STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET

Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

# Osnovi elektrotehnike 1 (I kolokvijum)

**A**

07.12.2019.

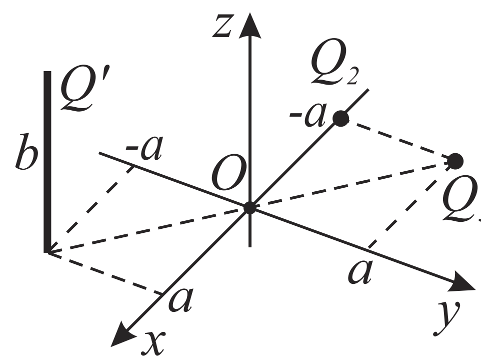
## ZADACI

**Zadatak 1.** Tanak, prav, štap dužine $b$, naelektrisan ravnomerno podužnom količinom naelektrisanja $Q'$, postavljen je upravno na $x$-$y$ ravan zadatog Dekartovog koordinatnog sistema. Jedan kraj štapa leži u $x$-$y$ ravni, na simetrali četvrtog kvadranta, kao na slici 1. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja, koji u tački $O$ (centar koordinatnog sistema) stvara štap.

b) Izračunati količine tačkastih naelektrisanja, $Q_2$ i $Q_3$, tako da rezultantni vektor jačine električnog polja u tački $O$ ima samo $z$ komponentu. Tačkasto naelektrisanje $Q_2$ se nalazi na $x$ osi, dok je tačkasto naelektrisanje $Q_3$ na simetrali drugog kvadranta $x$-$y$ ravni.

Brojni podaci su: $a = 1\ cm$, $b = a$, $Q' = 10\ \mu C/m$, $\varepsilon_0 = 8{,}85\cdot 10^{-12}\ F/m$.




Slika 1.

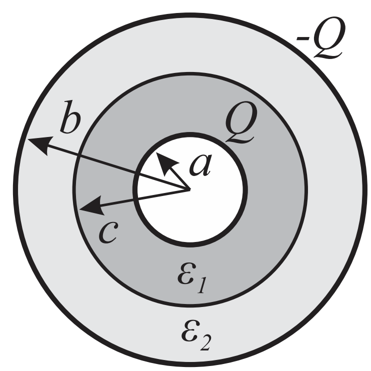
**Zadatak 2.** Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika, čvrstim permitivnosti $\varepsilon_1$ i tečnim permitivnosti $\varepsilon_2$. Poluprečnici elektroda kondenzatora su $a$ i $b$, dok je poluprečnik razdvojne površi dva dielektrika $c$. Kondenzator je kratkotrajno bio priključen na izvor napona, tako da je naelektrisan naelekrisanjem $Q$. Apsolutna vrednost površinske gustine vezanog naelektrisanja u drugom dielektriku, uz razdvojnu površ, je dva puta veća od apsolutne vrednosti površinske gustine vezanog naelektrisanja u prvom dielektriku, uz razdvojnu površ. Ako se kroz mali otvor na spoljašnjoj elektrodi ispusti tečni dielektrik, nakon odvajanja kondenzatora od izvora napona, jačina polja uz spoljašnju elektrodu poraste 4 puta.

a) Odrediti, u opštim brojevima, izraz za ekvivalentnu kapacitivnost kondenzatora.

b) Izračunati relativne permitivnosti dielektrika, $\varepsilon_{r1}$ i $\varepsilon_{r2}$.

c) Izračunati promenu kapacitivnosti kondenzatora koja nastaje nakon ispuštanja dielektrika permitivnosti $\varepsilon_2$.

Brojni podaci su: $a = 1\ mm$, $b = 3\ mm$, $c = 2\ mm$, $Q = 5\ nC$.




Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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# Osnovi elektrotehnike 1
# (I kolokvijum)

**B**

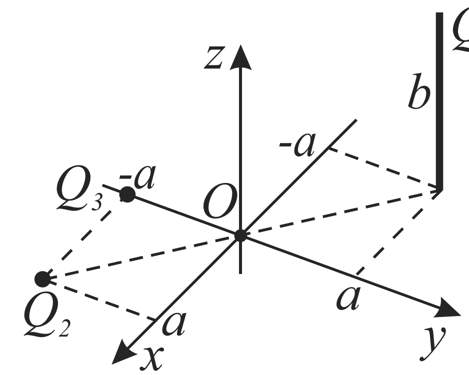
**07.12.2019.**

## ZADACI

**Zadatak 1.** Tanak, prav, štap dužine $b$, naelektrisan ravnomerno podužnom količinom naelektrisanja $Q'$, postavljen je upravno na $x$-$y$ ravan zadatog Dekartovog koordinatnog sistema. Jedan kraj štapa leži u $x$-$y$ ravni, na simetrali drugog kvadranta, kao na slici 1. Sredina je vazduh.

a) Izvesti, u opštim brojevima, izraz za vektor jačine električnog polja, koji u tački $O$ (centar koordinatnog sistema) stvara štap.
b) Izračunati količine tačkastih naelektrisanja, $Q_2$ i $Q_3$, tako da rezultantni vektor jačine električnog polja u tački $O$ ima samo $z$ komponentu. Tačkasto naelektrisanje $Q_2$ se nalazi na simetrali četvrtog kvadranta $x$-$y$ ravni, dok je tačkasto naelektrisanje $Q_3$ na $y$ osi.

Brojni podaci su: $a = 1{,}5\ cm$, $b = a$, $Q' = 20\ \mu C/m$, $\varepsilon_0 = 8{,}85\cdot 10^{-12}\ F/m$.

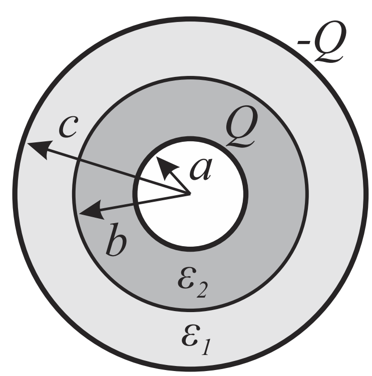



Slika 1.

**Zadatak 2.** Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika, tečnim permitivnosti $\varepsilon_1$ i čvrstim permitivnosti $\varepsilon_2$. Poluprečnici elektroda kondenzatora su $a$ i $c$, dok je poluprečnik razdvojne površi dva dielektrika $b$. Kondenzator je kratkotrajno bio priključen na izvor napona, tako da je naelektrisan naelekrisanjem $Q$. Apsolutna vrednost površinske gustine vezanog naelektrisanja u prvom dielektriku, uz razdvojnu površ, je tri puta veća od apsolutne vrednosti površinske gustine vezanog naelektrisanja u drugom dielektriku, uz razdvojnu površ. Ako se kroz mali otvor na spoljašnjoj elektrodi ispusti tečni dielektrik, nakon odvajanja kondenzatora od izvora napona, jačina polja uz spoljašnju elektrodu poraste 2 puta.

a) Odrediti, u opštim brojevima, izraz za ekvivalentnu kapacitivnost kondenzatora.
b) Izračunati relativne permitivnosti dielektrika, $\varepsilon_{r1}$ i $\varepsilon_{r2}$.
c) Izračunati promenu kapacitivnosti kondenzatora koja nastaje nakon ispuštanja dielektrika permitivnosti $\varepsilon_1$.

Brojni podaci su: $a = 1\ mm$, $b = 2\ mm$, $c = 3\ mm$, $Q = 1\ nC$.




Slika 2.

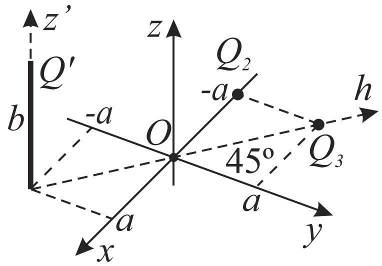
## PRAVILA POLAGANJA

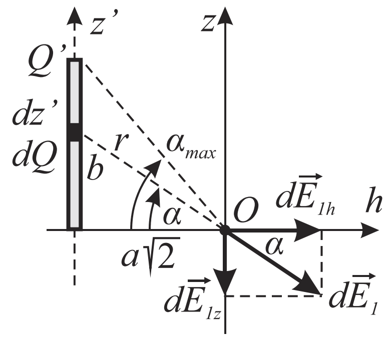
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

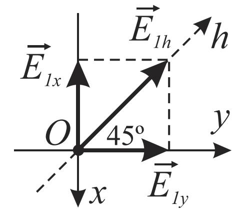
Prvi kolokvijum iz OET1, 07.12.2019.

**I-1** **A grupa**

a)




$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2} \qquad dE_{1h} = dE_1 \cos\alpha \qquad \left( dz' = \frac{r\,d\alpha}{\cos\alpha}, \qquad r = \frac{a\sqrt{2}}{\cos\alpha} \right)$$

$$dE_{1z} = dE_1 \sin\alpha$$

$$E_{1z} = \int\limits_{\text{duž štapa}} dE_{1z} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\sin\alpha = \int \frac{Q'\frac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\sin\alpha = \int \frac{Q'\frac{d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 \frac{a\sqrt{2}}{\cos\alpha}}\sin\alpha = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int_0^{\alpha_{\max}} \sin\alpha\, d\alpha$$

$$E_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\cos\alpha_{\max}\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{a\sqrt{2}}{\sqrt{b^2+2a^2}}\right) \qquad b = a \qquad \boxed{\vec{E}_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)\cdot\left(-\vec{i}_z\right)}$$

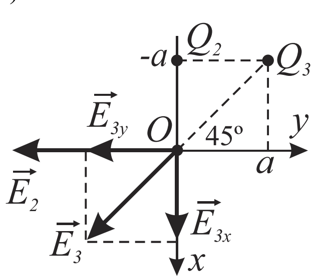
$$E_{1h} = \int\limits_{\text{duž štapa}} dE_{1h} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\cos\alpha = \int \frac{Q'\frac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = \int \frac{Q'\,d\alpha}{4\pi\varepsilon_0 \frac{a\sqrt{2}}{\cos\alpha}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int_0^{\alpha_{\max}} \cos\alpha\, d\alpha$$

$$E_{1h} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(\sin\alpha_{\max}-0\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{b}{\sqrt{b^2+2a^2}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}$$

$$E_{1x} = E_{1h}\sin 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1x} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\left(-\vec{i}_x\right)}$$

$$E_{1y} = E_{1h}\cos 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1y} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\vec{i}_y}$$

b)

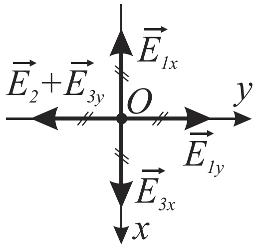



$$E_2 = \frac{Q_2}{4\pi\varepsilon_0 a^2} \qquad \vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0 a^2}\cdot\vec{i}_x$$

$$E_3 = \frac{Q_3}{4\pi\varepsilon_0\left(a\sqrt{2}\right)^2} = \frac{Q_3}{8\pi\varepsilon_0 a^2}$$

$$E_{3x} = E_3\sin 45^\circ = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2} \qquad \vec{E}_{3x} = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\cdot\vec{i}_x$$

$$E_{3y} = E_3\cos 45^\circ = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2} \qquad \vec{E}_{3y} = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_y\right)$$




$$\vec{E}_{Ox} = 0 \qquad \Rightarrow \qquad E_{1x} = E_{3x}$$

$$\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$Q_3 = \frac{2a}{\sqrt{2}\cdot\sqrt{3}}Q'$$

$$\boxed{Q_3 = 81,6\,nC}$$

$$\vec{E}_{Oy} = 0 \qquad \Rightarrow \qquad E_{1y} = E_2 + E_{3y}$$

$$\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} = \frac{Q_2}{4\pi\varepsilon_0 a^2} + \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$\left(\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} = \frac{Q_3}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\right) \quad iz\ uslova\ \vec{E}_{Ox} = 0$$

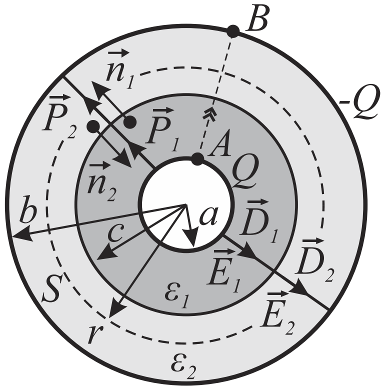
$$\Rightarrow \qquad \frac{Q_2}{4\pi\varepsilon_0 a^2} = 0$$

$$\boxed{Q_2 = 0\,C}$$

Prvi kolokvijum iz OET1, 07.12.2019.

**I-2** **A grupa**

a)




Granični uslov:

$D_{n1} = D_{n2} \qquad D_1 = D_2 = D$

$E_{t1} \neq E_{t2}$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_S D\, ds = Q$$

$$D\, 4r^2\pi = Q \qquad \boxed{D = \frac{Q}{4\pi r^2}}, \qquad a \le r \le b$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_1 r^2}}, \qquad a \le r \le c$$

$$\boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_2 r^2}}, \qquad c \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^c E_1\, dr + \int_c^b E_2\, dr = \int_a^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr + \int_c^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr = \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$U_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}}\frac{c-a}{a\,c} + \frac{1}{\varepsilon_{r2}}\frac{b-c}{b\,c}\right) \qquad \boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\frac{c-a}{a\,c} + \frac{1}{\varepsilon_{r2}}\frac{b-c}{b\,c}}}$$

b)

$Q = 5\ nC$

$$P = D - \varepsilon_0 E = D - \varepsilon_0 \frac{D}{\varepsilon} = \left(1 - \frac{1}{\varepsilon_r}\right) D$$

$$\sigma_{v1} = \vec{P}_1 \cdot \vec{n}_1 = P_1(r=c) = \left(1 - \frac{1}{\varepsilon_{r1}}\right) D(r=c) = \left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi c^2}$$

$$\sigma_{v2} = \vec{P}_2 \cdot \vec{n}_2 = -P_2(r=c) = -\left(1 - \frac{1}{\varepsilon_{r2}}\right) D(r=c) = -\left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi c^2}$$

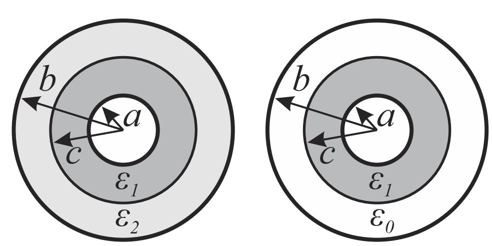
$$|\sigma_{v2}| = 2|\sigma_{v1}| \quad \Rightarrow \quad \left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi c^2} = 2\left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi c^2} \quad \Rightarrow \quad 1 - \frac{1}{\varepsilon_{r2}} = 2 - \frac{2}{\varepsilon_{r1}}$$

*Kada se ispusti tečni dielektrik* : $\varepsilon_2 \to \varepsilon_0 \qquad (Q = const.)$

$E_{novo}(r=b) = 4E(r=b)$

$$\frac{Q}{4\pi\varepsilon_0 b^2} = 4\frac{Q}{4\pi\varepsilon_2 b^2} \quad \Rightarrow \quad \varepsilon_2 = 4\varepsilon_0 \qquad \boxed{\varepsilon_{r2} = 4}$$

$$\varepsilon_{r1} = \frac{2}{\frac{1}{\varepsilon_{r2}} + 2 - 1} = \frac{2}{\frac{1}{4} + 2 - 1} = \frac{2}{\frac{5}{4}} = \frac{8}{5} \qquad \boxed{\varepsilon_{r1} = 1{,}6}$$



c)

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\frac{c-a}{a\,c}+\frac{1}{\varepsilon_{r2}}\frac{b-c}{b\,c}} = 0{,}35\ pF$$

$$Kada\ se\ ispusti\ te\check{c}ni\ dielektrik:\quad \varepsilon_2 \to \varepsilon_0 \quad (\varepsilon_{r2} \to 1)$$

$$C_{novo} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\frac{c-a}{a\,c}+\frac{1}{1}\cdot\frac{b-c}{b\,c}} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\frac{c-a}{a\,c}+\frac{b-c}{b\,c}} = 0{,}26\ pF$$

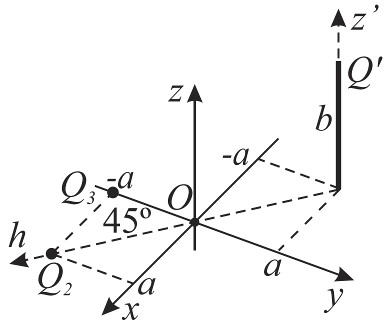
$$\Delta C = C_{novo} - C = 0{,}26\ pF - 0{,}35\ pF$$

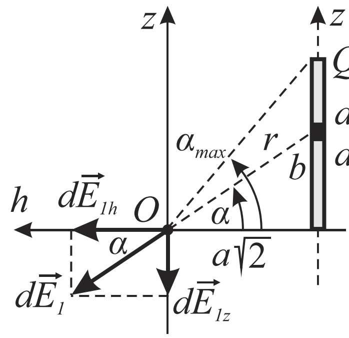
$$\boxed{\Delta C = -0{,}09\ pF}$$

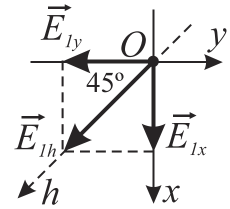
Prvi kolokvijum iz OET1, 07.12.2019.

**I-1** **B grupa**

a)




$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2} \qquad dE_{1h} = dE_1\cos\alpha \qquad \left( dz' = \frac{r\,d\alpha}{\cos\alpha}, \qquad r = \frac{a\sqrt{2}}{\cos\alpha} \right)$$

$$dE_{1z} = dE_1 \sin\alpha$$

$$E_{1z} = \int\limits_{\text{duž štapa}} dE_{1z} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\sin\alpha = \int \frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\sin\alpha = \int \frac{Q'\dfrac{d\alpha}{\cos\alpha}}{4\pi\varepsilon_0\dfrac{a\sqrt{2}}{\cos\alpha}}\sin\alpha = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int\limits_0^{\alpha_{\max}} \sin\alpha\,d\alpha$$

$$E_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\cos\alpha_{\max}\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{a\sqrt{2}}{\sqrt{b^2+2a^2}}\right) \qquad b = a \qquad \boxed{\vec{E}_{1z} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)\cdot\left(-\vec{i}_z\right)}$$

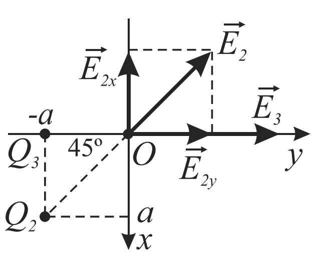
$$E_{1h} = \int\limits_{\text{duž štapa}} dE_{1h} = \int \frac{Q'\,dz'}{4\pi\varepsilon_0 r^2}\cos\alpha = \int \frac{Q'\dfrac{r\,d\alpha}{\cos\alpha}}{4\pi\varepsilon_0 r^2}\cos\alpha = \int \frac{Q'\,d\alpha}{4\pi\varepsilon_0\dfrac{a\sqrt{2}}{\cos\alpha}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\int\limits_0^{\alpha_{\max}} \cos\alpha\,d\alpha$$

$$E_{1h} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\left(\sin\alpha_{\max}-0\right) = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{b}{\sqrt{b^2+2a^2}} = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}$$

$$E_{1x} = E_{1h}\sin 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1x} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\vec{i}_x}$$

$$E_{1y} = E_{1h}\cos 45^\circ = \frac{Q'}{4\pi\varepsilon_0 a\sqrt{2}}\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{2} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}} \qquad \boxed{\vec{E}_{1y} = \frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}\cdot\left(-\vec{i}_y\right)}$$

b)

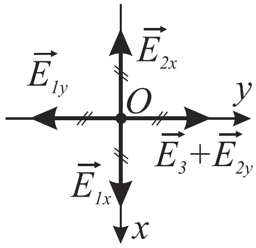



$$E_3 = \frac{Q_3}{4\pi\varepsilon_0 a^2} \qquad \vec{E}_3 = \frac{Q_3}{4\pi\varepsilon_0 a^2}\cdot\vec{i}_y$$

$$E_2 = \frac{Q_2}{4\pi\varepsilon_0\left(a\sqrt{2}\right)^2} = \frac{Q_2}{8\pi\varepsilon_0 a^2}$$

$$E_{2x} = E_2\sin 45^\circ = \frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2} \qquad \vec{E}_{2x} = \frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\cdot\left(-\vec{i}_x\right)$$

$$E_{2y} = E_2\cos 45^\circ = \frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2} \qquad \vec{E}_{2y} = \frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\cdot\vec{i}_y$$




$\vec{E}_{Ox}=0 \qquad \Rightarrow \qquad E_{1x}=E_{2x}$

$$\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}=\frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$Q_2=\frac{2a}{\sqrt{2}\cdot\sqrt{3}}Q'$$

$$\boxed{Q_2=244{,}9\ nC}$$

$\vec{E}_{Oy}=0 \qquad \Rightarrow \qquad E_{1y}=E_3+E_{2y}$

$$\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}=\frac{Q_3}{4\pi\varepsilon_0 a^2}+\frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$\left(\frac{Q'}{8\pi\varepsilon_0 a\sqrt{3}}=\frac{Q_2}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}\right)\quad \textit{iz uslova } \vec{E}_{Ox}=0$$

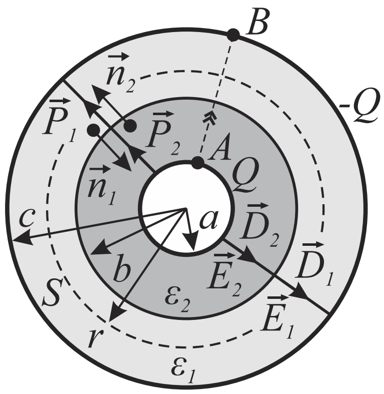
$$\Rightarrow \qquad \frac{Q_3}{4\pi\varepsilon_0 a^2}=0$$

$$\boxed{Q_3=0\ C}$$

Prvi kolokvijum iz OET1, 07.12.2019.

**I-2** **B grupa**

a)




Granični uslov:

$D_{n1} = D_{n2}$ $\qquad D_1 = D_2 = D$

$E_{t1} \neq E_{t2}$

$\oint_S \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S}$

$\int_S D\, ds = Q$

$D\, 4r^2\pi = Q$ $\qquad \boxed{D = \dfrac{Q}{4\pi r^2}}, \qquad a \le r \le c$

$\boxed{E_1 = \dfrac{D}{\varepsilon_1} = \dfrac{Q}{4\pi\varepsilon_1 r^2}}, \qquad b \le r \le c$

$\boxed{E_2 = \dfrac{D}{\varepsilon_2} = \dfrac{Q}{4\pi\varepsilon_2 r^2}}, \qquad a \le r \le b$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^c E\, dr = \int_a^b E_2\, dr + \int_b^c E_1\, dr = \int_a^b \frac{Q}{4\pi\varepsilon_2 r^2}\, dr + \int_b^c \frac{Q}{4\pi\varepsilon_1 r^2}\, dr = \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{b} - \frac{1}{c}\right)$$

$$U_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r2}}\frac{b-a}{a\,b} + \frac{1}{\varepsilon_{r1}}\frac{c-b}{b\,c}\right) \qquad \boxed{C = \frac{Q}{U_{AB}} = \frac{4\pi\varepsilon_0}{\dfrac{1}{\varepsilon_{r2}}\dfrac{b-a}{a\,b} + \dfrac{1}{\varepsilon_{r1}}\dfrac{c-b}{b\,c}}}$$

b)

$Q = 1\, nC$

$$P = D - \varepsilon_0 E = D - \varepsilon_0 \frac{D}{\varepsilon} = \left(1 - \frac{1}{\varepsilon_r}\right) D$$

$$\sigma_{v1} = \vec{P}_1 \cdot \vec{n}_1 = -P_1(r=b) = -\left(1 - \frac{1}{\varepsilon_{r1}}\right) D(r=b) = -\left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi b^2}$$

$$\sigma_{v2} = \vec{P}_2 \cdot \vec{n}_2 = P_2(r=b) = \left(1 - \frac{1}{\varepsilon_{r2}}\right) D(r=b) = \left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi b^2}$$

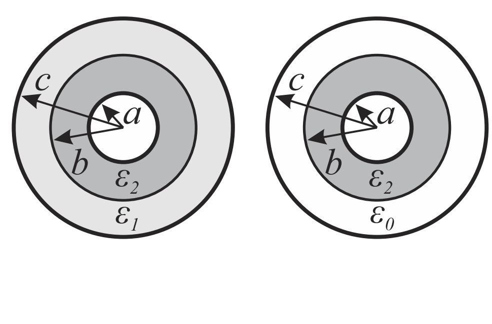
$$|\sigma_{v1}| = 3|\sigma_{v2}| \qquad \Rightarrow \qquad \left(1 - \frac{1}{\varepsilon_{r1}}\right)\frac{Q}{4\pi b^2} = 3\left(1 - \frac{1}{\varepsilon_{r2}}\right)\frac{Q}{4\pi b^2} \qquad \Rightarrow \qquad 1 - \frac{1}{\varepsilon_{r1}} = 3 - \frac{3}{\varepsilon_{r2}}$$

*Kada se ispusti tečni dielektrik :* $\varepsilon_1 \to \varepsilon_0 \qquad (Q = const.)$

$E_{novo}(r=c) = 2E(r=c)$

$$\frac{Q}{4\pi\varepsilon_0 c^2} = 2\frac{Q}{4\pi\varepsilon_1 c^2} \qquad \Rightarrow \qquad \varepsilon_1 = 2\varepsilon_0 \qquad \boxed{\varepsilon_{r1} = 2}$$

$$\varepsilon_{r2} = \frac{3}{\dfrac{1}{\varepsilon_{r1}} + 3 - 1} = \frac{3}{\dfrac{1}{2} + 3 - 1} = \frac{3}{\dfrac{5}{2}} = \frac{6}{5} \qquad \boxed{\varepsilon_{r2} = 1{,}2}$$



c)

$$C = \frac{Q}{U_{AB}} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r2}}\frac{b-a}{ab} + \frac{1}{\varepsilon_{r1}}\frac{c-b}{bc}} = 2,22\ pF$$

*Kada se ispusti tečni dielektrik :* $\varepsilon_1 \to \varepsilon_0 \quad (\varepsilon_{r1} \to 1)$

$$C_{novo} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r2}}\frac{b-a}{ab} + \frac{1}{1}\cdot\frac{c-b}{bc}} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r2}}\frac{b-a}{ab} + \frac{c-b}{bc}} = 1,91\ pF$$

$$\Delta C = C_{novo} - C = 1,91\ pF - 2,22\ pF$$

$$\boxed{\Delta C = -0,31\ pF}$$