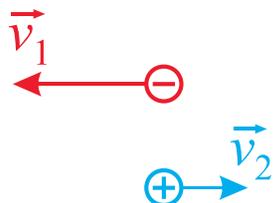


Petak, 03.12.2021.

Vežbe 16

Vremenski konstantne
električne struje

Zadatak 1. Izračunati vektor gustine struje u sredini gde se elektroni, koncentracije $N_1=10^{10} \text{ m}^{-3}$ kreću brzinom $v=30 \text{ cm/s}$, a šupljine dvostruko veće koncentracije tri puta manjom brzinom.

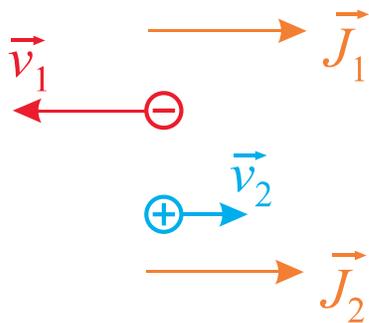


$$J = \sum_{i=1}^n N_i \cdot |Q_i| \cdot v_i$$

$$\vec{J} = N_1 \cdot Q_e \cdot v_1 \cdot (-\vec{i}_x) + N_2 \cdot |Q_e| \cdot v_2 \cdot \vec{i}_x$$

$$= N_1 \cdot |Q_e| \cdot v_1 \cdot \vec{i}_x + 2N_1 \cdot |Q_e| \cdot \frac{1}{3} v_1 \cdot \vec{i}_x$$

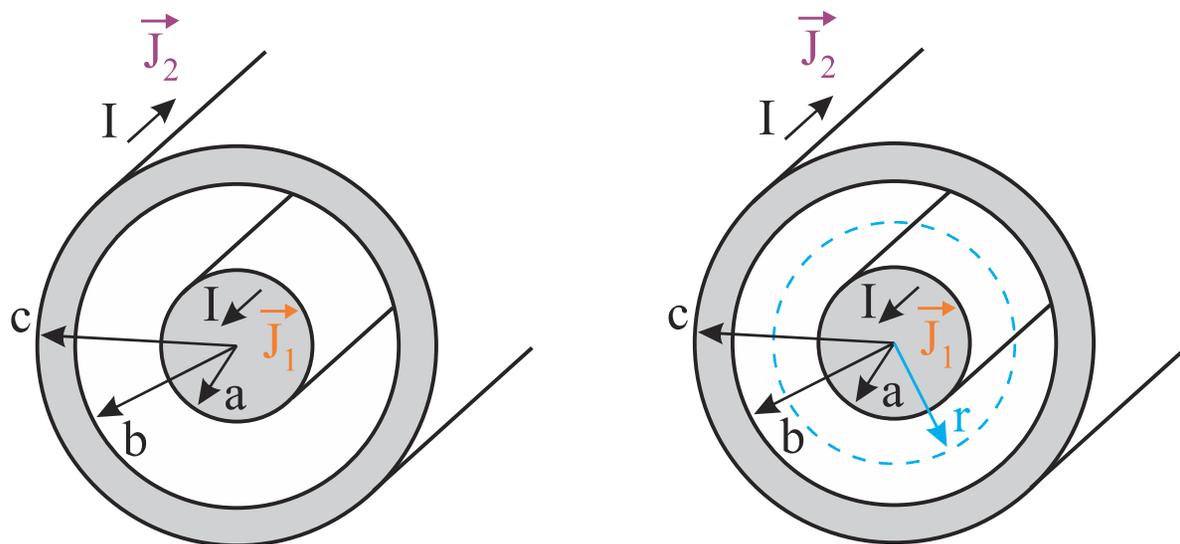
$$= \left(N_1 \cdot |Q_e| \cdot v_1 + 2N_1 \cdot |Q_e| \cdot \frac{1}{3} v_1 \right) \cdot \vec{i}_x = \frac{5}{3} N_1 \cdot |Q_e| \cdot v_1 \cdot \vec{i}_x$$



$$J = \frac{5}{3} \cdot 10^{10} \cdot 1,602 \cdot 10^{-19} \cdot 30 \cdot 10^{-2}$$

$$J = 0,8 \frac{\text{nA}}{\text{m}^2}$$

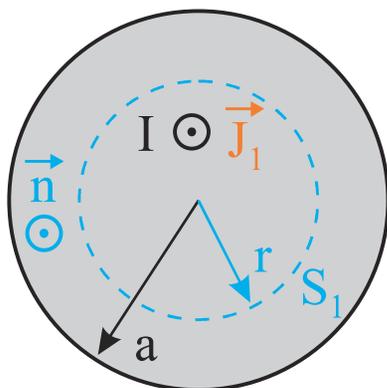
Zadatak 2. Na slici je prikazan deo dugog koaksijalnog kabla u čijim provodnicima postoji struja jačine I . Odrediti intenzitet struje kroz ravnu, kružnu površ S , poluprečnika r , za $0 < r < \infty$.



$$J_1 = \frac{I}{a^2 \pi}$$

$$J_2 = \frac{I}{(c^2 - b^2) \pi}$$

1) $r < a$

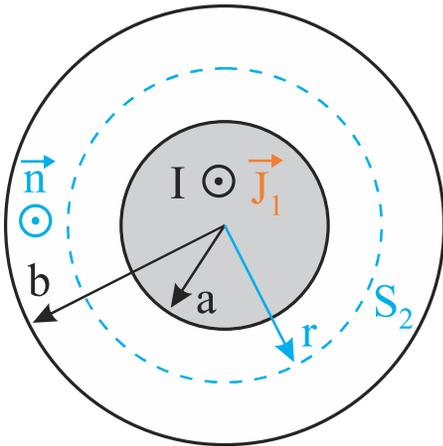


$$\int_S \vec{J} \cdot d\vec{S} = I$$

$$I(r) = \int_{S_1} \vec{J} \cdot d\vec{S} = J_1 \cdot S_1 = \frac{I}{a^2 \pi} \cdot r^2 \pi$$

$$I(r) = \frac{r^2}{a^2} \cdot I$$

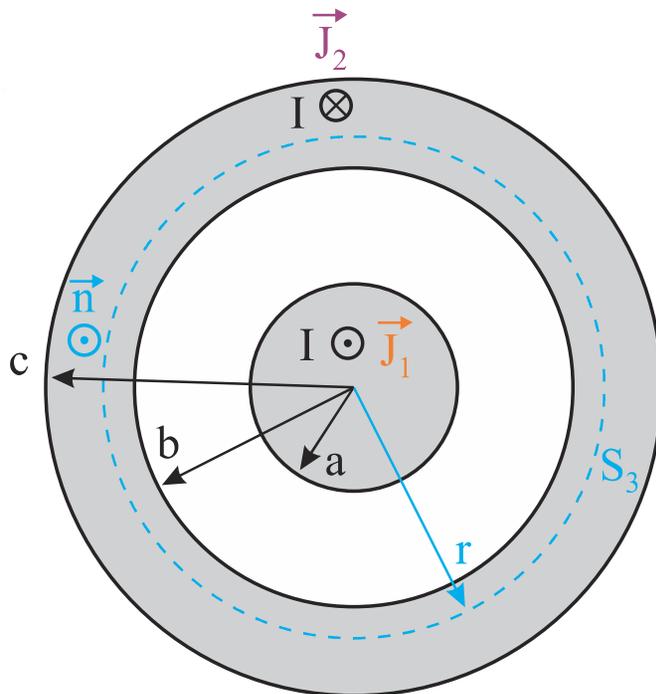
2) $a < r < b$



$$I(r) = \int_{S_2} \vec{J} \cdot d\vec{S} = J_1 \cdot S_2 = \frac{I}{a^2 \pi} \cdot a^2 \pi$$

$$I(r) = I$$

3) $b < r < c$

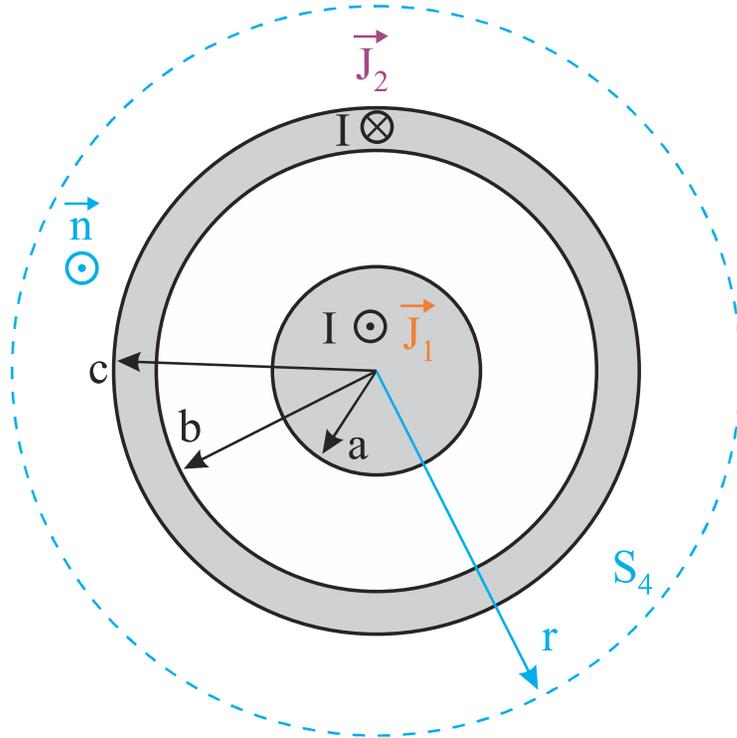


$$I(r) = \int_{S_3} \vec{J} \cdot d\vec{S} = J_1 \cdot a^2 \pi - J_2 \cdot (r^2 - b^2) \pi$$

$$= I - \frac{I}{(c^2 - b^2) \pi} \cdot (r^2 - b^2) \pi$$

$$I(r) = I - \frac{I}{(c^2 - b^2)} \cdot (r^2 - b^2)$$

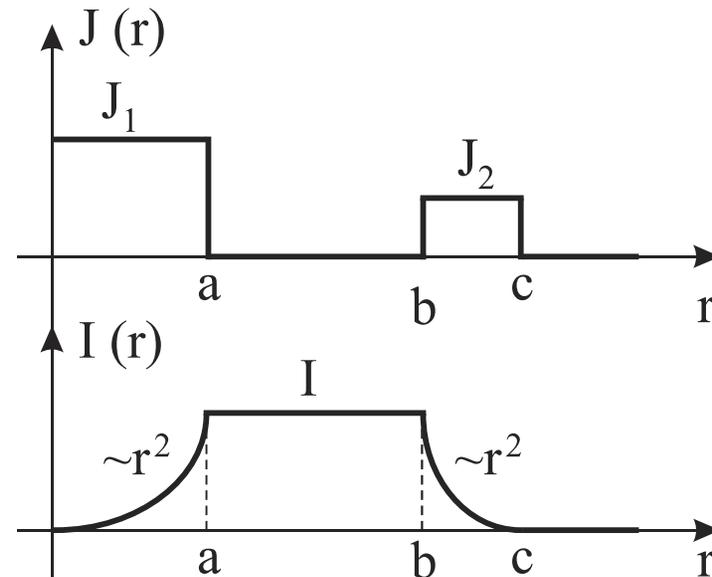
4) $r > c$



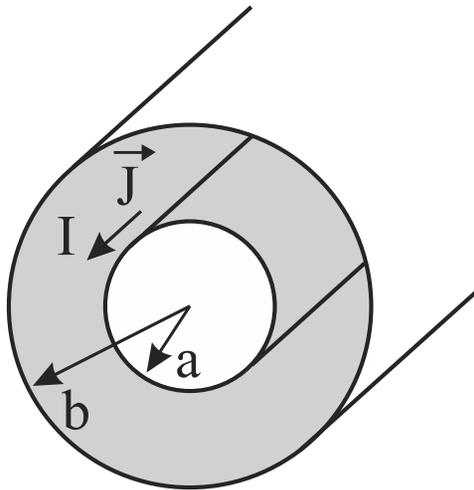
$$I(r) = \int_{S_4} \vec{J} \cdot d\vec{S} = J_1 \cdot a^2 \pi - J_2 \cdot (c^2 - b^2) \pi$$

$$= I - I$$

$$I(r) = 0$$



Zadatak 3. Izračunati intenzitet vektora gustine struje u dugoj bakarnoj cevi sa strujom $I=5$ A. Unutrašnji poluprečnik cevi je $a=10$ mm, a spoljašnji $b=12$ mm. Specifična otpornost bakra iznosi $\rho_{Cu}=1,6 \cdot 10^{-8}$ Ωm .



$$I = \int_{S_{pp}} \vec{J} \cdot \vec{dS} = J \cdot (b^2 \pi - a^2 \pi)$$

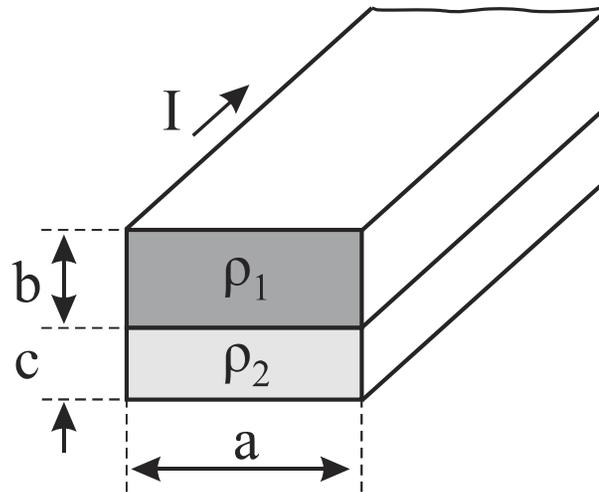
$$J = \frac{I}{(b^2 - a^2) \pi} = 36,19 \frac{kA}{m^2}$$

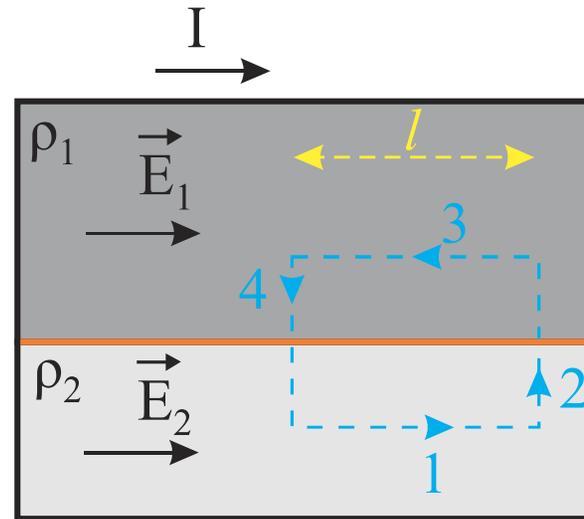
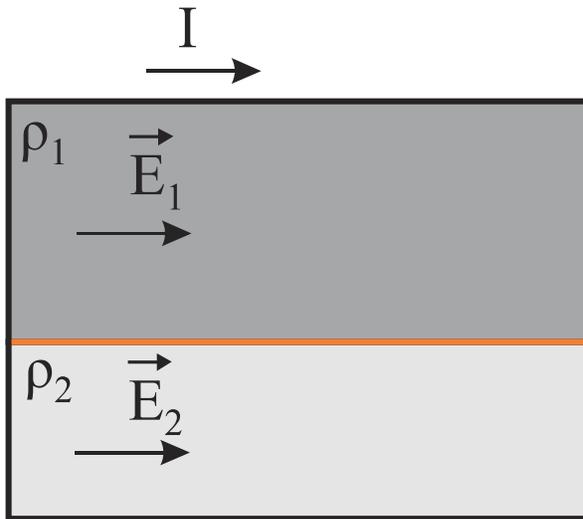
Zadatak 4. Izračunati gustinu snage Džulovih gubitaka u bakarnoj cevi iz prethodnog pitanja.

$$p_j = \left(\frac{dP}{dV} \right)_j = E \cdot J = \rho \cdot J^2 = \rho \cdot \left(\frac{I}{(b^2 - a^2) \pi} \right)^2 \quad \boxed{p_j = 20,96 \frac{W}{m^3}}$$

Zadatak 5. Na slici je prikazan deo dugog pravog nehomogenog provodnika sa strujom jačine I . Dokazati da je električno polje u provodniku homogeno.

- Skicirati (na posebnim crtežima) linije vektora J i E tako da njihova gustina odražava odnos intenziteta vektora u različitim materijalima.
- Odrediti odnos jačina struja kroz jedan i drugi njegov sloj.
- Odrediti odnos gustina snaga Džulovih gubitaka u dva sloja, ako je: $\rho_2 = 5\rho_1$, $b = 0,5a$ i $c = 0,25a$.





$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

$$E_2 \cdot l + 0 - E_1 \cdot l + 0 = 0$$

 \Rightarrow

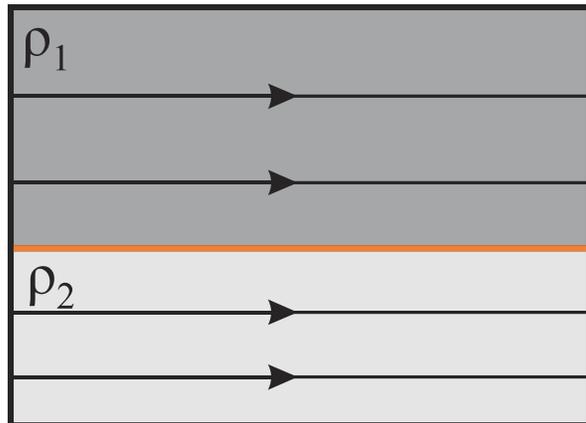
$$E_1 = E_2 = E$$

a)

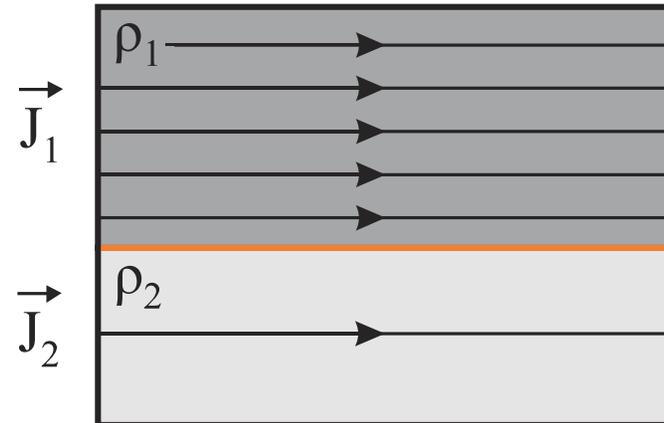
$$\vec{J} = \sigma \cdot \vec{E} = \frac{\vec{E}}{\rho} \quad \boxed{E_1 = E_2 = E}$$

$$J_1 = \frac{E}{\rho_1} \quad J_2 = \frac{E}{\rho_2} \quad \boxed{\rho_2 = 5\rho_1}$$

$$\frac{J_1}{J_2} = \frac{\frac{E}{\rho_1}}{\frac{E}{\rho_2}} = \frac{\rho_2}{\rho_1} = \frac{5\rho_1}{\rho_1} \quad \boxed{\frac{J_1}{J_2} = 5}$$



\vec{E}



\vec{J}

b)

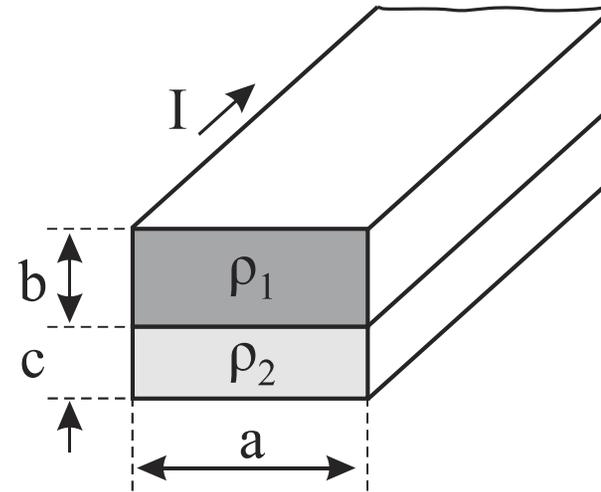
$$J_1 = \frac{I_1}{S_1}$$

$$J_2 = \frac{I_2}{S_2}$$

$$\boxed{\frac{J_1}{J_2} = 5}$$

$$\frac{I_1}{I_2} = \frac{J_1 \cdot S_1}{J_2 \cdot S_2} = 5 \cdot \frac{a \cdot b}{a \cdot c} = 5 \cdot \frac{0,5a}{0,25a}$$

$$\boxed{\frac{I_1}{I_2} = 10}$$



c)

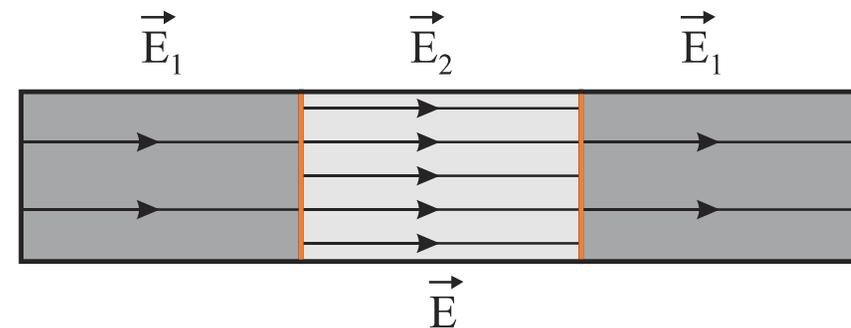
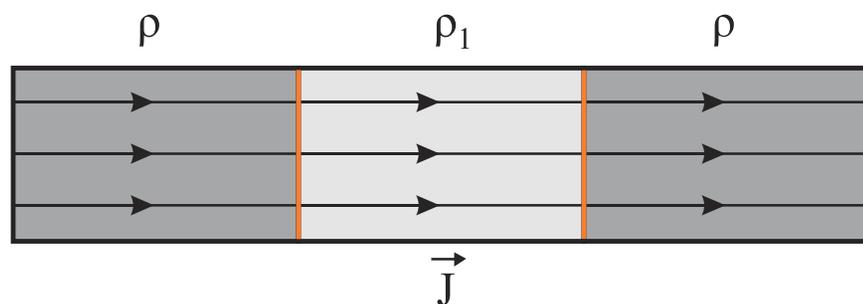
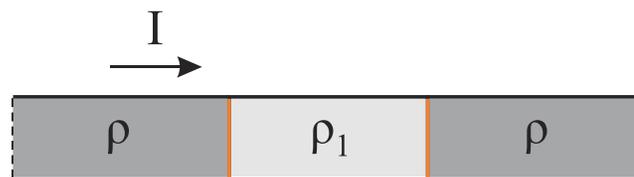
$$p_j = \frac{dP}{dV} = \rho \cdot J^2$$

$$\frac{p_{j1}}{p_{j2}} = \frac{\rho_1 \cdot J_1^2}{\rho_2 \cdot J_2^2} = \frac{\rho_1}{5\rho_1} \cdot 5^2$$

$$\boxed{\frac{p_{j1}}{p_{j2}} = 5}$$

Zadatak 6. Na slici je prikazan uzdužni presek dela pravolinijskog provodnika konstantnog poprečnog preseka površine S , načinjenog od dva materijala različite specifične otpornosti. Kroz provodnik se uspostavila struja jačine I . Skicirati (na posebnim crtežima) linije vektora \vec{J} i \vec{E} tako da njihova gustina odražava odnos intenziteta vektora u različitim materijalima.

$$\rho_1 = 10\rho$$



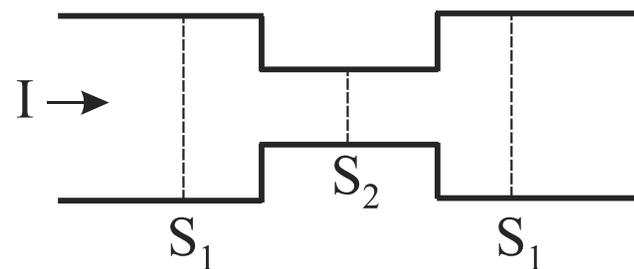
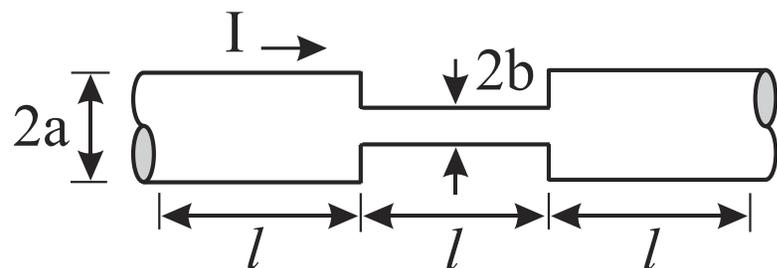
$$J = \frac{I}{S} \quad \Rightarrow \quad J_1 = J_2 = J$$

$$\vec{E} = \rho \cdot \vec{J}$$

$$E_1 = \rho \cdot J$$

$$E_2 = \rho_1 \cdot J = 10\rho \cdot J$$

Zadatak 7. Na slici je prikazan deo pravolinijskog provodnika načinjenog od bakra kroz koji postoji struja jačine I . Ako je ova struja veoma velika, obeležiti na crtežu koji deo će se prvi istopiti i objasniti zašto. (Princip rada topljivog osigurača).



$$J = \frac{I}{S}$$

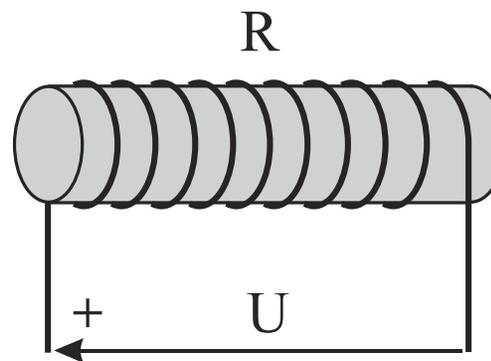
$$S_1 > S_2 \quad \Rightarrow \quad J_1 < J_2$$

$$\frac{p_1}{p_2} = \frac{\rho \cdot J_1^2}{\rho \cdot J_2^2} = \frac{\rho \cdot \left(\frac{I}{S_1}\right)^2}{\rho \cdot \left(\frac{I}{S_2}\right)^2} = \frac{S_2^2}{S_1^2} = \frac{(b^2 \pi)^2}{(a^2 \pi)^2} = \frac{b^4}{a^4}$$

$$\text{Za } a = 3b: \quad \frac{p_1}{p_2} = \frac{b^4}{(3b)^4} = \frac{1}{81} \quad \boxed{p_2 = 81 \cdot p_1}$$

Zadatak 8. Grejač snage $P=800\text{ W}$, koji će se priključiti na izvor napona $U=200\text{ V}$, treba napraviti od žice od kantala. Dozvoljen intenzitet vektora gustine struje kantala je 10 A/mm^2 , a specifična otpornost pri radnoj temperaturi $\rho=12\cdot 10^{-6}\ \Omega\text{m}$. Izračunati površinu poprečnog preseka i dužinu žice.

$$P = U \cdot I = \frac{U^2}{R} = R \cdot I^2$$



$$P = \frac{U^2}{R} \quad \Rightarrow \quad R = \frac{U^2}{P} = \frac{200^2}{800} = 50\ \Omega$$

$$P = R \cdot I^2 \quad \Rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{800}{50}} = \sqrt{16} = 4\text{ A}$$

$$J = \frac{I}{S} \quad \Rightarrow \quad S = \frac{I}{J} = \frac{4\text{ A}}{10\text{ A/mm}^2} = 0,4\text{ mm}^2$$

$$R = \rho \frac{\ell}{S} \quad \Rightarrow \quad \ell = \frac{SR}{\rho} = \frac{0,4 \cdot 10^{-6} \cdot 50}{12 \cdot 10^{-6}} \quad \boxed{\ell = 1,67\text{ m}}$$