STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

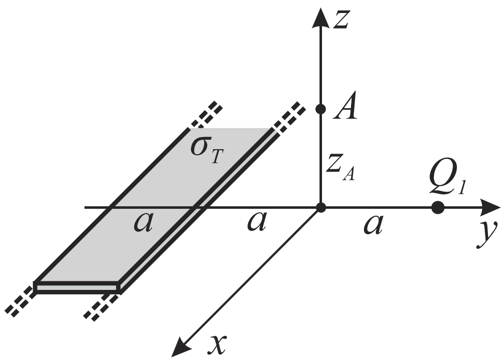
09.06.2020.

ZADACI

Zadatak 1. Tanka, veoma dugačka, traka od dielektrika, širine a, ravnomerno je naelektrisana površinskim naelektrisanjem σ_T. Traka je postavljena u x-y ravni Dekartovog koordinatnog sistema, paralelno sa x osom, kao što je prikazano na slici 1.

a) Odrediti izraz za vektor jačine električnog polja koji u tački A, koja se nalazi na z osi, na visini z_A, stavara naelektrisana traka.
b) Odrediti intenzitet sile kojom traka i tačkasto naelektrisanje, naelektrisano sa Q_1, deluju na malo probno naelektrisanje, Q_P, postavljeno u tačku A. Tačkasto naelektrisanje Q_1 se nalazi na y osi.

Brojni podaci su: $a = z_A = 1\ cm$, $\sigma_T = 10\ \mu C/m^2$, $Q_1 = 20\ nC$, $Q_P = 1\ pC$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



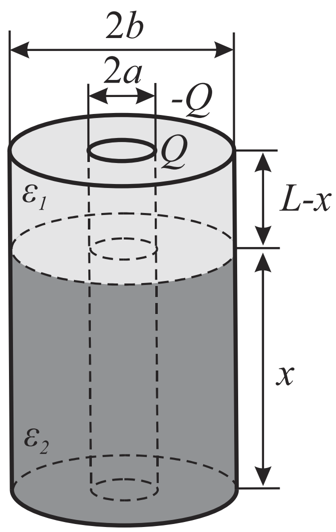
Slika 1.

$$\int tg\alpha\, d\alpha = -\ln|\cos\alpha|$$

Zadatak 2. Koaksijalni kabl poluprečnika elektroda a i b, dužine L, ispunjen je do visine x čvrstim dielektrikom relativne permitivnosti ε_{r2}, dok je ostatak kabla ispunjen tečnim dielektrikom relativne permitivnosti ε_{r1}. Kabl je priključen na izvor napona $U = 1\ kV$ i postavljen u vertikalni položaj, kao što je prikazano na slici 2.

a) Izvesti izraz za kapacitivnost kabla.
b) Ako se, kroz odgovarajući otvor na kablu, nivo tečnog dielektrika u njemu smanji za 50%, kapacitivnost kondenzatora se smanji za 10%. Odrediti nivo čvrstog dielektrika, x.
c) Izračunati količinu naelektrisanja na oblogama kondenzatora, nakon smanjenja nivoa tečnog dielektrika za 50%.

Brojni podaci su: $a = 1\ mm$, $b = 2{,}7a$, $L = 10\ cm$, $\varepsilon_{r1} = 2$, $\varepsilon_{r2} = 3$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

09.06.2020.

ZADACI

Zadatak 1. U kolu vremenski konstantne struje sa slike 1 izračunati:

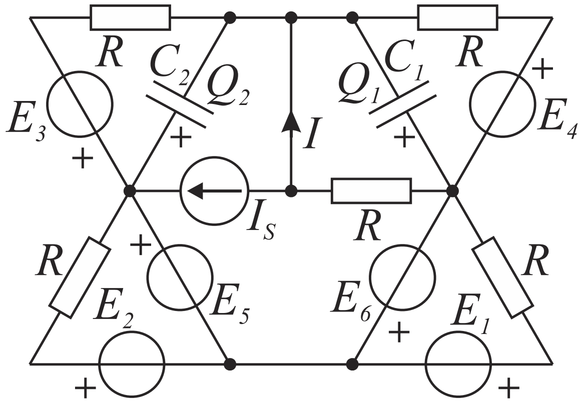
a) naelektrisanja kondenzatora Q_1 i Q_2,

b) jačinu struje I,

c) snagu generatora *ems* E_6.

Pri rešavanju kola primeniti metodu sa najmanjim brojem jednačina.

Brojni podaci: $R = 10\ \Omega$, $I_S = 0{,}5\ A$, $E_1 = 2\ V$, $E_2 = 8\ V$, $E_3 = 6\ V$, $E_4 = 3\ V$, $E_5 = E_6 = 4\ V$, $C_1 = C_2 = 5\ \mu F$.



Slika 1.

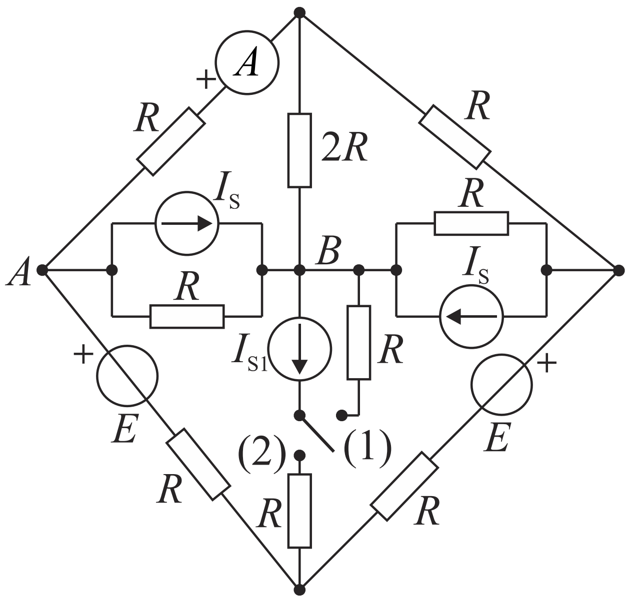
Zadatak 2. Kada se u simetričnoj mreži sa slike 2, preklopnik prebaci iz položaja (1) u položaj (2), napon između tačaka A i B se poveća za $\Delta U_{AB} = 7\ V$.

a) Primenjujući teoremu superpozicije, izračunati jačinu struje strujnog generatora, I_{S1}.

b) Izračunati snagu strujnog generatora I_{S1}, i snagu naponskog generatora *ems* E, kada je preklopnik u položaju (2).

c) Izračunati pokazivanje idealnog ampermetra, kada je preklopnik u položaju (2).

Brojni podaci su: $R = 35\ \Omega$, $E = 6\ V$, $I_S = 100\ mA$.



Slika 2.

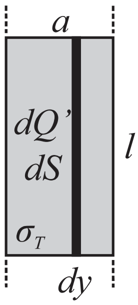
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje dva sata.

Ispit iz OET1, 09.06.2020.

I-1

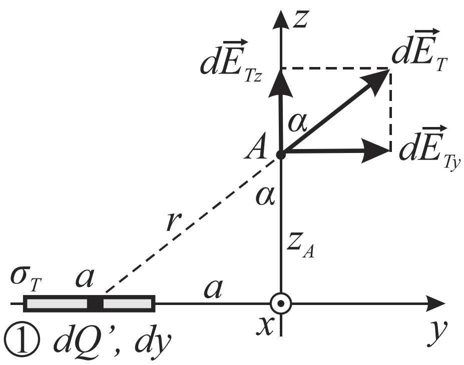
a)



$$dQ = \sigma_T \; dS = \sigma \; l \; dy \qquad (l \to \infty)$$

$$\frac{dQ}{l} = \sigma_T \; dy$$

$$dQ' = \sigma_T \; dy$$

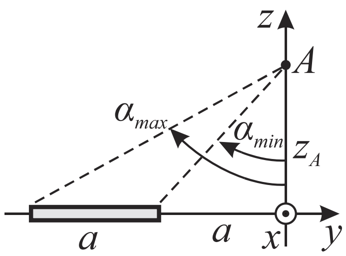


$$dE_T = \frac{dQ'}{2\pi\varepsilon_0 r} = \frac{\sigma_T \; dy}{2\pi\varepsilon_0 r}$$

$$dE_{Ty} = dE_T \sin\alpha = \frac{\sigma_T \; dy}{2\pi\varepsilon_0 r}\sin\alpha = \frac{\sigma_T \dfrac{r\,d\alpha}{\cos\alpha}}{2\pi\varepsilon_0 r}\sin\alpha = \frac{\sigma_T}{2\pi\varepsilon_0} tg\alpha \; d\alpha$$

$$E_{Ty} = \int dE_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\int_{\alpha_{\min}}^{\alpha_{\max}} tg\alpha \; d\alpha = \frac{\sigma_T}{2\pi\varepsilon_0}\left(-\ln|\cos\alpha|\right)\Big|_{\alpha_{\min}}^{\alpha_{\max}}$$

$$E_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\ln|\cos\alpha_{\min}| - \ln|\cos\alpha_{\max}|\right) = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{|\cos\alpha_{\min}|}{|\cos\alpha_{\max}|}$$



$$E_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\dfrac{z_A}{\sqrt{z_A^{\,2} + a^2}}}{\dfrac{z_A}{\sqrt{z_A^{\,2} + (2a)^2}}} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\dfrac{a}{\sqrt{a^2 + a^2}}}{\dfrac{a}{\sqrt{a^2 + (2a)^2}}} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\sqrt{5}}{\sqrt{2}}$$

$$\boxed{E_{Ty} = \frac{\sigma_T}{2\pi\varepsilon_0}\ln\frac{\sqrt{5}}{\sqrt{2}}} \qquad \boxed{\vec{E}_{Ty} = E_{Ty}\cdot\vec{i}_y}$$

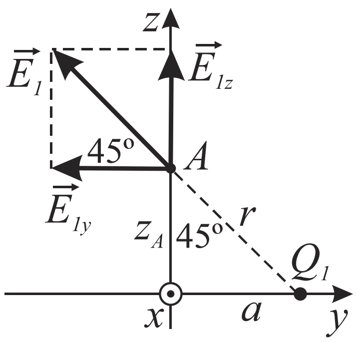
$$dE_{Tz} = dE_T \cos\alpha = \frac{\sigma_T \; dy}{2\pi\varepsilon_0 r}\cos\alpha = \frac{\sigma_T \dfrac{r\,d\alpha}{\cos\alpha}}{2\pi\varepsilon_0 r}\cos = \frac{\sigma_T}{2\pi\varepsilon_0} d\alpha$$

$$E_{Tz} = \int dE_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha = \frac{\sigma_T}{2\pi\varepsilon_0}\alpha\Big|_{\alpha_{\min}}^{\alpha_{\max}} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(\alpha_{\max} - \alpha_{\min}\right)$$

$$E_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(arctg\frac{a+a}{z_A} - arctg\frac{a}{z_A}\right) E_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(arctg\,2 - arctg\,1\right)$$

$$\boxed{E_{Tz} = \frac{\sigma_T}{2\pi\varepsilon_0}\left(arctg\,2 - arctg\,1\right)} \qquad \boxed{\vec{E}_{Tz} = E_{Tz}\cdot\vec{i}_z}$$

b)



$$E_1 = \frac{Q_1}{4\pi\varepsilon_0 r^2} = \frac{Q_1}{4\pi\varepsilon_0\left(a\sqrt{2}\right)^2} = \frac{Q_1}{8\pi\varepsilon_0 a^2}$$

$$E_{1y} = E_1 \cos 45° = \frac{Q_1}{8\pi\varepsilon_0 a^2}\cdot\frac{\sqrt{2}}{2} \qquad \boxed{E_{1y} = \frac{\sqrt{2}\,Q_1}{16\pi\varepsilon_0 a^2}} \qquad \boxed{\vec{E}_{1y} = E_{1y}\cdot\left(-\vec{i}_y\right)}$$

$$E_{1z} = E_1 \sin 45° = \frac{Q_1}{8\pi\varepsilon_0 a^2}\cdot\frac{\sqrt{2}}{2} \qquad \boxed{E_{1z} = \frac{\sqrt{2}\,Q_1}{16\pi\varepsilon_0 a^2}} \qquad \boxed{\vec{E}_{1z} = E_{1z}\cdot\vec{i}_z}$$

$$\vec{E}_A = \left(\frac{\sigma_T}{2\pi\varepsilon_0} \ln\frac{\sqrt{5}}{\sqrt{2}} - \frac{\sqrt{2}\,Q_1}{16\pi\varepsilon_0 a^2} \right)\cdot \vec{i}_y + \left(\frac{\sigma_T}{2\pi\varepsilon_0}\left(arctg\,2 - arctg\,1\right) + \frac{\sqrt{2}\,Q_1}{16\pi\varepsilon_0 a^2} \right)\cdot \vec{i}_z = 0,55\,\frac{MV}{m}\cdot\left(-\vec{i}_y\right) + 0,69\,\frac{MV}{m}\cdot\vec{i}_z$$

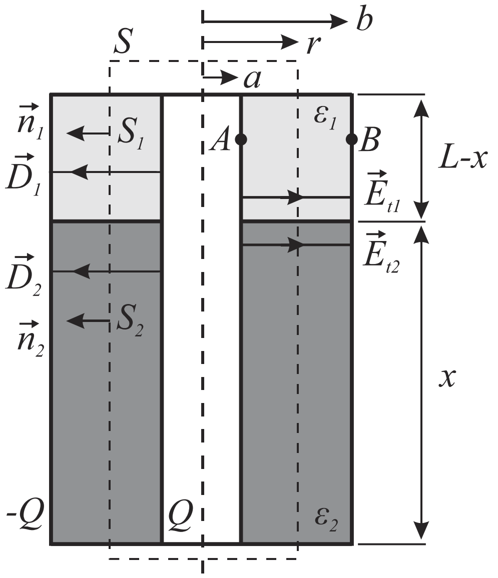
$$F_{na\,Q_P} = Q_P\,E_A = Q_P\,\sqrt{{E_{Ay}}^2 + {E_{Az}}^2} = 1\cdot 10^{-12}\cdot\sqrt{\left(0,55\cdot 10^6\right)^2 + \left(0,69\cdot 10^6\right)^2} = 1\cdot 10^{-12}\cdot 0,88\cdot 10^6 = 0,88\cdot 10^{-6}\ N$$

$$\boxed{F_{na\,Q_P} = 0,88\ \mu N}$$

<u>Ispit iz OET1, 09.06.2020.</u>

I-2

a)



Granični uslov:

$$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$$

$$D_{n1} = D_{n2} = 0$$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{u\,S}$$

$$\int_{S_{OM}} D\,ds = Q$$

$$\int_{OM_1} D_1\,ds + \int_{OM_2} D_2\,ds = Q$$

$$D_1\,2\pi r\left(L-x\right) + D_2\,2\pi r\,x = Q \qquad \left(D_1 = \varepsilon_1 E, \quad D_2 = \varepsilon_2 E\right)$$

$$\boxed{E = \frac{Q}{\left[\varepsilon_1\left(L-x\right)+\varepsilon_2 x\right]2\pi r}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\,dr = \int_a^b \frac{Q}{\left[\varepsilon_1\left(L-x\right)+\varepsilon_2 x\right]2\pi r}dr = \frac{Q}{\left[\varepsilon_1\left(L-x\right)+\varepsilon_2 x\right]2\pi}\ln\frac{b}{a}$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{\left[\varepsilon_1\left(L-x\right)+\varepsilon_2 x\right]2\pi}{\ln\frac{b}{a}} = \frac{\left(4L+2x\right)\pi\varepsilon_0}{\ln\frac{b}{a}}} \qquad \varepsilon_1 = 2\varepsilon_0, \qquad \varepsilon_2 = 3\varepsilon_0$$

b)

$$\left(L-x\right)^{NOVO} = \frac{L-x}{2} \qquad \Rightarrow \qquad C^{NOVO} = 0{,}9C$$

$$C^{NOVO} = \frac{\left[\varepsilon_0\frac{L-x}{2}+\varepsilon_1\frac{L-x}{2}+\varepsilon_2 x\right]2\pi}{\ln\frac{b}{a}} = \frac{\left(3L+3x\right)\pi\varepsilon_0}{\ln\frac{b}{a}}$$

$$\frac{\left(3L+3x\right)\pi\varepsilon_0}{\ln\frac{b}{a}} = 0{,}9\cdot\frac{\left(4L+2x\right)\pi\varepsilon_0}{\ln\frac{b}{a}}$$

$$3L+3x = 0{,}9\cdot\left(4L+2x\right)$$

$$3L+3x = 3{,}6L+1{,}8x$$

$$1{,}2x = 0{,}6L \qquad \boxed{x = \frac{L}{2} = 5\,cm}$$

c)

$$C^{NOVO} = \frac{\left(3L+3x\right)\pi\varepsilon_0}{\ln\frac{b}{a}} = \frac{\left(3L+3\cdot\frac{L}{2}\right)\pi\varepsilon_0}{\ln\frac{b}{a}} = \frac{\frac{9L}{2}\pi\varepsilon_0}{\ln\frac{b}{a}} = 12{,}51\,pF$$

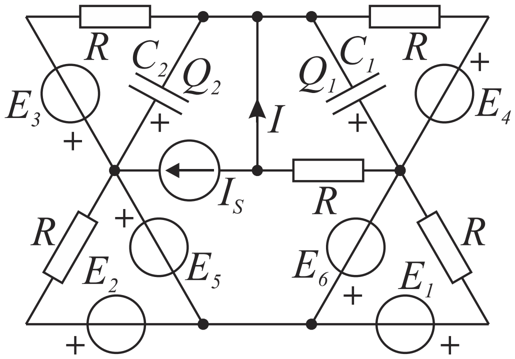
$$Q^{NOVO} = C^{NOVO}U = 12{,}51\cdot10^{-12}\cdot1\cdot10^{3} = 12{,}51\cdot10^{-9}\,C$$

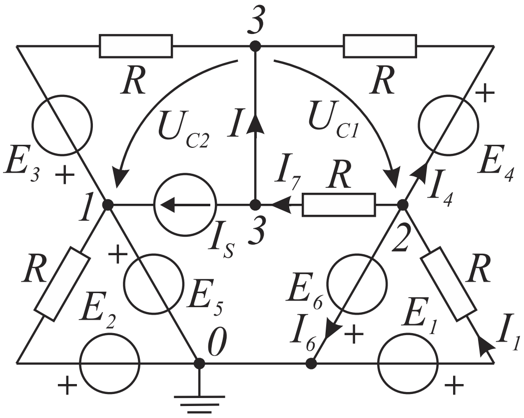
$$\boxed{Q^{NOVO} = 12{,}51\,nC}$$

Ispit iz OET1, 09.06.2020.

II-1

a)





$V_1 = E_5 = 4V$

$V_2 = -E_6 = -4V$

$$V_3\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)-V_1\left(\frac{1}{R}\right)-V_2\left(\frac{1}{R}+\frac{1}{R}\right)=-I_S-\frac{E_3}{R}+\frac{E_4}{R}$$

$$V_3\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)-4\cdot\left(\frac{1}{10}\right)-(-4)\cdot\left(\frac{1}{10}+\frac{1}{10}\right)=-0{,}5-\frac{6}{10}+\frac{3}{10} \quad /\cdot 10$$

$$3\cdot V_3-4\cdot 1+4\cdot 2=-5-6+3 \quad \Rightarrow \quad V_3=-4V$$

$$\boxed{Q_1 = C_1 U_{C1} = (V_2 - V_3)C_1 = 0\,\mu C}$$

$$\boxed{Q_2 = C_2 U_{C2} = (V_1 - V_3)C_2 = 40\,\mu C}$$

b)

$I = I_7 - I_S$

$$I_7 = \frac{V_2 - V_3}{R} = 0\,A$$

$$\boxed{I = -I_S = -0{,}5\,A}$$

c)

$P_{E_6} = E_6 I_6$

$I_6 = I_1 - I_7 - I_4$

$$I_1 = \frac{-V_2 + E_1}{R} = 0{,}6\,A$$

$$I_4 = \frac{V_2 - V_3 + E_4}{R} = 0{,}3\,A$$

$I_6 = 0{,}3\,A$

$$\boxed{P_{E_6} = 4\cdot 0{,}3 = 1{,}2W}$$

Ispit iz OET1, 09.06.2020.

II-2

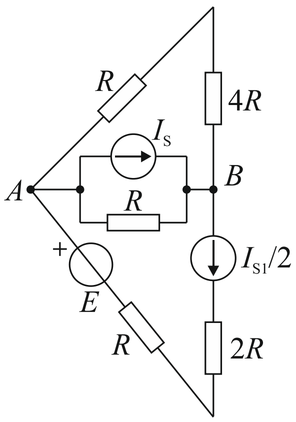
a)

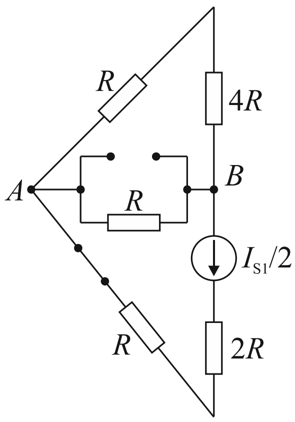
$$\boxed{\begin{matrix}\textit{Svi}\\ \textit{generatori}\end{matrix}} = \boxed{\begin{matrix}\textit{Svi}\\ \textit{sem } I_{S1}\end{matrix}} + \boxed{\begin{matrix}\textit{Samo}\\ I_{S1}\end{matrix}}$$

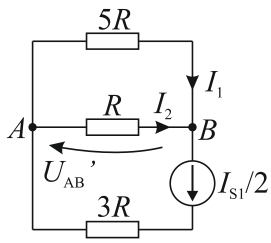
(2) (1)

$$U_{AB}^{(2)} = U_{AB}^{(1)} + U_{AB}' \qquad \Rightarrow \qquad \Delta U_{AB} = U_{AB}^{(2)} - U_{AB}^{(1)} = U_{AB}' = 7\,V$$

Kolo je simetrično.







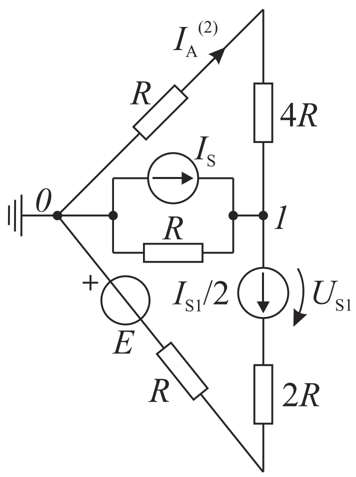
$$I_1 = \frac{U_{AB}'}{5R} = \frac{7}{5\cdot 35} = 40\ mA$$

$$I_2 = \frac{U_{AB}'}{R} = \frac{7}{35} = 200\ mA$$

$$\frac{I_{S1}}{2} = I_1 + I_2 = 240\,mA$$

$$\boxed{I_{S1} = 480\,mA}$$

b)



$$V_0 = 0\,V$$

$$V_1\left(\frac{1}{5R} + \frac{1}{\infty} + \frac{1}{R} + \frac{1}{3R+\infty}\right) = I_S - \frac{I_{S1}}{2}$$

$$V_1\left(\frac{1}{5\cdot 35} + \frac{1}{35}\right) = 100\,m - 240\,m \qquad /\cdot 5\cdot 35$$

$$6V_1 = -24{,}5 \qquad \Rightarrow \qquad V_1 = -4{,}08\,V$$

$$U_{S1} = 2R\frac{I_{S1}}{2} + R\frac{I_{S1}}{2} - E + U_{01} = 23{,}283\,V$$

$$P_{S1} = U_{S1}\,I_{S1} = 23{,}283\cdot 480\,m$$

$$\boxed{P_{S1} = 11{,}176\,W}$$

$$P_E = E\,I_E = E\frac{I_{S1}}{2}$$

$$\boxed{P_E = 1{,}44\,W}$$

c)

$$I_A^{(2)} = \frac{V_0 - V_1}{R + 4R} = \frac{4{,}08}{5\cdot 35}$$

$$\boxed{I_A^{(2)} = 23{,}31\,mA}$$