STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **06.09.2022.**

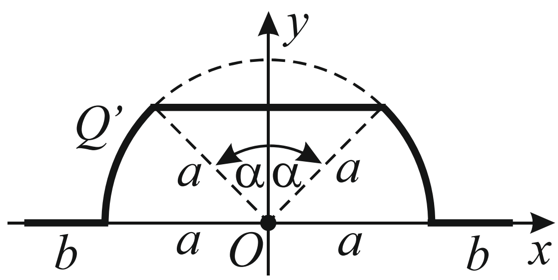
ZADACI

Zadatak 1. Tanak štap od izolacionog materijala, savijen kao što je prikazano na slici 1, naelektrisan je ravnomerno podužnom gustinom naelektrisanja Q'. Štap se sastoji od tri pravolinijska segmenta i dva simetrična segmenta u obliku luka poluprečnika a. Dva pravolinijska segmenta su dužine b. Cela struktura leži u x-y ravni Dekartovog koordinatnog sistema. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja u koordinatnom početku (tačka O), koji potiče od naelektrisanog štapa.

b) Izračunati intenzitet tog vektora.

Brojni podaci su: $a = 1\ cm$, $b = 0{,}5\ cm$, $\alpha = 45°$, $Q' = 1\ nC/m$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



Slika 1.

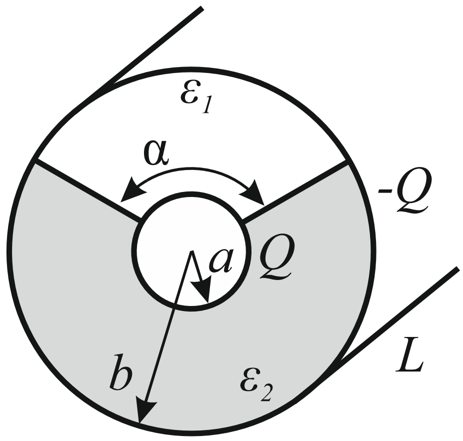
Zadatak 2. Na slici 2 je prikazan koaksijalni kabl, dužine L, ispunjen sa dva sloja dielektrika: tečni – relativne permitivnosti ε_{r1} i čvrsti – relativne permitivnosti ε_{r2}. Poluprečnici elektroda ovog kabla su a i b. Elektrode kabla su naelektrisane naelektrisanjem Q i $-Q$.

a) Izvesti, u opštim brojevima, izraz za kapacitivnost kabla.

b) Izračunati ugao α, ako je kapacitivnost onog dela kabla ispunjenog sa tečnim dielektrikom (relativne permitivnosti ε_{r1}) duplo veća od kapacitivnosti onog dela kabla ispunjenog sa čvrstim dielektrikom (relativne permitivnosti ε_{r2}).

c) Odrediti najveći napon na koji sme da se priključi ovaj kabl.

Brojni podaci su: $L = 1\ m$, $a = 3\ mm$, $b = 2{,}7a$, $\varepsilon_{r1} = 6$, $\varepsilon_{r2} = 1{,}5$, $E_{\check{C}1} = 73\ kV/cm$, $E_{\check{C}2} = 64\ kV/cm$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET	Osnovi elektrotehnike 1 (II kolokvijum)	K2
Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs		06.09.2022.

ZADACI

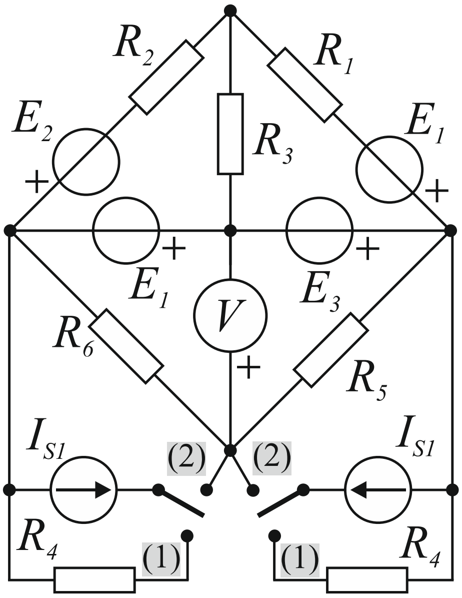
Zadatak 1. Kada se u mreži sa slike 1 oba preklopnika istovremeno prebace iz položaja (1) u položaj (2), pokazivanje idealnog voltmetra se poveća za 3,5 V.

a) Primenjujući teoremu superpozicije, izračunati jačinu struje strujnih generatora, I_{S1}.

b) Izračunati snagu na otporniku otpornosti R_5, kada su preklopnici u položaju (2).

c) Izračunati pokazivanje idealnog voltmetra, kada su preklopnici u položaju (2).

Brojni podaci su: $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 1\ \Omega$, $R_6 = 7\ \Omega$, $E_1 = 5\ V$, $E_2 = 2{,}2\ V$, $E_3 = 13{,}4\ V$.



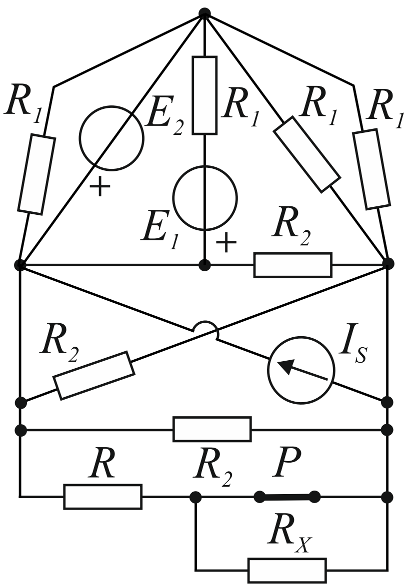
Slika 1.

Zadatak 2. U električnom kolu vremenski konstantnih struja prikazanom na slici 2, kada je prekidač P zatvoren, na otporniku otpornosti R se razvija najveća moguća snaga.

a) Primenom Tevenenove teoreme i metode potencijala čvorova odrediti otpornost otpornika R, kao i najveću moguću snagu na njemu, kada je prekidač P zatvoren.

b) Odrediti otpornost otpornika R_X, ako se zna da nakon otvaranja prekidača P, snaga na otporniku otpornosti R opadne za 50%.

Brojni podaci su: $R_1 = 1\ k\Omega$, $R_2 = 0{,}5\ k\Omega$, $E_1 = 5\ V$, $E_2 = 3\ V$, $I_S = 3\ mA$.



Slika 2.

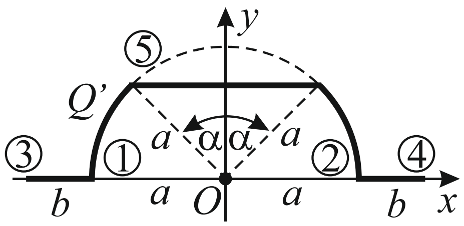
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

<u>Ispit iz OET1, 06.09.2022.</u>

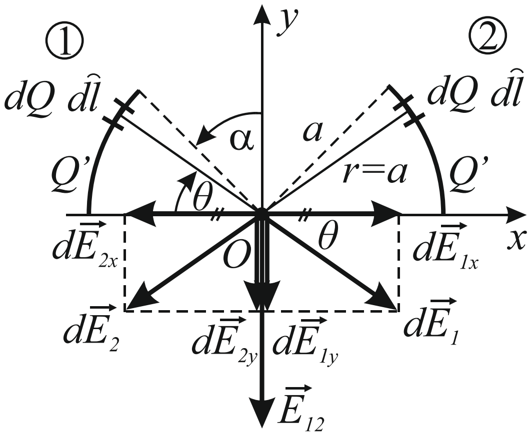
I-1

a)



Zbog simetrije je:

$$\vec{E}_3 + \vec{E}_4 = 0 \quad \Rightarrow \quad \boxed{\vec{E}_{34} = 0}$$



Zbog simetrije je:

$$d\vec{E}_{1x} + d\vec{E}_{2x} = 0 \quad \Rightarrow \quad \boxed{\vec{E}_{12x} = 0}$$

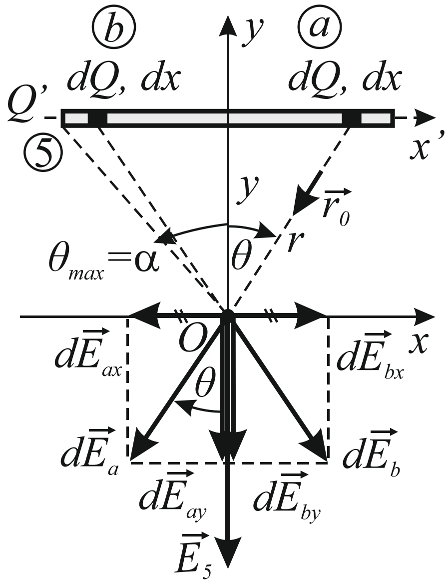
$$dE_{1y} = dE_1 \sin\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\sin\theta$$

$$dE_{1y} = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\sin\theta = \frac{Q'}{4\pi\varepsilon_0 a}\sin\theta\,d\theta$$

$$E_{12} = 2\int\limits_{po\ luku} dE_{1y} = 2\,\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}-\alpha}\sin\theta\,d\theta = \frac{Q'}{2\pi\varepsilon_0 a}\int_0^{\frac{\pi}{4}}\sin\theta\,d\theta$$

$$E_{12} = \frac{Q'}{2\pi\varepsilon_0 a}\left(\cos 0 - \cos\frac{\pi}{4}\right) = \frac{Q'}{2\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{2}}{2}\right)$$

$$E_{12} = \frac{Q'}{4\pi\varepsilon_0 a}\left(2-\sqrt{2}\right) \qquad \boxed{\vec{E}_{12} = \frac{Q'}{4\pi\varepsilon_0 a}\left(2-\sqrt{2}\right)\cdot\left(-\vec{i}_y\right)}$$



$$\overrightarrow{dE}_{ax} + \overrightarrow{dE}_{bx} = 0 \quad \Rightarrow \quad \boxed{\vec{E}_{5x} = 0}$$

$$dE_{ay} = dE_a\cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'dx}{4\pi\varepsilon_0 r^2}\cos\theta \qquad \left(dx = \frac{r\,d\theta}{\cos\theta}\right)$$

$$dE_{ay} = \frac{Q'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'}{4\pi\varepsilon_0\dfrac{y}{\cos\theta}}d\theta \qquad \left(r = \frac{y}{\cos\theta}\right)$$

$$E_5 = 2\int\limits_{\substack{po\ pola\\ štapa}} dE_{ay} = 2\frac{Q'}{4\pi\varepsilon_0 y}\int_0^{\theta_{max}=\alpha}\cos\theta\,d\theta = \frac{Q'}{2\pi\varepsilon_0 y}\left(\sin\alpha - \sin 0\right)$$

$$E_5 = \frac{Q'}{2\pi\varepsilon_0\dfrac{a\sqrt{2}}{2}}\cdot\frac{\sqrt{2}}{2} = \frac{Q'}{2\pi\varepsilon_0 a} \qquad \boxed{\vec{E}_5 = \frac{Q'}{2\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right)}$$

$$\boxed{\vec{E}_O = \vec{E}_{12} + \vec{E}_5 = \left[\frac{Q'}{4\pi\varepsilon_0 a}\left(2-\sqrt{2}\right) + \frac{Q'}{2\pi\varepsilon_0 a}\right]\cdot\left(-\vec{i}_y\right)}$$

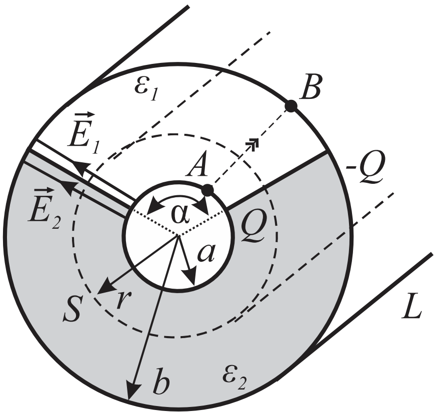
b)

$$E_O = \frac{Q'}{4\pi\varepsilon_0 a}\left(2-\sqrt{2}\right) + \frac{Q'}{2\pi\varepsilon_0 a}$$

$$\boxed{E_O = 2{,}33\,\frac{kV}{m}}$$

I-2

a)



Granični uslov:

$$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$$

$$D_{n1} = D_{n2} = 0$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\cancel{\int_{S_{B1}} \vec{D} \cdot \vec{ds}}^{\,0} + \cancel{\int_{S_{B2}} \vec{D} \cdot \vec{ds}}^{\,0} + \int_{S_{OM}} \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\measuredangle\left(\vec{D}, \vec{n}_{B1}\right) = 90^\circ \qquad \measuredangle\left(\vec{D}, \vec{n}_{B2}\right) = 90^\circ \qquad \measuredangle\left(\vec{D}, \vec{n}_{OM}\right) = 0^\circ$$

$$\int_{S_{OM1}} D_1\, ds + \int_{S_{OM2}} D_2\, ds = Q$$

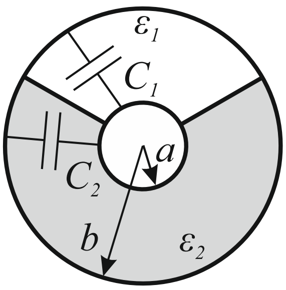
$$D_1\, \alpha\, r\, L + D_2 \left(2\pi - \alpha\right) r\, L = Q \qquad D_1 = \varepsilon_1 E \qquad D_2 = \varepsilon_2 E$$

$$\boxed{E = \frac{Q}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) r\, L}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) r\, L} dr = \frac{Q}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) L} \ln \frac{b}{a}$$

$$C = \frac{Q}{U_{AB}} \qquad \boxed{C = \frac{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) L}{\ln \frac{b}{a}}}$$

b)



$$C = \frac{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) L}{\ln \frac{b}{a}} = \frac{\varepsilon_1\, \alpha\, L}{\ln \frac{b}{a}} + \frac{\varepsilon_2 \left(2\pi - \alpha\right) L}{\ln \frac{b}{a}} = C_1 + C_2$$

$$\frac{C_1}{C_2} = \frac{\dfrac{\varepsilon_1\, \alpha\, L}{\ln \frac{b}{a}}}{\dfrac{\varepsilon_2 \left(2\pi - \alpha\right) L}{\ln \frac{b}{a}}} = \frac{\varepsilon_1\, \alpha}{\varepsilon_2 \left(2\pi - \alpha\right)} = \frac{6\varepsilon_0\, \alpha}{1{,}5\,\varepsilon_0 \left(2\pi - \alpha\right)} = \frac{6\alpha}{1{,}5\left(2\pi - \alpha\right)} = 2$$

$$6\alpha = 3\left(2\pi - \alpha\right) = 6\pi - 3\alpha$$

$$9\alpha = 6\pi \qquad \boxed{\alpha = \frac{2\pi}{3} = 120^\circ}$$

c)

$$E_{\max} = \frac{Q_{\max}}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) a\, L} \le \min\left\{E_{\check{C}1}, E_{\check{C}2}\right\} = E_{\check{C}2}$$

$$Q_{\max} \le E_{\check{C}2} \left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) a\, L$$

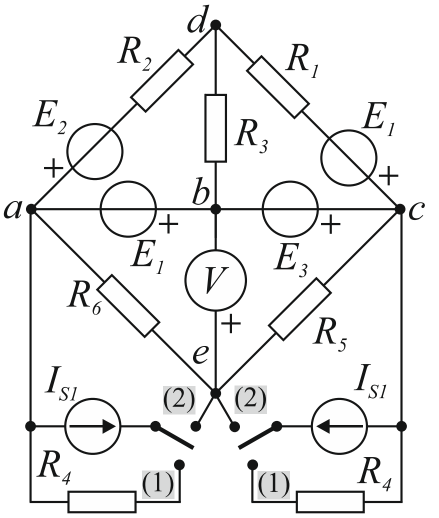
$$U_{\max} = \frac{Q_{\max}}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) L} \ln \frac{b}{a} = \frac{E_{\check{C}2} \left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) a\, L}{\left(\varepsilon_1\, \alpha + \varepsilon_2 \left(2\pi - \alpha\right)\right) L} \ln \frac{b}{a} = E_{\check{C}2}\; a\; \cancel{\ln \frac{b}{a}}^{\,1}$$

$$\boxed{U_{\max} = 19{,}2\, kV}$$

<u>Ispit iz OET1, 06.09.2022.</u>

II-1

a)

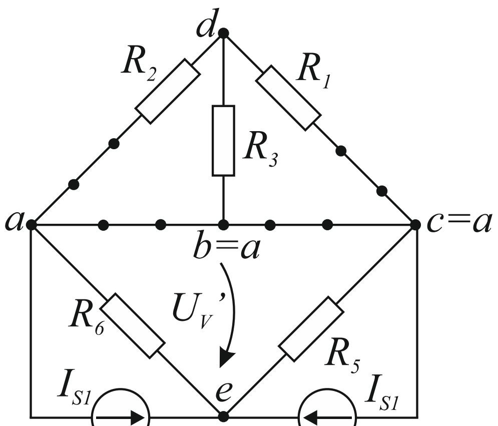


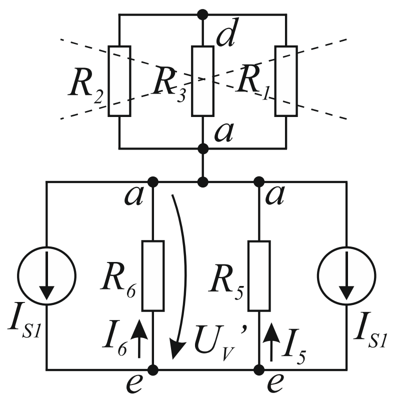
$$\boxed{\text{Svi generatori}} = \boxed{\text{Svi sem dva } I_{S1}} + \boxed{\text{Samo dva } I_{S1}}$$

(2) (1)

$$U_V^{(2)} = U_V^{(1)} + U_V'$$

$$\Delta U_V = U_V^{(2)} - U_V^{(1)} = U_V' = 3{,}5\,V$$





$$I_6 = \frac{U_V'}{R_6} = \frac{3{,}5}{7} = 0{,}5\ A$$

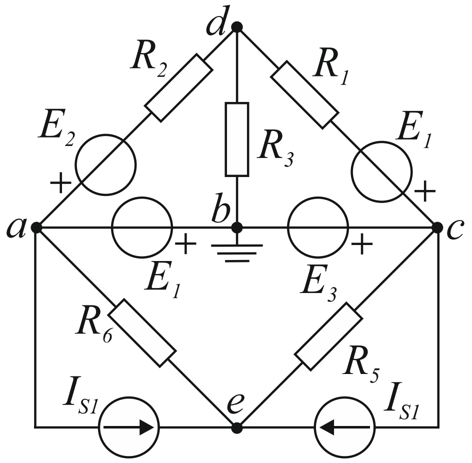
$$I_5 = \frac{U_V'}{R_5} = \frac{3{,}5}{1} = 3{,}5\ A$$

$$-I_{S1} + I_6 + I_5 - I_{S1} = 0$$

$$2\,I_{S1} = I_6 + I_5 = 0{,}5 + 3{,}5 = 4\ A$$

$$\boxed{I_{S1} = 2\ A}$$

b)



$$V_b = 0\,V, \qquad V_a = -E_1 = -5\,V \qquad V_c = E_3 = 13{,}4\,V$$

$$V_d\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - V_a\left(\frac{1}{R_2}\right) - V_c\left(\frac{1}{R_1}\right) = -\frac{E_1}{R_1} - \frac{E_2}{R_2}$$

$$V_e\left(\frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{\infty}+\frac{1}{\infty}\right) - V_a\left(\frac{1}{R_6}+\frac{1}{\infty}\right) - V_c\left(\frac{1}{R_5}+\frac{1}{\infty}\right) = I_{S1} + I_{S1}$$

$$V_d\left(\frac{1}{4}+\frac{1}{3}+\frac{1}{6}\right) - (-5)\cdot\left(\frac{1}{3}\right) - 13{,}4\cdot\left(\frac{1}{4}\right) = -\frac{5}{4} - \frac{2{,}2}{3} \qquad /\cdot 12$$

$$V_e\left(\frac{1}{1}+\frac{1}{7}\right) - (-5)\cdot\left(\frac{1}{7}\right) - 13{,}4\cdot\left(\frac{1}{1}\right) = 2 + 2 \qquad /\cdot 7$$

$$\left.\begin{aligned} 9\,V_d &= -3{,}6 \\ 8\,V_e &= 116{,}8 \end{aligned}\right\} \Rightarrow \quad V_d = -0{,}4\,V \qquad V_e = 14{,}6\,V$$

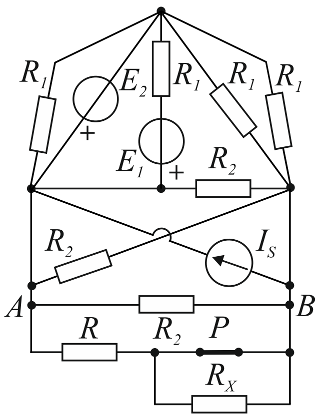
$$P_{R5}^{(2)} = \frac{U_{R5}^{\,2}}{R_5} = \frac{U_{ec}^{\,2}}{R_5} = \frac{(V_e - V_c)^2}{R_5} = \frac{(14{,}6 - 13{,}4)^2}{1} \qquad \boxed{P_{R5}^{(2)} = 1{,}44\,W}$$

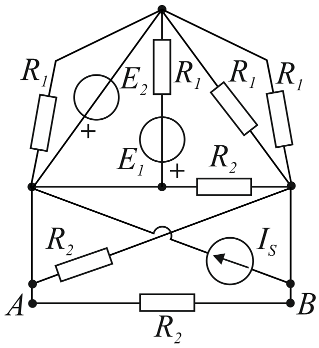
$$U_V^{(2)} = U_{eb} = V_e - V_b = V_e \qquad \boxed{U_V^{(2)} = 14{,}6\,V}$$

<u>Ispit iz OET1, 06.09.2022.</u>

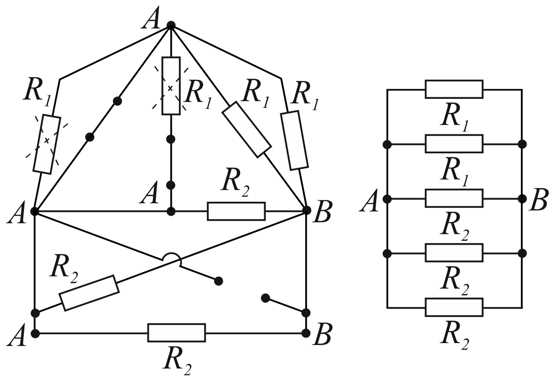
II-2

a)





R_T:

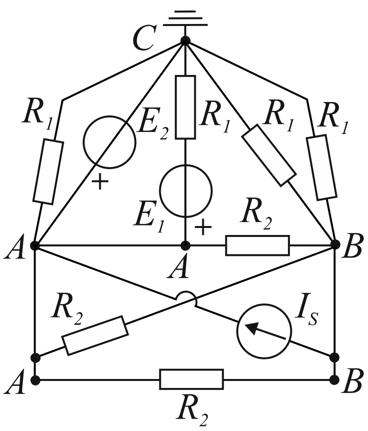


$$R_T = R_{AB} = R_1 \,\|\, R_1 \,\|\, R_2 \,\|\, R_2 \,\|\, R_2$$

$$R_T = 1k \,\|\, 1k \,\|\, 0{,}5k \,\|\, 0{,}5k \,\|\, 0{,}5k = 0{,}125\; k\Omega$$

$$\boxed{R_T = 125\,\Omega}$$

E_T:



$$V_C = 0\,V, \qquad V_A = E_2 = 3\,V$$

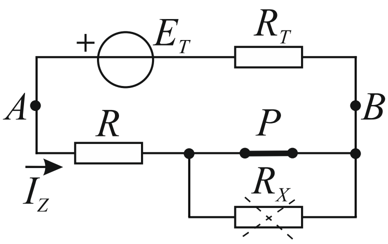
$$V_B\left(\frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{\infty}\right) - V_A\left(\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{R_2}+\frac{1}{\infty}\right) = -I_S$$

$$V_B\left(\frac{1}{1k}+\frac{1}{1k}+\frac{1}{0{,}5k}+\frac{1}{0{,}5k}+\frac{1}{0{,}5k}\right) - 3\cdot\left(\frac{1}{0{,}5k}+\frac{1}{0{,}5k}+\frac{1}{0{,}5k}\right) = -3m \qquad /\cdot 1k$$

$$8V_B = 15 \qquad V_B = 1{,}875\,V$$

$$E_T = U_{AB} = V_A - V_B = 3 - 1{,}875$$

$$\boxed{E_T = 1{,}125\,V}$$

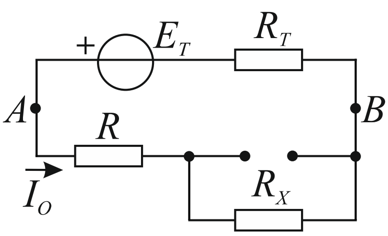


$$\boxed{R = R_T = 125\,\Omega}$$

$$I_Z = \frac{E_T}{R_T + R} = \frac{1{,}125}{250} = 4{,}5\; mA$$

$$P_R^{\max} = R\, I_Z^2 = 125\cdot(4{,}5\,m)^2 \qquad \boxed{P_R^{\max} = 2{,}53\; mW}$$

b)



$$P_R^O = \frac{P_R^{\max}}{2} = 1{,}27\; mW$$

$$I_O = \sqrt{\frac{P_R^O}{R}} = \sqrt{\frac{1{,}27\,m}{125}} = 3{,}19\; mA$$

$$I_O = \frac{E_T}{R_T + R + R_X}$$

$$R_X = \frac{E_T}{I_O} - R_T - R = \frac{1{,}125}{3{,}19\,m} - 125 - 125 \qquad \boxed{R_X = 102{,}66\,\Omega}$$