2}}{2} \qquad \boxed{\vec{E}_{3x} = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\vec{i}_x}$$

$$E_{3y} = E_{3h}\sin 45^\circ = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2} - \frac{2\sqrt{13}}{13}\right)\cdot\frac{\sqrt{2}}{2} \qquad \boxed{\vec{E}_{3y} = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\left(-\vec{i}_y\right)}$$

$$\boxed{\vec{E}_A = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\vec{i}_x + \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\left(-\vec{i}_y\right) + \frac{Q_2}{10\sqrt{5}\,\pi\varepsilon_0 a^2}\cdot\vec{i}_z}$$

b)

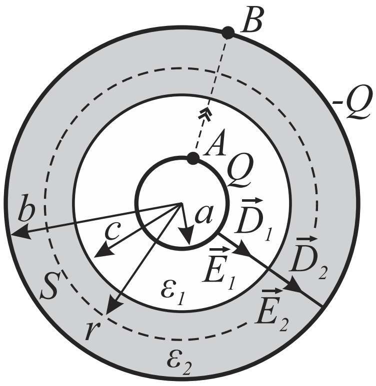
$$E_{Ax} = E_{Ay} = E_{Az} \quad \Rightarrow \quad \frac{Q_3'}{4\pi\varepsilon_0 a}\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right) = \frac{Q_2}{10\sqrt{5}\,\pi\varepsilon_0 a^2}$$

$$Q_2 = Q_3'\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\frac{5\sqrt{5}\,a}{2} = 1\cdot 10^{-9}\cdot\left(\frac{1}{2} - \frac{\sqrt{26}}{13}\right)\cdot\frac{5\sqrt{5}\cdot 1\cdot 10^{-2}}{2} = 6{,}02\cdot 10^{-12}\ C \qquad \boxed{Q_2 = 6{,}02\ pC}$$

<u>Prvi kolokvijum iz OET1, 03.12.2022.</u>

I-2

a)



Granični uslov:

$$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$$

$$E_{t1} = E_{t2} = 0$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_S D\, ds = Q$$

$$D\, 4r^2\pi = Q \qquad \boxed{D = \frac{Q}{4\pi r^2}}, \qquad a \le r \le b$$

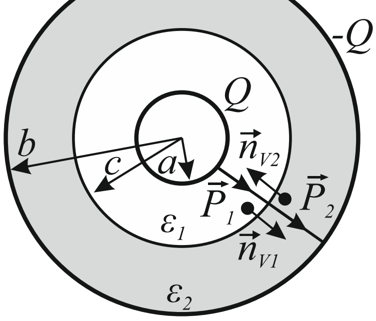
$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_1 r^2}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_2 r^2}}, \qquad c \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr + \int_c^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$U_{AB} = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}\right) \qquad \boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi}{\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}}}$$

b)



$$P_1 = D - \varepsilon_0 E_1 = D - \varepsilon_0 \frac{D}{\varepsilon_1} = \left(1 - \frac{1}{\varepsilon_{r1}}\right) D = \left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi r^2}$$

$$P_2 = D - \varepsilon_0 E_2 = D - \varepsilon_0 \frac{D}{\varepsilon_2} = \left(1 - \frac{1}{\varepsilon_{r2}}\right) D = \left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi r^2}$$

$$\sigma_{V1} = \vec{P}_1 \cdot \vec{n}_{V1} = P_1(c) = \left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi c^2}$$

$$\sigma_{V2} = \vec{P}_2 \cdot \vec{n}_{V2} = -P_2(c) = -\left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi c^2}$$

$$Q_V = \left(\sigma_{V1} + \sigma_{V2}\right) 4\pi c^2 = \left[\left(1 - \frac{1}{\varepsilon_{r1}}\right) - \left(1 - \frac{1}{\varepsilon_{r2}}\right)\right]\frac{Q}{4\pi c^2}\, 4\pi c^2 = \left(1 - \frac{1}{\varepsilon_{r1}} - 1 + \frac{1}{\varepsilon_{r2}}\right) Q = \left(\frac{1}{\varepsilon_{r2}} - \frac{1}{\varepsilon_{r1}}\right) Q$$

$$\left(\frac{1}{\varepsilon_{r2}} - \frac{1}{2\varepsilon_{r2}}\right) Q = \frac{1}{6} Q \qquad \boxed{\varepsilon_{r2} = 3} \qquad \boxed{\varepsilon_{r1} = 6}$$

c)

$$\left.\begin{aligned} E_{1\max} &= \frac{Q_{1\max}}{4\pi\varepsilon_1 a^2} \le E_{\check{C}1} \quad \Rightarrow \quad Q_{1\max} = E_{\check{C}1}\, 4\pi\varepsilon_1 a^2 = 3{,}47\ nC \\ E_{2\max} &= \frac{Q_{2\max}}{4\pi\varepsilon_2 c^2} \le E_{\check{C}2} \quad \Rightarrow \quad Q_{2\max} = E_{\check{C}2}\, 4\pi\varepsilon_2 c^2 = 10{,}4\ nC \end{aligned}\right\} \Rightarrow \quad Q_{\max} = \min\left\{Q_{1\max}, Q_{2\max}\right\} = 3{,}47\ nC$$

$$U_{\max} = \frac{Q_{\max}}{4\pi}\left(\frac{1}{\varepsilon_1}\frac{c-a}{ac} + \frac{1}{\varepsilon_2}\frac{b-c}{bc}\right) \qquad \boxed{U_{\max} = 4{,}34\ kV}$$