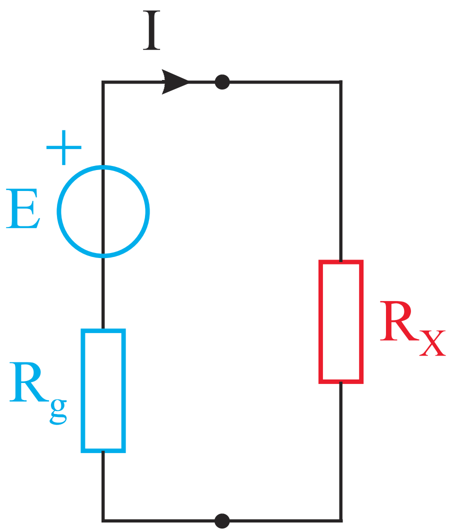
Petak, 10.12.2021.

Vežbe 18

Vremenski konstantne električne struje

Uslov prenosa maksimalne snage:
(snaga pretvaranja električne energije u toplotu)



$$I = \frac{E}{R_g + R_X}$$

$$P_{RX} = R_X \cdot I^2 = R_X \cdot \left(\frac{E}{R_g + R_X} \right)^2 = \frac{R_X}{\left(R_g + R_X\right)^2} \cdot E^2$$

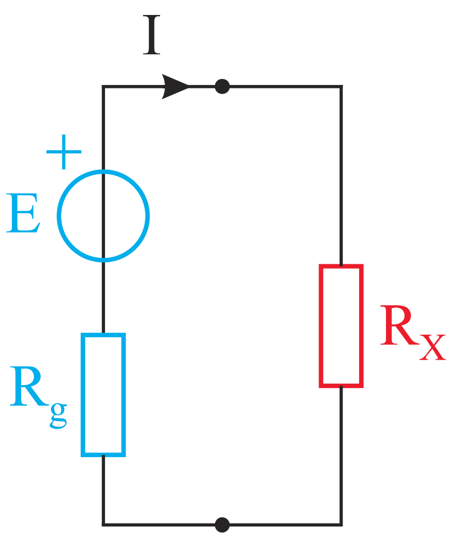
$$\frac{\partial P_{RX}}{\partial R_X} = \frac{1 \cdot \left(R_g + R_X\right)^2 - R_X \cdot 2 \cdot \left(R_g + R_X\right) \cdot 1}{\left(R_g + R_X\right)^4} \cdot E^2$$

$$\frac{\partial P_{RX}}{\partial R_X} = 0 \quad \Rightarrow \quad P_{RX}^{\ max}$$

$$\frac{\partial P_{RX}}{\partial R_X} = 0 \qquad \left(R_g + R_X\right)^2 - 2 \cdot R_X \cdot \left(R_g + R_X\right) = 0$$

$$\left(R_g + R_X\right) \cdot \left(R_g + R_X - 2 \cdot R_X\right) = 0$$

$$R_g - R_X = 0 \qquad \boxed{R_X = R_g}$$



Za $R_X = R_g$:

$$P_{RX} = P_{RX}^{max} = \frac{R_g}{\left(R_g + R_g\right)^2} \cdot E^2$$

$$\boxed{P_{RX}^{max} = \frac{E^2}{4 \cdot R_g}}$$

$$P_{Rg} = R_g \cdot I^2 = R_g \cdot \left(\frac{E}{R_g + R_X}\right)^2 = \frac{R_g}{\left(R_g + R_X\right)^2} \cdot E^2$$

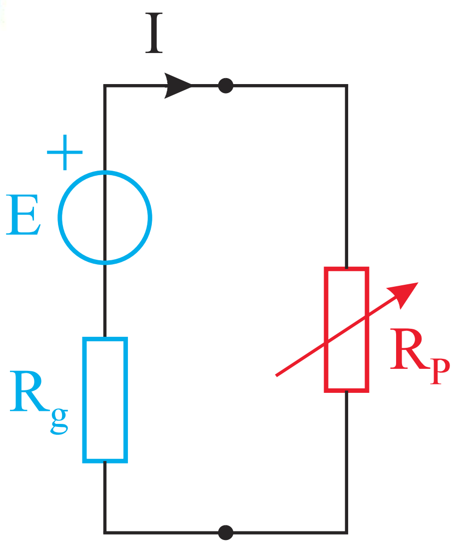
$$P_E = E \cdot I = E \cdot \frac{E}{R_g + R_X} = \frac{1}{R_g + R_X} \cdot E^2$$

Zadatak 1. Prijemnik promenljive otpronosti R_P je priključen na generator elektromotorne sile $E=6$ V i unutrašnje otpornosti $R_g=2\ \Omega$. Otpornost prijemnika se menja u granicama od $0\ \Omega \leq R_P \leq 10\ \Omega$.

a) Na istom dijagramu prikazati grafički, kako se u funkciji otpornosti R_P menjaju: snaga prijemnika P_{RP}, snaga generatora P_E i snaga Džulovih gubitaka u generatoru P_{Rg}.

b) Skicirati grafik promene jačine struje u kolu u funkciji otpornosti R_P.

a)



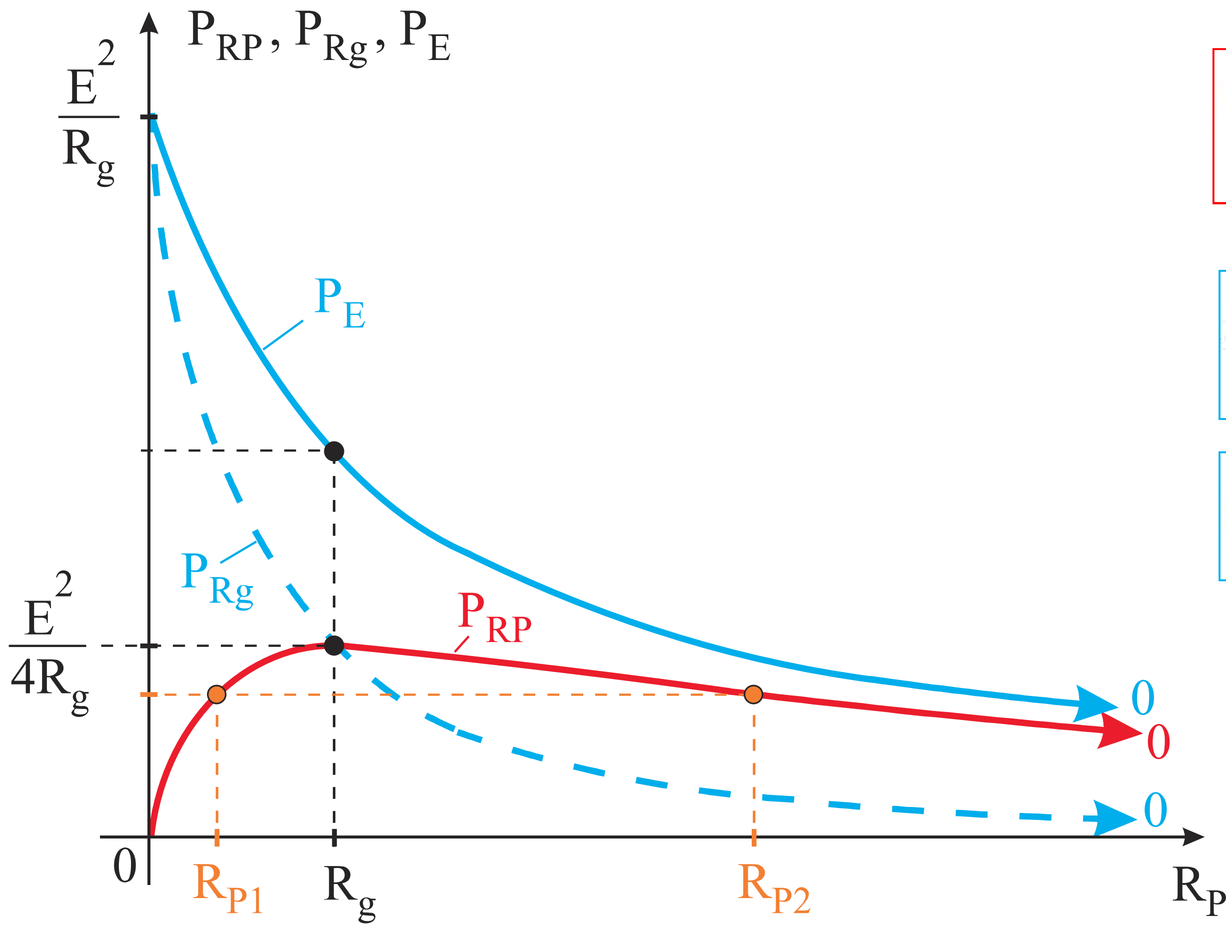
$$I=\frac{E}{R_g+R_P}$$

$$P_{RP}=R_P\cdot I^2=R_P\cdot\left(\frac{E}{R_g+R_P}\right)^2=\frac{R_P}{\left(R_g+R_P\right)^2}\cdot E^2$$

$$P_{Rg}=R_g\cdot I^2=R_g\cdot\left(\frac{E}{R_g+R_P}\right)^2=\frac{R_g}{\left(R_g+R_P\right)^2}\cdot E^2$$

$$P_E=E\cdot I=E\cdot\frac{E}{R_g+R_P}=\frac{1}{R_g+R_P}\cdot E^2$$

R_P [Ω]	0	1	2	3	4	6	8	10
P_{RP} [W]	0	4	4,5	4,32	4	3,375	2,88	2,5
P_{Rg} [W]	18	8	4,5	2,88	2	1,125	0,72	0,5
P_E [W]	18	12	9	7,2	6	4,5	3,6	3
I [A]	3	2	1,5	1,2	1	0,75	0,6	0,5

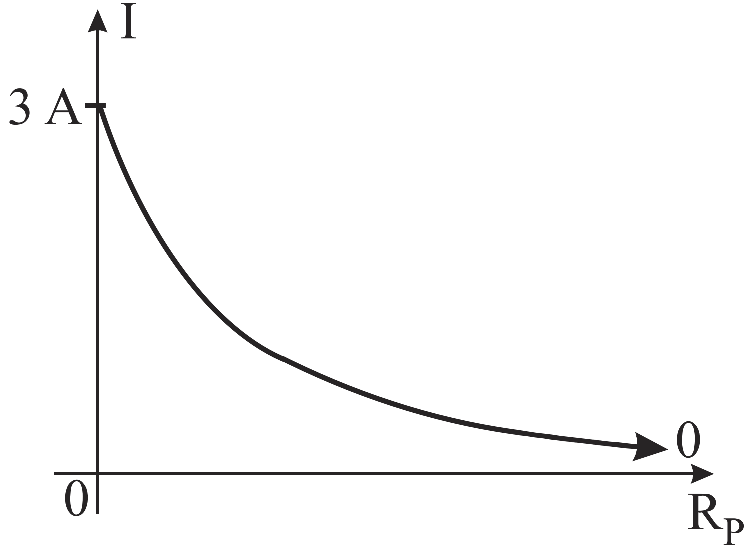


$$P_{RP} = \frac{R_P}{(R_g + R_P)^2} \cdot E^2$$

$$P_{Rg} = \frac{R_g}{(R_g + R_P)^2} \cdot E^2$$

$$P_E = \frac{1}{R_g + R_P} \cdot E^2$$

b)



Zadatak 2. Stepen iskorišćenja.

$$\boxed{P_{RP} = \frac{R_P}{\left(R_g + R_P\right)^2} \cdot E^2} \qquad \boxed{P_{Rg} = \frac{R_g}{\left(R_g + R_P\right)^2} \cdot E^2} \qquad \boxed{P_E = \frac{1}{R_g + R_P} \cdot E^2}$$

$$\eta = \frac{P_{korisno}}{P_{utrošeno}} = \frac{P_{RP}}{P_E} = \frac{\dfrac{R_P}{\left(R_g + R_P\right)^2} \cdot E^2}{\dfrac{1}{R_g + R_P} \cdot E^2} = \frac{R_P}{R_g + R_P}$$

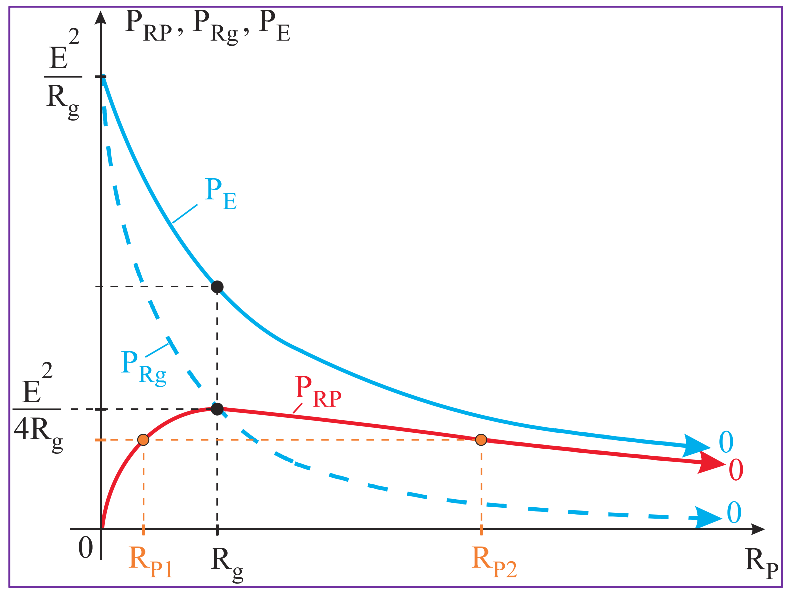
$$R_g = 2\,\Omega$$

1.) $R_P = 1\,\Omega$ $\qquad \eta = \dfrac{1}{2+1} = 0,33$ $\qquad \boxed{\eta(\%) = 33\,\%}$

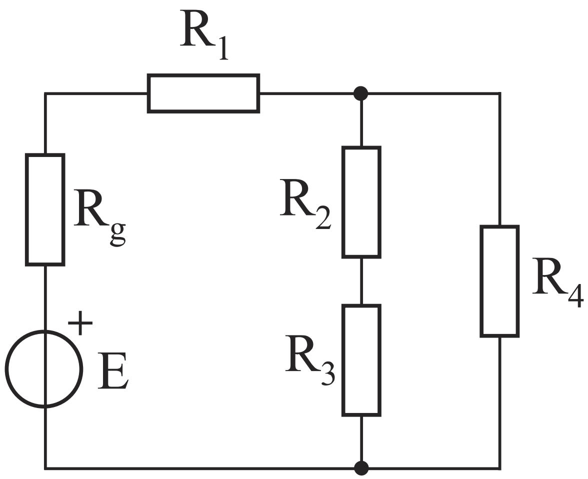
2.) $R_P = 2\,\Omega$ $\qquad \eta = \dfrac{2}{2+2} = 0,5$ $\qquad \boxed{\eta(\%) = 50\,\%}$

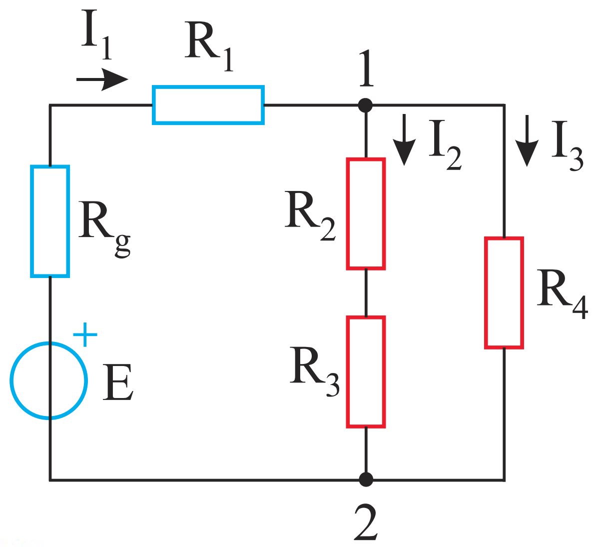
3.) $R_P = 4\,\Omega$ $\qquad \eta = \dfrac{4}{2+4} = 0,66$ $\qquad \boxed{\eta(\%) = 66\,\%}$

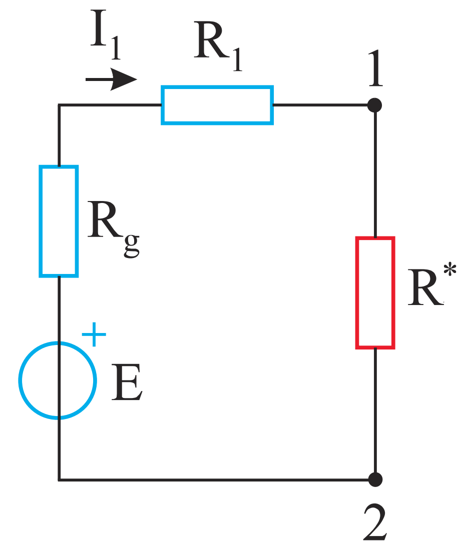
R_P=4 Ω daje efikasnije prenos energije nego R_P=1 Ω.



Zadatak 3. Odrediti R_4 u mreži prikazanoj na slici, tako da ukupna snaga Džulovih gubitaka u otpornicima R_2, R_3 i R_4 bude najveća moguća. Odrediti sve vrednosti otpornosti i snagu cele grupe. $E=110$ V, $R_g=0{,}5\ \Omega$, $R_1=25\ \Omega$, $R_2=10\ \Omega$, $R_3=35\ \Omega$.







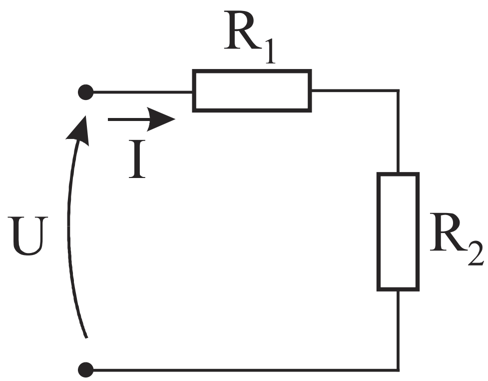
$$R^* = R_4 \,||\, (R_2 + R_3) = \frac{R_4 \cdot (R_2 + R_3)}{R_4 + (R_2 + R_3)} = ?$$

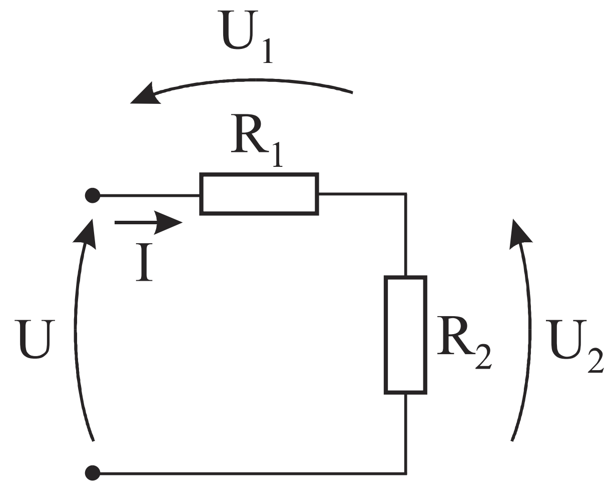
Prilagođenje po snazi: $\boxed{R_g + R_1 = R^*}$ $\Rightarrow$ $R^* = 25{,}5\ \Omega$

$$\frac{R_4 \cdot (10+35)}{R_4 + (10+35)} = 25{,}5 \quad \Rightarrow \quad \boxed{R_4 = 58{,}85\ \Omega}$$

$$P_{R^*} = R^* \cdot I_1^2 = R^* \cdot \left(\frac{E}{(R_1 + R_g) + R^*} \right)^2 = R^* \cdot \frac{E^2}{(2R^*)^2} = \frac{E^2}{4R^*} = \frac{110^2}{4 \cdot 25{,}5} \qquad \boxed{P_{R^*} = 118{,}6\, W}$$

Naponski razdelnik:



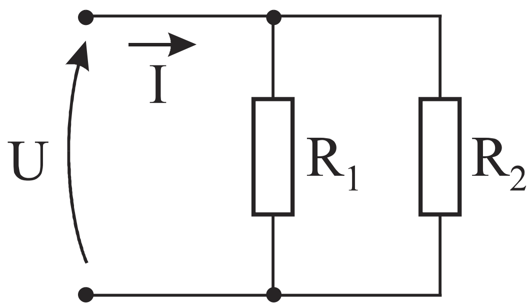


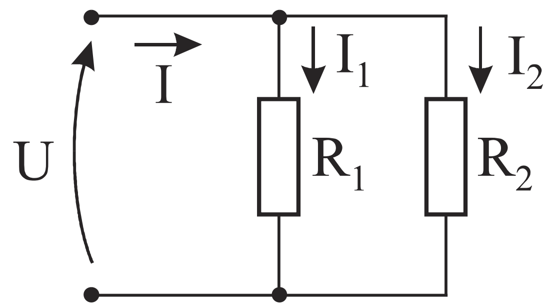
$$I = \frac{U}{R_1 + R_2}$$

$$U_1 = R_1 \cdot I = R_1 \cdot \frac{U}{R_1 + R_2} \qquad \boxed{U_1 = \frac{R_1}{R_1 + R_2} \cdot U}$$

$$U_2 = R_2 \cdot I = R_2 \cdot \frac{U}{R_1 + R_2} \qquad \boxed{U_2 = \frac{R_2}{R_1 + R_2} \cdot U}$$

Strujni razdelnik:





$$U = (R_1 \parallel R_2) \cdot I$$

$$I_1 = \frac{U}{R_1} = \frac{(R_1 \parallel R_2)}{R_1} \cdot I = \frac{\frac{R_1 \cdot R_2}{R_1 + R_2}}{R_1} \cdot I$$

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} \cdot I}$$

$$I_2 = \frac{U}{R_2} = \frac{(R_1 \parallel R_2)}{R_2} \cdot I = \frac{\frac{R_1 \cdot R_2}{R_1 + R_2}}{R_2} \cdot I$$

$$\boxed{I_2 = \frac{R_1}{R_1 + R_2} \cdot I}$$