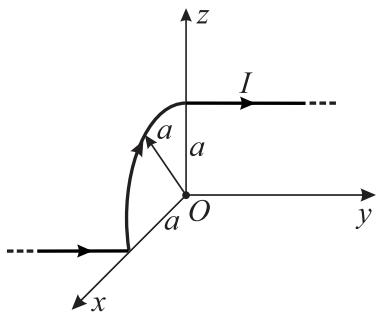
STUDIJSKI PROGRAM MERENJE I REGULACIJA		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (I kolokvijum)	**K1** **01.10.2022.**

ZADACI

Zadatak 1. Veoma dugačak žičani provodnik sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji iz dva pravolinijska i jednog lučnog segmenta, dužine četvrtine obima kruga poluprečnika a. Jedan pravolinijski segment leži u ravni x-y, drugi u ravni y-z, Dekartovog pravouglog koordinatnog sistema, dok se lučni segment nalazi u ravni x-z. Odrediti vektor magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti su: $I = 10$ A, $a = 10$ cm, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



Slika 1.

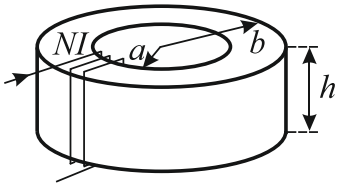
Zadatak 2. Na slici 2a je prikazano debelo torusno jezgro, načinjeno od materijala čija je magnetska karakteristika data na slici 2b. Na jezgro je ravnomerno i gusto namotan pobudni namotaj sa N zavojaka u kojima je uspostavljena struja jačine $I = 9{,}42$ A. Odrediti:

a) broj zavojaka, N, tako da jedna trećina jezgra bude u zasićenju,

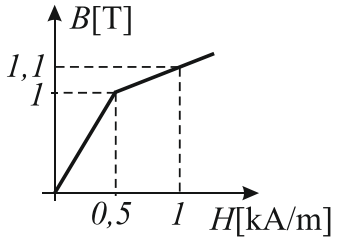
b) fluks vektora magnetske indukcije kroz poprečni presek jezgra, pri broju zavojaka određenom pod a),

c) samoinduktivnost torusnog namotaja.

Dimenzije jezgra su: $a = 2$ cm, $b = 5$ cm, $h = 2{,}1$ cm.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MERENJE I REGULACIJA		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (II kolokvijum)	**K2** **01.10.2022.**

ZADACI

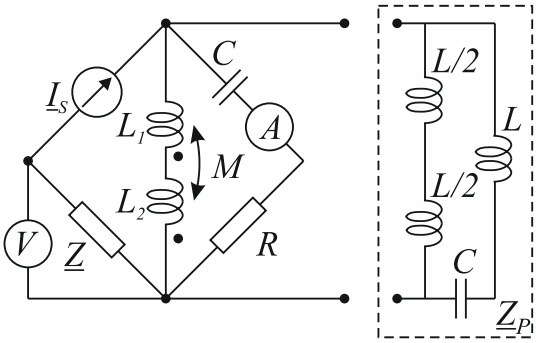
Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Odrediti pokazivanja idealnih mernih instrumenata, kada je prijemnik priključen na ulazni deo kola, pri čemu strujni generator $\underline{I}_S$ radi na većoj rezonantnoj učestanosti.

c) Odrediti kompleksnu snagu strujnog generatora $\underline{I}_S$ za slučaj pod c).

Brojni podaci su: $L = 10\ mH$, $C = 40\ nF$, $R = 1\ k\Omega$, $\underline{Z} = (5 + j3)\ k\Omega$, $\underline{I}_S = (1 - j)\ mA$, $L_1 = 2L$, $L_2 = 3L$, $k = 0{,}82$.



Slika 1.

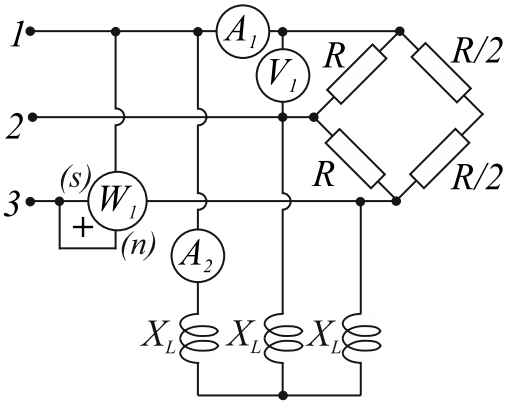
Zadatak 2. Simetričan trofazni prijemnik, prikazan na slici 2, priključen je na mrežu faznog napona prve faze $U_1 = 230\ V$.

a) Odrediti pokazivanja idealnih mernih instrumenata.

b) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.

c) Odrediti aktivnu i reaktivnu snagu trofaznog prijemnika sastavljenog od kalemova.

Brojni podaci su: $R = 60\ \Omega$, $X_L = 40\ \Omega$.



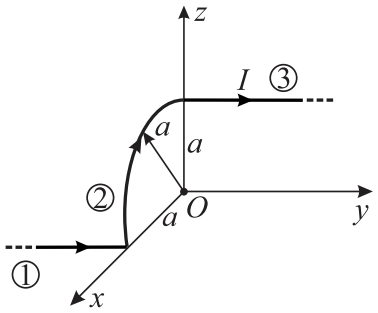
Slika 2.

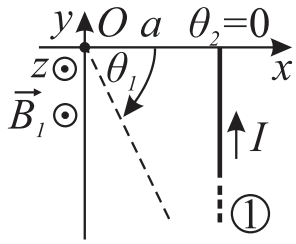
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MiR ISPIT 01.10.2022.

K1 Z1

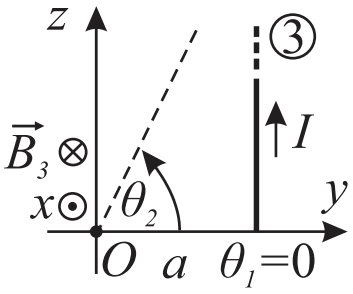




$$d = a, \qquad \theta_1 \to -\frac{\pi}{2}, \qquad \theta_2 = 0$$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[0-(-1)\right] = \frac{\mu_0 I}{4\pi a}$$

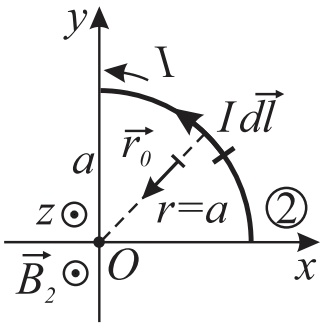
$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_z}$$



$$d = a, \qquad \theta_1 = 0, \qquad \theta_2 \to \frac{\pi}{2}$$

$$B_3 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[1-0\right] = \frac{\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_3 = \frac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_x\right)}$$



$$\vec{dB}_2 = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_2 = \int dB_2 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{1}{4}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}\frac{1}{2}a\pi = \frac{\mu_0 I}{8a}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{8a}\cdot\left(-\vec{i}_y\right)}$$

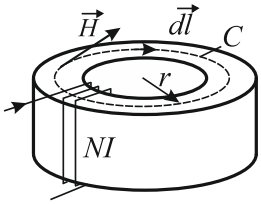
$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_x\right) + \frac{\mu_0 I}{8a}\cdot\left(-\vec{i}_y\right) + \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_z}$$

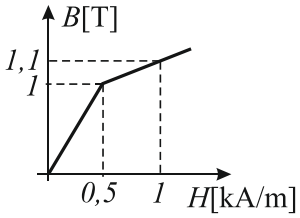
$$\vec{B}_O = 10\ \mu\text{T}\cdot\left(-\vec{i}_x\right) + 15{,}71\ \mu\text{T}\cdot\left(-\vec{i}_y\right) + 10\ \mu\text{T}\cdot\vec{i}_z$$

$$\left|\vec{B}_O\right| = \sqrt{(10\ \mu\text{T})^2 + (15{,}71\ \mu\text{T})^2 + (10\ \mu\text{T})^2} \qquad \boxed{\left|\vec{B}_O\right| = 21{,}14\ \mu\text{T}}$$

K1 Z2

a)





$$\oint_C \vec{H}\cdot\vec{dl} = NI$$

$$H\,2\pi r = NI$$

$$H = \frac{NI}{2\pi r} \qquad a \le r \le b$$

$$I = 9{,}42\ \text{A}$$

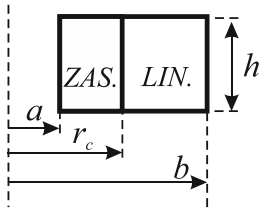
$$r_c = a + \frac{1}{3}(b-a) = 3\ \text{cm}$$

$$\frac{NI}{2\pi r_c} = H_K \quad \rightarrow \quad N = \frac{H_K\,2\pi r_c}{I} = \frac{500\cdot 2\pi\cdot 0{,}03}{9{,}42}$$

$$\boxed{N = 10\ \text{zav}}$$

b)

Jednačina prave drugog dela karakteristike: $B = \frac{0{,}1}{500}H + 0{,}9$



$$\Phi = \int_a^{r_c} \frac{0{,}1}{500}\frac{NI}{2\pi r}dr\,h + 0{,}9\cdot(r_c - a)h + \int_{r_c}^{b} \mu\frac{NI}{2\pi r}dr\,h = \frac{0{,}1}{500}\frac{NI}{2\pi}h\ln\frac{r_c}{a} + 0{,}9\cdot(r_c-a)h + \frac{1}{500}\frac{NI}{2\pi}h\ln\frac{b}{r_c}$$

$$\Phi = \frac{0{,}1}{500}\frac{10\cdot 9{,}24}{2\pi}\cdot 0{,}021\cdot\ln\frac{0{,}03}{0{,}02} + 0{,}9\cdot(0{,}03-0{,}02)\cdot 0{,}021 + \frac{1}{500}\frac{10\cdot 9{,}24}{2\pi}\cdot 0{,}021\cdot\ln\frac{0{,}05}{0{,}03}$$

$$\Phi = 25{,}04\ \mu\text{Wb} + 189\ \mu\text{Wb} + 315{,}51\mu\text{Wb}$$

$$\boxed{\Phi = 529{,}55\,\mu\text{Wb}}$$

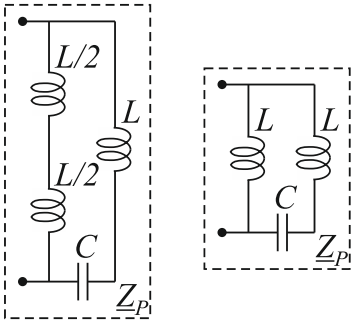
c)

$$\boxed{L = \frac{\Phi_{uk}}{I} = \frac{N\Phi}{I} = \frac{10\cdot 529{,}55\mu}{9{,}24} = 573{,}11\,\mu\text{H}}$$

MiR ISPIT 01.10.2022.

K2 Z1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_P = \underline{Z}_1 \,||\, j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C} j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC - \omega^2 LC}{j\omega C}}$$

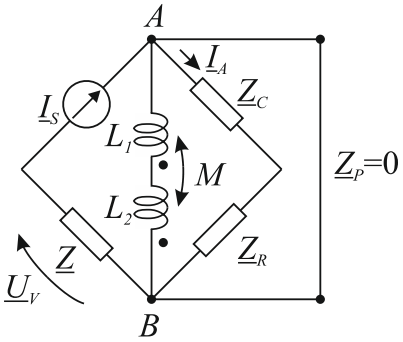
$$\underline{Z}_P = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC} \qquad \boxed{\underline{Z}_P = j\frac{\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}}$$

$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad \omega L = 0 \qquad \boxed{\omega_{r1} = 0}$

$1-\omega^2 LC = 0 \qquad \boxed{\omega_{r2} = \frac{1}{\sqrt{LC}}}$

$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad 1-2\omega^2 LC = 0 \qquad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}} \qquad \boxed{\omega_{ar2} \to \infty}$

b)



$$\omega_{r2} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4 \ \frac{rad}{s} = \omega$$

$\underline{Z}_P = 0$

$\underline{U}_{AB} = 0 \quad \Rightarrow \quad \underline{I}_A = 0$

$\boxed{I_A = |\underline{I}_A| = 0\ A}$

$\underline{U}_V = \underline{Z}\,\underline{I}_S = (5+j3)k\cdot(1-j)m = 5+3+j3-j5 = (8-j2)\,V$

$U_V = |\underline{U}_V| = \sqrt{8^2+(-2)^2} \qquad \boxed{U_V = 8{,}25\,V}$

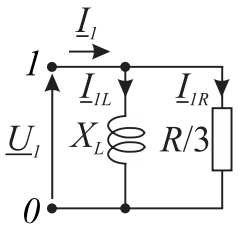
c)

$\underline{U}_S = \underline{U}_V$

$\underline{S}_S = \underline{U}_S \underline{I}_S^{\,*} = (8-j2)(1+j)m = (10+j6)\,mVA$

K2 Z2

a)



$$\underline{I}_{1R} = \frac{\underline{U}_1}{R/3} = \frac{230\,e^{j0°}}{20} = 11{,}5\,e^{j0°}\ A = 11{,}5\ A$$

$$\underline{I}_{1L} = \frac{\underline{U}_1}{jX_L} = \frac{230\,e^{j0°}}{j40} = \frac{230\,e^{j0°}}{40\,e^{j90°}} = 5{,}75\,e^{-j90°}\ A = -j5{,}75\ A$$

$$\underline{I}_1 = \underline{I}_{1R} + \underline{I}_{1L} = (11{,}5 - j5{,}75)\ A = 12{,}85\,e^{-j26{,}57}\ A$$

$$\underline{I}_2 = \underline{I}_1\,e^{-j120°} = 12{,}85\,e^{-j146{,}57°}\ A$$

$$\underline{I}_3 = \underline{I}_1\,e^{-j240°} = 12{,}85\,e^{-j266{,}57°}\ A$$

$$\boxed{I_{A1} = |\underline{I}_{1R}| = 11{,}5A}$$

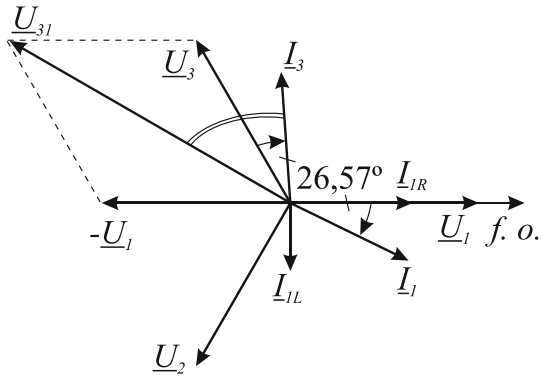
$$\boxed{I_{A2} = |\underline{I}_{1L}| = 5{,}75\ A}$$

$$\boxed{U_{V1} = |\underline{U}_{12}| = \sqrt{3}\,|\underline{U}_1| = \sqrt{3}\cdot 230 = 398{,}37\ V}$$

$$P_{W1} = \mathrm{Re}\left\{\underline{U}_{31}\,\underline{I}_3^{*}\right\} = U_{31}\,I_3\cos\sphericalangle\left(\underline{U}_{31}, \underline{I}_3\right) = \sqrt{3}\,U_1\,I_1\cos(30° + 26{,}57°) = \sqrt{3}\cdot 230\cdot 12{,}85\cdot\cos 56{,}57°$$

$$\boxed{P_{W1} = 2821{,}28\ W}$$

b)



c)

$$P_{3L} = 0\ W$$

$$Q_{3L} = 3X_L I_{A2}{}^2 = 3967{,}5\ VAr$$