Prostoperiodične struje
II deo

Kompleksni domen, rešavanje električnih kola, kompleksna snaga.

Kompleksni brojevi

$i = \sqrt{-1}$: imaginarni broj u matematici $\quad$ $j = \sqrt{-1}$: imaginarni broj u elektrotehnici

Na primer $\quad \sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{-1}\sqrt{4} = \mathrm{j}2$

Kompleksni broj je u potpunosti definisan svojim realnim i imaginarnim delom.

Kompleksni brojevi mogu da se predstave u kompleksnoj ravni određenoj dvema osama: Re – realna osa, Im – imaginarna osa

$\underline{A} = a + \mathrm{j}b$

a – realni deo

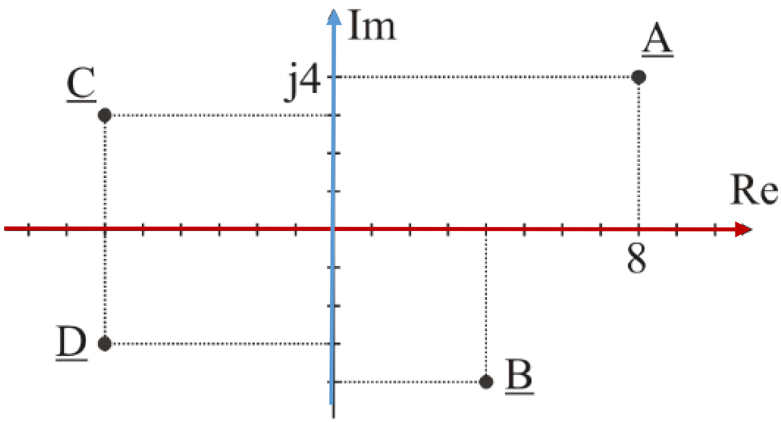
b – imaginarni deo

$\underline{A} = 8 + \mathrm{j}4$

$\underline{B} = 4 - \mathrm{j}4$

$\underline{C} = -6 + \mathrm{j}3$

$\underline{D} = -6 - \mathrm{j}3$



Kompleksni brojevi

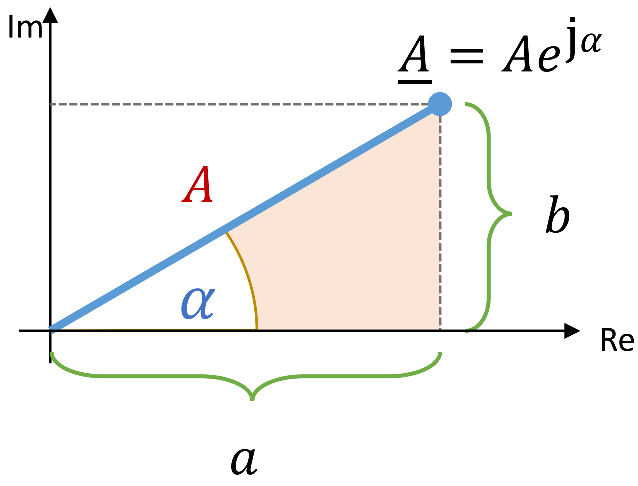
$\underline{A} = a + \mathrm{j}b$, algebarski oblik

$\underline{A} = Ae^{j\alpha}$, eksponencijalni oblik

A – moduo
α – argument

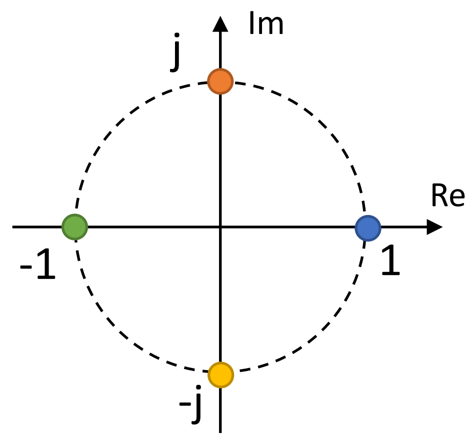
Za predstavljanje kompleksnih brojeva može da se koristi moduo i argument.

Transformacija iz algebarskog u eksponencijalni pomoću pravouglog trougla



$$A = \sqrt{a^2 + b^2} \qquad \alpha = \mathrm{arctg}(b/a)$$

Jedinični krug i eksponencijalni oblik kompleksnih brojeva: 1, j, -1 i -j



$$1 = e^{\mathrm{j}0}$$

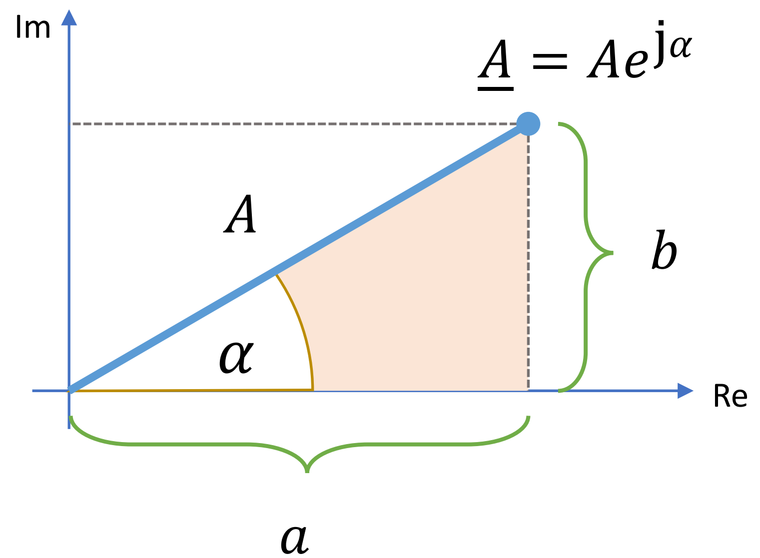
$$\mathrm{j} = e^{\mathrm{j}\frac{\pi}{2}}$$

$$-1 = e^{\mathrm{j}\pi}$$

$$-\mathrm{j} = e^{-\mathrm{j}\frac{\pi}{2}}$$

Kompleksni brojevi

Prebacivanje iz eksponencijalnog u algebarski oblik.



$\underline{A} = Ae^{j\alpha}$ ⇒ $\underline{A} = a + jb$

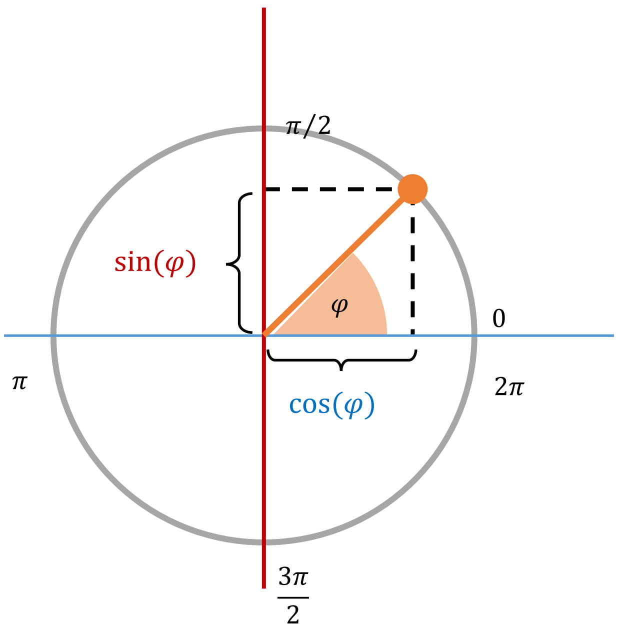
Ojlerov obrazac

$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$\underline{A} = A(\cos\alpha + j\sin\alpha)$$

$$a = A\cos\alpha, \quad b = A\sin\alpha$$

Sinus i kosinus



$\varphi[rad]$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\varphi[°]$	0	30	45	60	90	180	270	360
$cos\varphi$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$sin\varphi$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0

$$\varphi[°] = \frac{\varphi[rad]}{\pi} 180°$$

$$1\ rad = 57{,}3°$$

$$\pi = 3{,}1416\ \mathrm{rad}$$

$$\frac{\pi}{3} = 1{,}047\ \mathrm{rad}$$

Kompleksni brojevi,

$$\underline{X} = a + \mathrm{j}b, \quad \underline{Y} = c + \mathrm{j}d$$

$$\boxed{\underline{X} + \underline{Y} = (a + c) + \mathrm{j}(b + d)}$$

Primer

$$\underline{X} = -3 + \mathrm{j}4, \quad \underline{Y} = 6 - \mathrm{j}8$$

$$\underline{X} + \underline{Y} = (-3 + 6) + \mathrm{j}(4 - 8)$$

$$= (3 - \mathrm{j}4)$$

$$\underline{X} = a + \mathrm{j}b, \quad \underline{Y} = c + \mathrm{j}d$$

$$\boxed{\underline{X} - \underline{Y} = (a - c) + \mathrm{j}(b - d)}$$

Primer

$$\underline{X} = -3 + \mathrm{j}4, \quad \underline{Y} = 6 - \mathrm{j}8$$

$$\underline{X} - \underline{Y} = (-3 - 6) + \mathrm{j}\big(4 - (-8)\big)$$

$$= (-9 + \mathrm{j}12)$$

Kompleksni brojevi, množenje

$\underline{X} = a + \mathrm{j}b, \quad \underline{Y} = c + \mathrm{j}d$

$$\underline{X} \cdot \underline{Y} = (a + \mathrm{j}b)(c + \mathrm{j}d)$$

$$= ac + \mathrm{j}bc + \mathrm{j}ad + \mathrm{j}^2 bd$$

$$= (ac - bd) + \mathrm{j}(bc + ad)$$

$$\mathrm{j}^2 bd = -bd$$

Primer

$\underline{X} = -3 + \mathrm{j}4, \quad \underline{Y} = 6 - \mathrm{j}8$

$$\underline{X} \cdot \underline{Y} = (-3) \cdot 6 + \mathrm{j}4 \cdot 6$$
$$+(-3) \cdot (-\mathrm{j}8) + \mathrm{j}4 \cdot (-\mathrm{j}8)$$
$$= -18 + \mathrm{j}24 + \mathrm{j}24 - \mathrm{j}^2 32$$
$$= (-18 + 32) + \mathrm{j}(24 + 24)$$
$$= 14 + \mathrm{j}48$$

Množenje u eksponencijalnom obliku.

$\underline{X} = Xe^{\mathrm{j}\alpha}, \underline{Y} = Ye^{\mathrm{j}\beta}$

$$\underline{X} \cdot \underline{Y} = X \cdot Ye^{\mathrm{j}(\alpha+\beta)}$$

Kompleksni brojevi, konjugacija

$$\underline{X} = (a + \mathrm{j}b) \rightarrow \underline{X}^* = (a - \mathrm{j}b)$$

$$\underline{X} = Xe^{\mathrm{j}\alpha} \rightarrow \underline{X}^* = Xe^{-\mathrm{j}\alpha}$$

$$X = \sqrt{a^2 + b^2}$$

$$\underline{XX}^* = a^2 + b^2$$

$$\underline{XX}^* = X^2$$

Primer

$$\underline{X} = 6 - \mathrm{j}8 \quad \rightarrow \quad \underline{X}^* = 6 + \mathrm{j}8$$

$$\underline{X} = 10e^{-\mathrm{j}53^\circ} \rightarrow \underline{X}^* = 10e^{\mathrm{j}53^\circ}$$

$$\begin{aligned}\underline{XX}^* &= (6 - \mathrm{j}8)(6 + \mathrm{j}8) \\ &= 36 - \mathrm{j}48 + \mathrm{j}48 - \mathrm{j}^2 64 \\ &= 36 + 64 = 100\end{aligned}$$

$$\begin{aligned}\underline{XX}^* &= 10e^{-\mathrm{j}53^\circ} 10e^{\mathrm{j}53^\circ} \\ &= 100e^{\mathrm{j}0^\circ} = 100\end{aligned}$$

Kompleksni brojevi, deljenje

$\underline{X} = a + \mathrm{j}b, \quad \underline{Y} = c + \mathrm{j}d$

$$\boxed{\frac{\underline{X}}{\underline{Y}} = \frac{\underline{X}}{\underline{Y}} \cdot \frac{\underline{Y}^*}{\underline{Y}^*} = \frac{(a + \mathrm{j}b) \cdot (c - \mathrm{j}d)}{(c + \mathrm{j}d) \cdot (c - \mathrm{j}d)}}$$

$$\boxed{\frac{\underline{X}}{\underline{Y}} = \frac{e + \mathrm{j}f}{g} = \frac{e}{g} + \mathrm{j}\frac{f}{g}}$$

Primer

$\underline{X} = 2 + \mathrm{j}4, \quad \underline{Y} = 6 - \mathrm{j}8$

$$\frac{\underline{X}}{\underline{Y}} = \frac{(2 + \mathrm{j}4) \cdot (6 + \mathrm{j}8)}{(6 - \mathrm{j}8) \cdot (6 + \mathrm{j}8)} =$$

$$= \frac{12 + \mathrm{j}16 + \mathrm{j}24 - 32}{36 + 64} =$$

$$= \frac{-20 + \mathrm{j}40}{100} = -0{,}2 + \mathrm{j}0{,}4$$

Deljenje u eksponencijalnom obliku.

$\underline{X} = Xe^{j\alpha}, \underline{Y} = Ye^{j\beta}$

$$\boxed{\underline{X}/\underline{Y} = \frac{X}{Y} e^{\mathrm{j}(\alpha - \beta)}}$$

Kompleksni predstavnik pp veličine

- PP veličina: $u(t) = U_m \cos(\omega t + \theta) = \sqrt{2}U \cos(\omega t + \theta)$
  - Efektivna vrednost: U
  - Početna faza: θ
- Kompleksni predstavnik: $\underline{U} = U e^{\mathrm{j}\theta}$
  - Moduo: U
  - Argument: θ
- Izražava se u istim jedinicama kao i pp. veličina koju predstavlja

PRIMER

$u(t) = 28{,}2 cos(\omega t - \pi/3)\ \mathrm{V}$

$U_m = 28{,}2\mathrm{V},\ U = \frac{28{,}2}{\sqrt{2}} = 20\mathrm{V}$

$\theta = -\pi/3, \quad \underline{U} = 20e^{-\mathrm{j}\pi/3}\mathrm{V}$

PRIMER

$i(t) = 10\sqrt{2} cos(\omega t + \pi/4)\mathrm{A}$

$I_m = 10\sqrt{2}\mathrm{A} \qquad I = 10\mathrm{A}$

$\psi = \pi/4 \qquad \underline{I} = 10e^{\mathrm{j}\pi/4}\mathrm{A}$

Kompleksni predstavnik pp veličine

Napon, jačina struje i elektromotorna sila naponskog generatora.

$$u(t) = \sqrt{2}U cos(\omega t + \theta)$$

$$i(t) = \sqrt{2}I cos(\omega t + \psi)$$

$$e(t) = \sqrt{2}E cos(\omega t + \theta)$$

$$\underline{U} = U e^{\mathrm{j}\theta}$$

$$\underline{I} = I e^{\mathrm{j}\psi}$$

$$\underline{E} = E e^{\mathrm{j}\theta}$$

$$\underline{U} = U\mathrm{cos}\theta + \mathrm{j}U sin\theta$$

$$\underline{I} = I\mathrm{cos}\psi + \mathrm{j}I sin\psi$$

$$\underline{E} = E\mathrm{cos}\theta + \mathrm{j}E sin\theta$$

PRIMER

$$u(t) = 28{,}2 cos(\omega t - \pi/3)\ \mathrm{V}$$

$$\underline{U} = 20e^{-\mathrm{j}\pi/3}\ \mathrm{V} =$$
$$= \left(20\cos\left(-\frac{\pi}{3}\right) + \mathrm{j}20\sin\left(-\frac{\pi}{3}\right)\right)\ \mathrm{V}$$

$$\underline{U} = \left(10 - \mathrm{j}10\sqrt{3}\right)\ \mathrm{V}$$

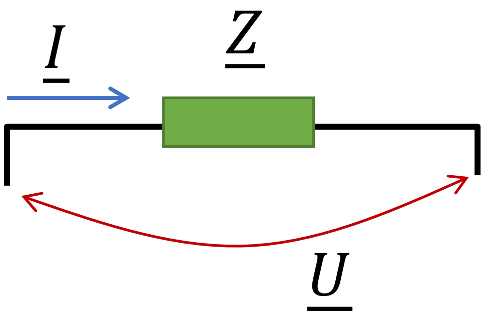
PRIMER

$$i(t) = 10\sqrt{2} cos(\omega t + \pi/4)\mathrm{A}$$

$$\underline{I} = 10e^{\mathrm{j}\pi/4}\mathrm{A} =$$
$$= \left(10\cos\left(\frac{\pi}{4}\right) + \mathrm{j}10\sin\left(\frac{\pi}{4}\right)\right)\ \mathrm{A}$$

$$\underline{I} = \left(5\sqrt{2} + \mathrm{j}5\sqrt{2}\right)\ \mathrm{A}$$

Impedansa prijemnika, $\underline{Z}$



Prelaskom u kompleksni domen koristimo kompleksne predstavnike napona i struje.

Količnik kompleksnog napona i kompleksne struje predstavlja IMPEDANSU prijemnika. Jedinica za impedansu je om (Ω).

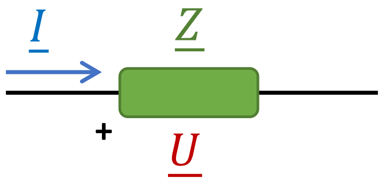
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} \quad [\Omega]$$

Ponekad se umesto impedanse koristi ADMITANSA. To je količnik kompleksne struje i kompleksnog napona. Jedinica za admitansu je simens (S).

$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{1}{\underline{Z}} \quad [S]$$

Impedansa prijemnika

Svaki prijemnik ima neku impedansu.
To je kompleksni broj.



$$\underline{Z} = \frac{\underline{U}}{\underline{I}} \quad [\Omega]$$

Može da se zapiše na dva načina:

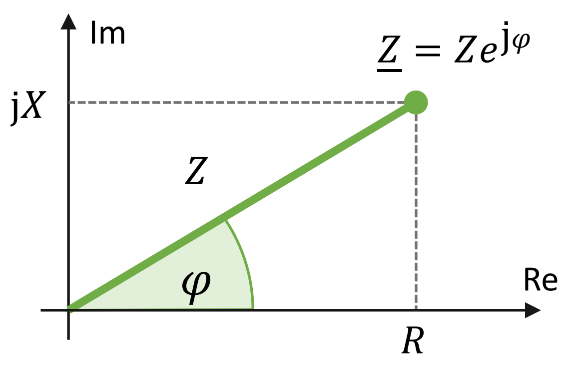
1. Iz eksponencijanog oblika: $\underline{Z} = \dfrac{Ue^{\mathrm{j}\theta}}{Ie^{\mathrm{j}\psi}} = \dfrac{U}{I}e^{\mathrm{j}(\theta-\psi)} = Ze^{\mathrm{j}\varphi}$ vidimo da je:

Moduo impedanse $Z = \dfrac{U}{I}$ Argument impedanse $\varphi = \theta - \psi$

2. Algebarski oblik: $\underline{Z} = R + \mathrm{j}X$ pri čemu je:

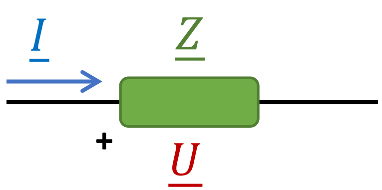
$R = Z\cos(\varphi)$ R - Rezistansa

$X = Zsin(\varphi)$ X - Reaktansa



Primer

Ako su vremenski oblici napona i struje prijemnika dati izrazima:
$u(t) = 28{,}2\cos(\omega t)$ V i $i(t) = 10\sqrt{2}\cos(\omega t + \pi/4)$ A odrediti impedansu prijemnika.



$$\underline{U} = 20e^{\mathrm{j}0} \qquad \underline{I} = 10e^{\mathrm{j}\frac{\pi}{4}}$$

$$\underline{Z} = \frac{Ue^{\mathrm{j}\theta}}{Ie^{\mathrm{j}\psi}} = \frac{U}{I}e^{\mathrm{j}(\theta-\psi)} = \frac{20}{10}e^{\mathrm{j}\left(0-\frac{\pi}{4}\right)} = 2e^{-\mathrm{j}\frac{\pi}{4}}\Omega$$

$$R = Z\cos(\varphi) = 2\cos\left(-\tfrac{\pi}{4}\right) = \sqrt{2}$$

$$X = Z\sin(\varphi) = 2\sin\left(-\tfrac{\pi}{4}\right) = -\sqrt{2} \qquad \underline{Z} = \left(\sqrt{2} - \mathrm{j}\sqrt{2}\right)\Omega$$

Impedanse osnovnih prijemnika

$\underline{Z} = R + \mathrm{j}X$

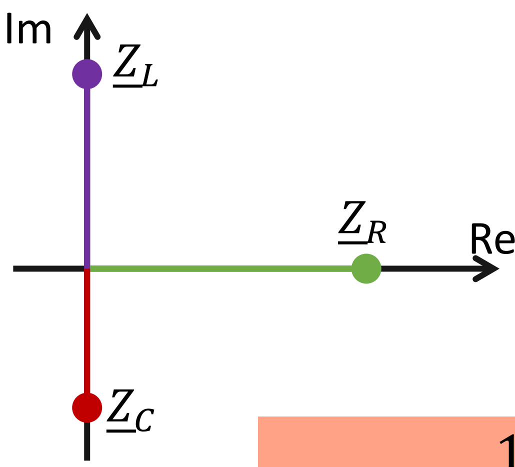
$$X_L = \omega L$$

X_L - reaktansa kalema

Otpornik $\quad \boxed{\underline{Z}_R = R}$

Kalem $\quad \boxed{\underline{Z}_L = \mathrm{j}\omega L}$

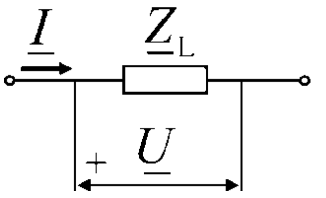
Kondenzator $\quad \boxed{\underline{Z}_C = -\mathrm{j}\frac{1}{\omega C}}$



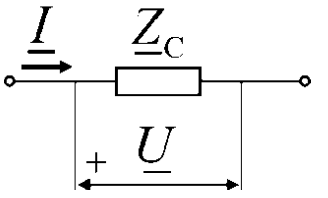
$$X_C = -\frac{1}{\omega C}$$

X_C - reaktansa kondenzatora

Primer



- $L = 15\text{mH}, \omega = 2000\text{rad/s}, \underline{Z}_L = ?$
- $C = 10\mu\text{F}, \omega = 2000\text{rad/s}, \underline{Z}_C = ?$

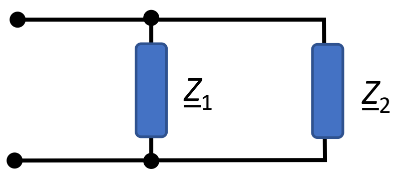


$$\underline{Z}_L = \mathrm{j}\omega L = \mathrm{j}2000 \cdot 15 \cdot 10^{-3} = \mathrm{j}30\Omega$$

$$\underline{Z}_C = -\mathrm{j}\frac{1}{\omega C} = -\mathrm{j}\frac{1}{2000 \cdot 10 \cdot 10^{-6}} = -\mathrm{j}\frac{10^6}{2 \cdot 10^4} = -\mathrm{j}50\Omega$$

Veze impedansi

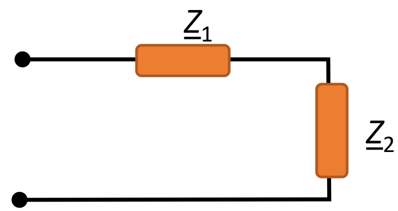
Paralelna veza



$$\frac{1}{\underline{Z}_e} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}$$

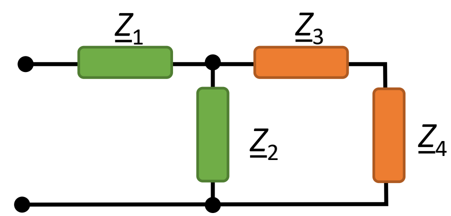
$$\underline{Z}_e = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

Redna veza

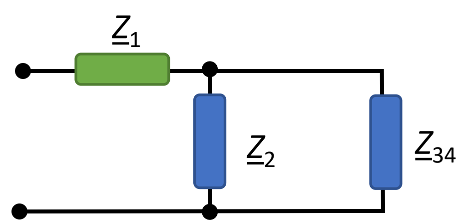


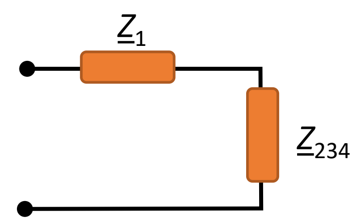
$$\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2$$

Mešovita veza



postepeno zamenjivanje rednih i paralelnih veza ekvivalentnim impedansama





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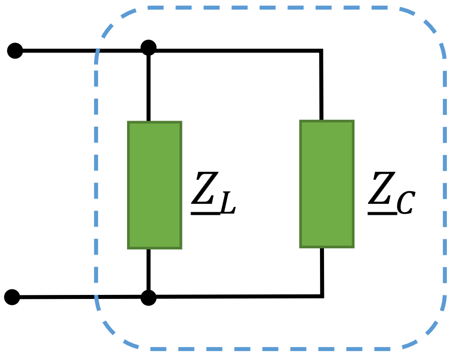


Odrediti ekvivalentnu impedansu ako je:

a) $\underline{Z}_L = \mathrm{j}20\Omega$, $\underline{Z}_C = -\mathrm{j}40\Omega$

b) $\underline{Z}_L = \mathrm{j}40\Omega$, $\underline{Z}_C = -\mathrm{j}20\Omega$

a) $$\underline{Z}_e = \frac{\underline{Z}_L \underline{Z}_C}{\underline{Z}_L + \underline{Z}_C}$$

$$\underline{Z}_e = \frac{\mathrm{j}20(-\mathrm{j}40)}{\mathrm{j}20 + (-\mathrm{j}40)}$$

$$\underline{Z}_e = \frac{800}{-\mathrm{j}20} = \mathrm{j}40\Omega$$

b) $$\underline{Z}_e = \frac{\underline{Z}_L \underline{Z}_C}{\underline{Z}_L + \underline{Z}_C} = -\mathrm{j}40\Omega$$

$$\underline{Z}_e = \frac{\mathrm{j}40(-\mathrm{j}20)}{\mathrm{j}40 + (-\mathrm{j}20)}$$

$$\underline{Z}_e = \frac{800}{\mathrm{j}20} = -\mathrm{j}40\Omega$$

Primer redne veze impedansi

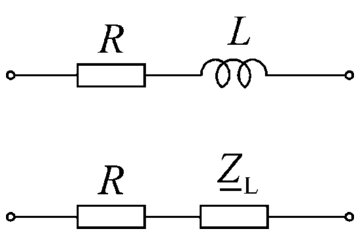
Potrošač modelovan kao redna veza otpornika otpornosti $50\,\Omega$ i kalema induktivnosti $100\,\mathrm{mH}$ priključen je na napon efektivne vrednosti $230\,\mathrm{V}$ i frekvencije $50\,\mathrm{Hz}$.

Izračunati:

a) kompleksnu impedansu potrošača,

b) moduo impedanse potrošača i

c) argument ove impedanse.



Rešenje.

a) Kružna frekvencija: $\omega = 2\pi f = 2\pi \cdot 50 = 314\,\mathrm{rad/s}$.

Impedansa kalema: $\underline{Z}_{\mathrm{L}} = \mathrm{j}\omega L = \mathrm{j}314 \cdot 0{,}1 = \mathrm{j}31{,}4\,\Omega$.

Impedansa potrošača: $\underline{Z} = R + \underline{Z}_{\mathrm{L}} = (50 + \mathrm{j}31{,}4)\,\Omega$.

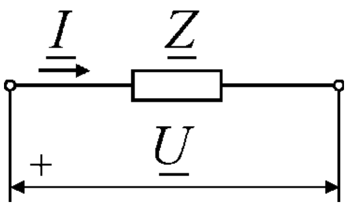
b) Moduo impedance: $Z = \sqrt{Z_{\mathrm{Re}}^2 + Z_{\mathrm{Im}}^2} = \sqrt{50^2 + 31{,}4^2} = 59\,\Omega$.

c) Argument impedance: $\varphi = \mathrm{arctg}\dfrac{Z_{\mathrm{Im}}}{Z_{\mathrm{Re}}} = \mathrm{arctg}\dfrac{31{,}4}{50} = 0{,}56$.

Primer

Kroz prijemnik impedanse $\underline{Z} = (3 + \mathrm{j}4)\,\Omega$ postoji kompleksna struja $\underline{I} = (10 - \mathrm{j}8)\,\mathrm{A}$. Kružna frekvencija je $\omega = 120\pi\,\mathrm{rad/s}$. Odrediti:

a) kompleksni napon na prijemniku,
b) efektivnu i maksimalnu vrednost ovog napona,
c) argument napona,
d) napisati vremenski oblik napona na prijemniku.



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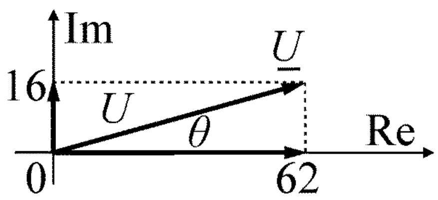
a) Kompleksni napon: $\underline{U} = \underline{Z}\,\underline{I} = (3 + \mathrm{j}4)(10 - \mathrm{j}8) = (62 + \mathrm{j}16)\,\mathrm{V}$.

b) Efektivna vrednost: $U = \sqrt{U_{\mathrm{Re}}^2 + U_{\mathrm{Im}}^2} = \sqrt{62^2 + 16^2} = 64\,\mathrm{V}$.

Maksimalna vrednost: $U_{\mathrm{m}} = \sqrt{2}\,U = 64\sqrt{2} = 90{,}5\,\mathrm{V}$.

c) Argument: $\theta = \mathrm{arctg}\dfrac{U_{\mathrm{Im}}}{U_{\mathrm{Re}}} = \mathrm{arctg}\dfrac{16}{62} = 0{,}25$.



d) Standardni oblik (vremenski domen): $u(t) = U_{\mathrm{m}}\cos(\omega t + \theta)$.

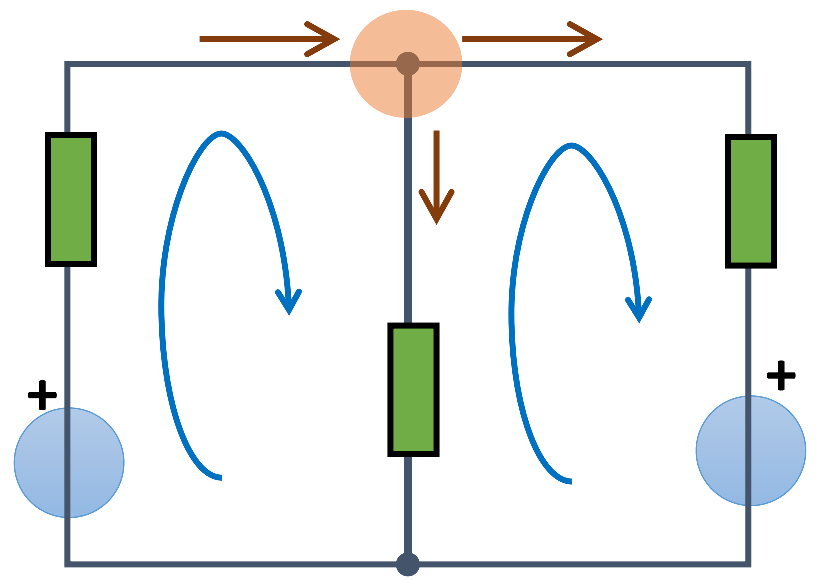
Zamenom $U_{\mathrm{m}} = 90{,}5\,\mathrm{V}$, $\omega = 120\pi\,\mathrm{rad/s}$ i $\theta = 0{,}25$:

$u(t) = 90{,}5\cos(120\pi t + 0{,}25)\,\mathrm{V}$.

Rešavanje kola u kompleksnom domenu

- Kirhofovi zakoni:
  - pravila o predznacima su ista kao u kolima VKS.

$$\sum \underline{U} = 0 \qquad \sum \underline{I} = 0$$



- Za kolo sa $n_č$ čvorova i n_g grana:
  - ukupan broj jednačina je n_g
  - od toga $(n_č - 1)$ po I KZ, i
  - $n_g - (n_č - 1)$ po II KZ

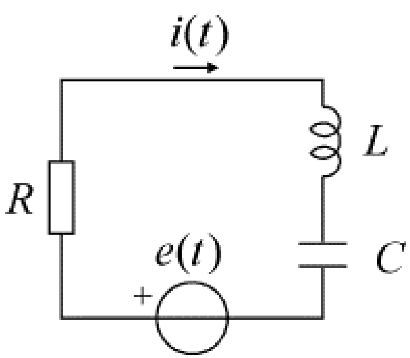
Postupak rešavanja

Vremenski domen

Zadato:

R, L, C

$e(t) = \sqrt{2}E\cos(\omega t + \theta)$



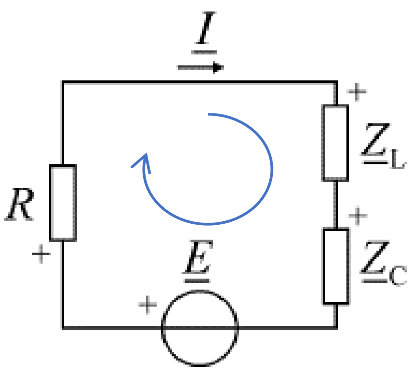
Kompleksni domen

Računa se:

$\underline{Z}_L = \mathrm{j}\omega L,$

$\underline{Z}_C = -\mathrm{j}\dfrac{1}{\omega C},$

$\underline{E} = Ee^{j\theta}$

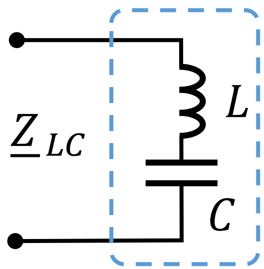


II KZ: $-\underline{E} + R\underline{I} + \underline{Z}_L\underline{I} + \underline{Z}_C\underline{I} = 0$

$$\underline{I} = \frac{\underline{E}}{R + \underline{Z}_L + \underline{Z}_C}$$

Rezonancija

Ekvivalentna impedansa može biti jednaka nuli. Najjednostavniji primer: redna LC veza.



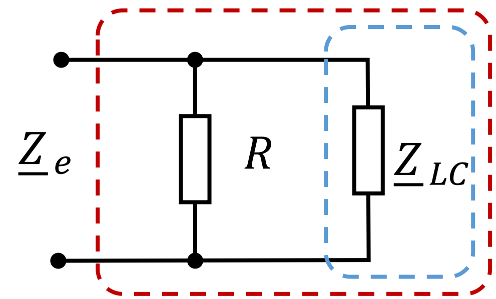
$$\underline{Z}_{LC} = \underline{Z}_L + \underline{Z}_C = 0$$

$$\underline{Z}_L = -\underline{Z}_C$$

Rezonantna kružna učestanost

$$\omega = \frac{1}{\sqrt{LC}}$$

Primer

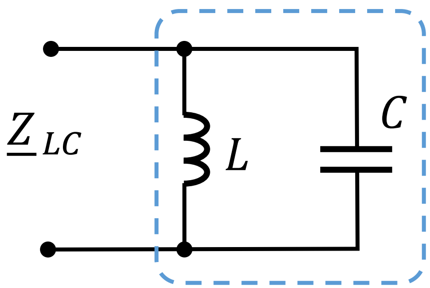


$$\underline{Z}_e = \frac{R \cdot \underline{Z}_{LC}}{R + \underline{Z}_{LC}} = \frac{R \cdot 0}{R + 0} = 0$$

U rezonanciji je zbir napona na kalemu i kondenzatoru jednak nuli iako svaki od napona može biti velik.

Antirezonancija

Ekvivalentna impedansa može biti beskonačna. Najjednostavniji primer: paralelna LC veza.

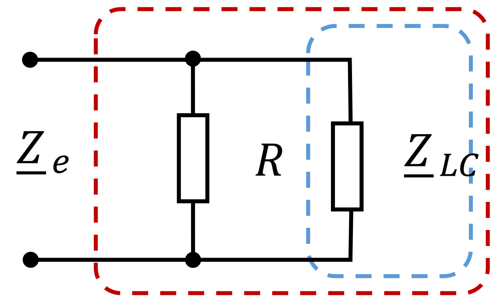


$$\underline{Z}_{LC} = \frac{\underline{Z}_L \underline{Z}_C}{\underline{Z}_L + \underline{Z}_C} = \infty$$

$$\underline{Z}_L = -\underline{Z}_C$$

Antirezonantna kružna učestanost $\omega = \frac{1}{\sqrt{LC}}$

Primer



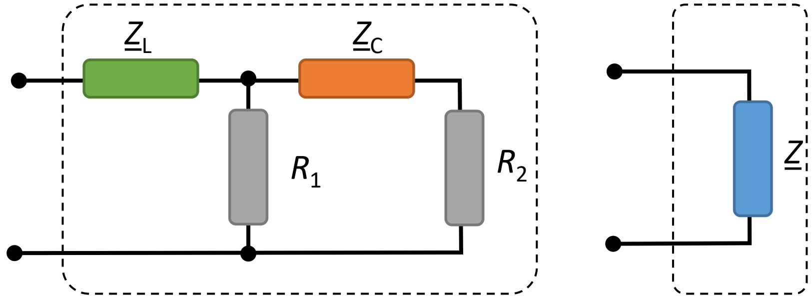
$$\underline{Z}_e = \frac{R \cdot \underline{Z}_{LC}}{R + \underline{Z}_{LC}} = \frac{R \cdot \infty}{R + \infty} = R$$

U antirezonanciji je zbir struja kalema i kondenzatora jednak nuli iako svaka od struja može biti velika.

Rezistansa i reaktansa

Svaki prijemnik ima neku impedansu. Neka je ekvivalentna impedansa prijemnika $\underline{Z}$.

Realni deo impedanse se zove REZISTANSA, a imaginarni deo REAKTANSA.



$$\underline{Z} = R + \mathrm{j}X$$

Rezistansa: $R = \mathrm{Re}\{\underline{Z}\}\ [\Omega]$

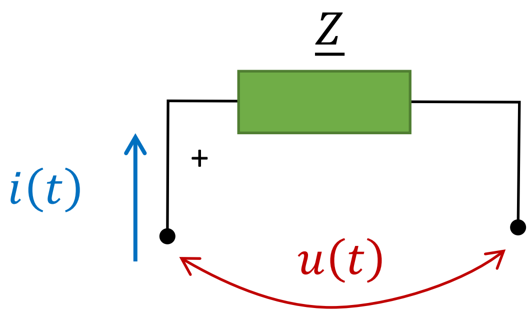
Reaktansa: $X = \mathrm{Im}\{\underline{Z}\}\ [\Omega]$

$X > 0$ prijemnik induktivnog karaktera Na primer $\underline{Z} = (10 + \mathrm{j}20)\Omega$

$X < 0$ prijemnik kapacitivnog karaktera Na primer $\underline{Z} = (30 - \mathrm{j}40)\Omega$

Snaga prijemnika

Posmatrajmo vremenski oblik napona
i struje prijemnika impedanse Z.



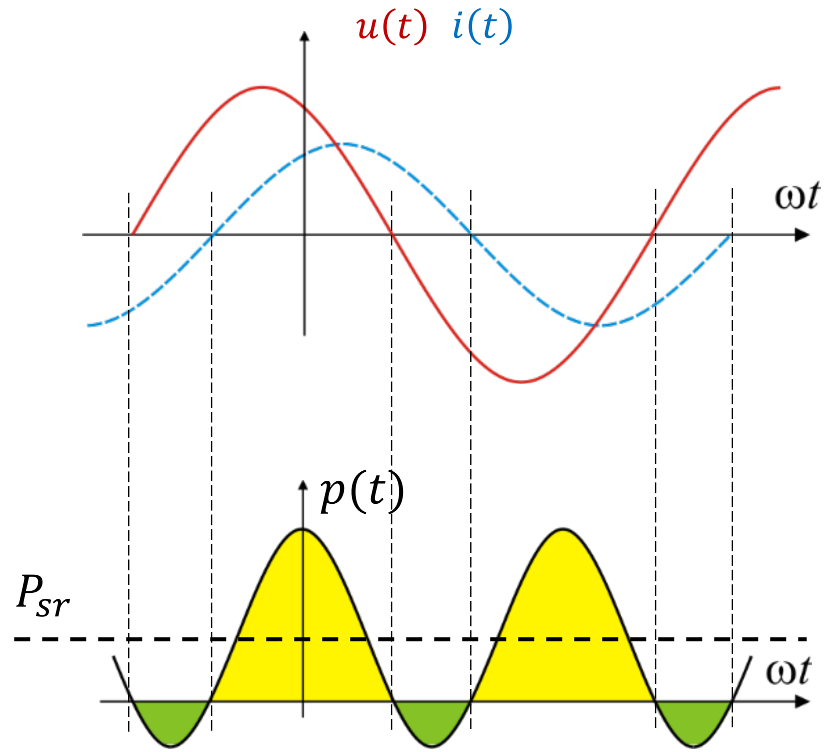
$$u(t) = \sqrt{2}Ucos(\omega t + \theta)$$

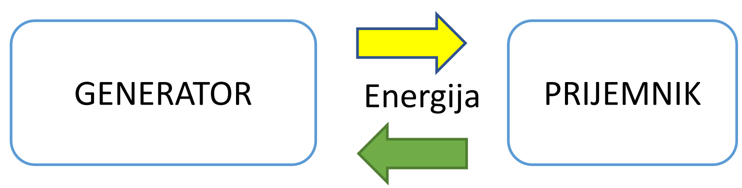
$$i(t) = \sqrt{2}Icos(\omega t + \psi)$$

Trenutna snaga: $p(t) = u(t)i(t)$

Srednja snaga: $P_{sr} = UIcos\varphi$ [W]

Zavisi od U, I, i fazne razlike: $\varphi = \theta - \psi$





Kada je $p(t) > 0$ energija ide prema prijemniku

Kada je $p(t) < 0$ energija ide prema generatoru.

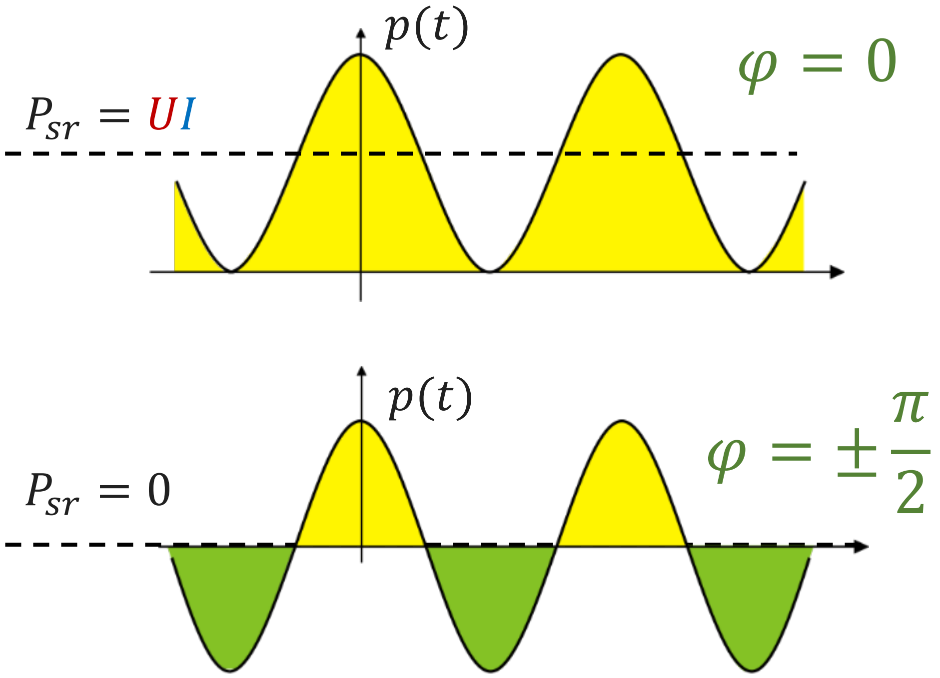
Srednja (aktivna) snaga prijemnika

Srednja (aktivna) snaga:

$$P = UIcos\varphi \text{ [W]}$$

Zavisi od fazne razlike, φ

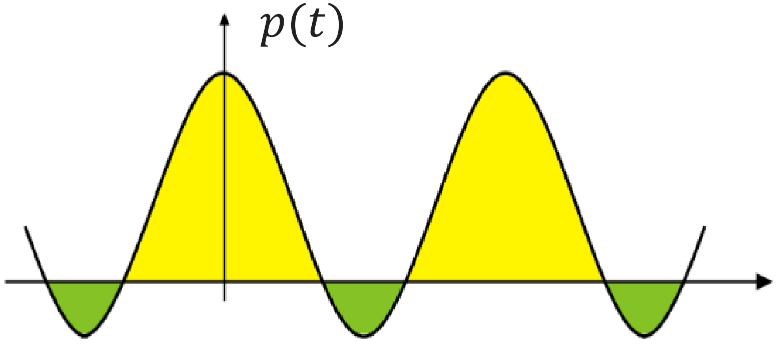
- Na otporniku, $\varphi = 0$
- Na kalemu, $\varphi = \frac{\pi}{2}$
- Na kondenzatoru, $\varphi = -\frac{\pi}{2}$



Snaga prijemnika

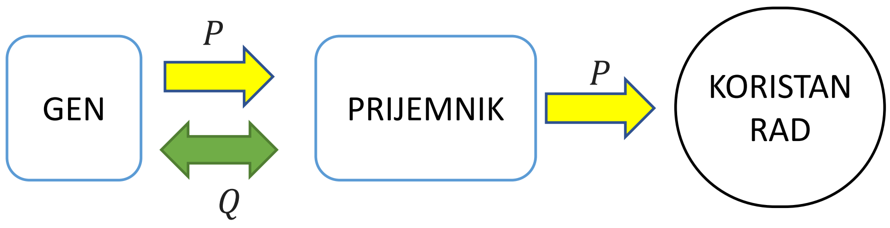
Aktivna snaga: $P = UIcos\varphi$ [W]

Reaktivna snaga: $Q = UIsin\varphi$ [var]



Reaktivna snaga se periodično razmenjuje između generatora i prijemnika.

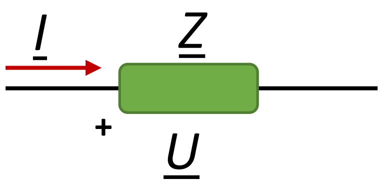
Prividna snaga: $S = UI$ [VA]



Faktor snage: $cos\varphi = \frac{P}{S}$

Odnos Aktivne i Prividne snage.

Kompleksna snaga prijemnika, $\underline{S}$



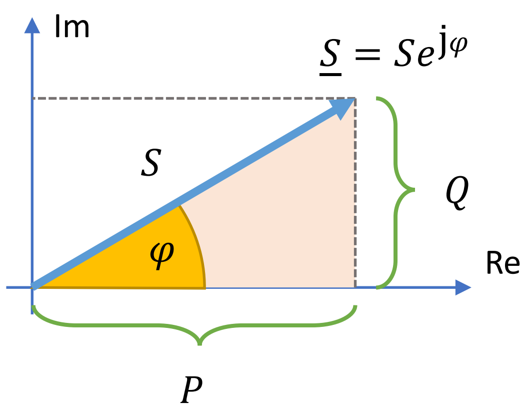
$$\underline{U} = \underline{Z}\,\underline{I} \qquad \underline{S} = \underline{U}\,\underline{I}^*$$

Kompleksna snaga prijemnika je proizvod kompleksnog napona i konjugovano kompleksne struje prijemnika.

To je kompleksan broj koji može da se zapiše u oba oblika:

$\underline{S} = P + \mathrm{j}Q\ [\mathrm{VA}]$ Algebarski oblik

$\underline{S} = Se^{\mathrm{j}\varphi}\ [\mathrm{VA}]$ Eksponencijalni oblik



Aktiva snaga: $P[W] = \mathrm{Re}\{\underline{S}\}$

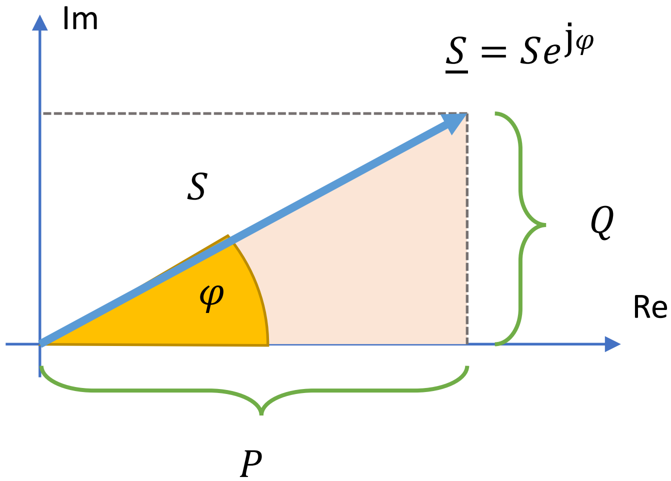
Reaktivna snaga: $Q[var] = \mathrm{Im}\{\underline{S}\}$

Prividna snaga: $S\ [VA]$

$$S = \sqrt{P^2 + Q^2}\ [VA]$$

Kompleksna snaga prijemnika, S̲

Prebacivanje iz jednog oblika u drugi oblik



$S, \varphi \Rightarrow P, Q$

$$P = S\cos\varphi \;\; [\mathrm{W}]$$

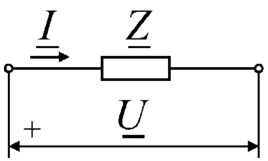
$$Q = S\sin\varphi \;\; [\mathrm{var}]$$

$P, Q \Rightarrow S, \varphi$

$$S = \sqrt{P^2 + Q^2} \; [\mathrm{VA}]$$

$$\varphi = \mathrm{arctg}\left(\frac{Q}{P}\right)$$

Snage – primer 1



Kroz prijemnik impedanse $\underline{Z} = (10 - \mathrm{j}5)\,\Omega$ postoji struja $\underline{I} = (4 + \mathrm{j}3)\,\mathrm{A}$.

Odrediti njegovu
a) kompleksnu snagu,
b) aktivnu snagu,
c) reaktivnu snagu,
d) prividnu snagu,
e) faktor snage.

a) $\underline{U} = \underline{Z}\,\underline{I} = (10 - j5)(4 + j3) = (55 + j10)\ \mathrm{V}$

$\underline{S} = \underline{U}\,\underline{I}^* = (55 + j10)(4 - j3) = (250 - j125)\ \mathrm{VA}$

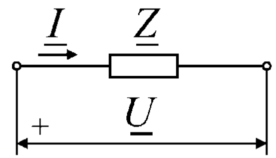
b) $P = \mathrm{Re}\{\underline{S}\} = 250\ \mathrm{W}$

c) $Q = \mathrm{Im}\{\underline{S}\} = -125\ \mathrm{var}$

d) $S = \sqrt{P^2 + Q^2} = \sqrt{250^2 + 125^2} = 280\ \mathrm{var}$

e) $cos\varphi = P/S = 250/280 = 0{,}9$

Snage – primer 2



Kroz prijemnik impedanse $\underline{Z} = (20 + \mathrm{j}10)\,\Omega$ postoji struja $\underline{I} = (4 + \mathrm{j}3)\,\mathrm{A}$.

Odrediti njegovu

a) kompleksnu snagu,

b) aktivnu snagu,

c) reaktivnu snagu,

d) prividnu snagu,

e) faktor snage.

a) $\underline{U} = \underline{Z}\,\underline{I} = (20 + \mathrm{j}10)(4 + \mathrm{j}3) = (50 + \mathrm{j}100)\ \mathrm{V}$

$\underline{S} = \underline{U}\,\underline{I}^{*} = (50 + \mathrm{j}100)(4 - \mathrm{j}3) = (500 + \mathrm{j}250)\ \mathrm{VA}$

b) $P = \mathrm{Re}\{\underline{S}\} = 500\ \mathrm{W}$

c) $Q = \mathrm{Im}\{\underline{S}\} = 250\ \mathrm{var}$

d) $S = \sqrt{P^2 + Q^2} = \sqrt{500^2 + 250^2} = 559\ \mathrm{var}$

e) $cos\varphi = P/S = 500/559 = 0{,}9$

Pitanja

1. Ako su X i Y realni i imaginarni deo kompleksnog broja, koliki je moduo tog broja?
2. Da li realni deo kompleksnog broja može da bude negativan?
3. Da li moduo kompleksnog broja može da bude negativan?
4. Šta je impedansa?
5. Ako su efektivne vrednosti napona i jačine struje, 220V i 2A, koliki je moduo impedanse?
6. Šta predstavlja argument impedanse u odnosu na početne faze napona i jačine struje?
7. Ako je početna faza jačine struje kroz kalem $\psi = \pi/2$ koliko iznosi početna faza napona?
8. Kada je impedansa prijemnika pretežno induktivna?
9. Šta je rezistana a šta reaktansa prijemnika? U kojim jedinicama se izražavaju?
10. Od čega zavisi srednja snaga prijemnika? Da li ova snaga može da bude negativna?
11. Koliko iznosi srednja snaga kalema?
12. Da li je aktivna snaga isto što i srednja snaga?
13. Šta je kompleksna snaga? Šta je aktivna, a šta reaktivna snaga?
14. Ako je aktivna snaga 80 W a reaktivna 60 var, koliko iznosi prividna snaga?