STUDIJSKI PROGRAM MEHATRONIKA

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **12.10.2019.**

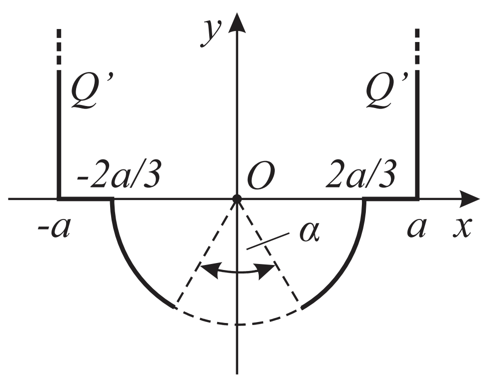
ZADACI

Zadatak 1. Na slici 1 su prikazane dve tanke žičane strukture načinjene od izolacionog materijala, naelektrisane ravnomerno istim podužnim naelektrisanjem Q'. Strukture se nalaze u x-y ravni pravouglog Dekartovog koordinatnog sistema. Sredina je vazduh.

a) Izvesti u opštim brojevima izraz za vektor jačine električnog polja u tački O, koji stvaraju obe strukture.

b) Koliki treba da bude ugao α da bi intenzitet vektora jačine električnog polja u tački O bio jednak nuli.

Brojni podaci su: $Q' = 5$ nC/m, $a = 3$ cm, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m.



Slika 1.

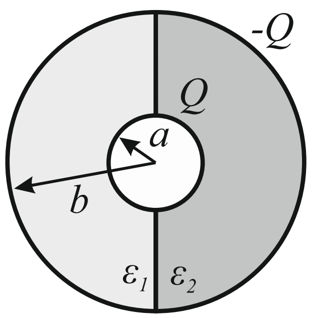
Zadatak 2. Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika, čvrstim permitivnosti ε_1 i tečnim permitivnosti ε_2. Poluprečnici elektroda kondenzatora su a i b.

a) Odrediti, u opštim brojevima, izraz za ekvivalentnu kapacitivnost kondenzatora.

b) Izračunati permitivnost tečnog dielektrika, ε_2, ako se nakon njegovog potpunog ispuštanja, kapacitivnost kondenzatora smanji dva puta.

c) Izračunati promenu energije sadržane u kondenzatoru koja nastaje nakon ispuštanja tečnog dielektrika. Kondenzator je sve vreme odvojen od izvora napajanja.

Brojni podaci su: $a = 2$ mm, $b = 5$ mm, $\varepsilon_1 = 5\varepsilon_0$, $Q = 1{,}2$ μC.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM MEHATRONIKA

K	TET	Osnovi elektrotehnike 1 (II kolokvijum)	K2
Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs			12.10.2019.

ZADACI

Zadatak 1. Mrežu vremenski konstantnih struja, prikazanu na slici 1, transformisati na sledeći način:

a) Deo mreže između tačaka A i B predstaviti ekvivalentnim otpornikom R_{AB},

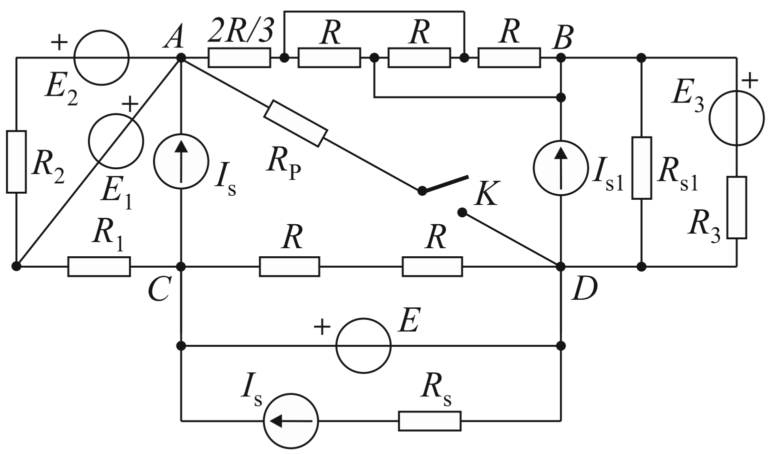
b) Deo mreže između tačaka A i C predstaviti ekvivalentnim Tevenenovim generatorom E_{T1}, R_{T1},

c) Deo mreže između tačaka C i D predstaviti ekvivalentnim Tevenenovim generatorom E_{T2}, R_{T2},

d) Deo mreže između tačaka B i D predstaviti ekvivalentnim Tevenenovim generatorom E_{T3}, R_{T3},

e) Analizirajući uprošćenu mrežu, proveriti, da li bi otpornik $R_p = 3\ \Omega$, maksimalne snage $P_{Rpmax} = 3$ W, pregoreo nakon zatvaranja prekidača K.

Brojni podaci su: $R = 4\ \Omega$, $R_1 = R_S = 2\ \Omega$, $R_2 = R_{S1} = 3\ \Omega$, $R_3 = 6\ \Omega$, $I_S = 1$ A, $I_{S1} = 2$ A, $E = 2$ V, $E_1 = 4$ V, $E_2 = E_3 = 3$ V.



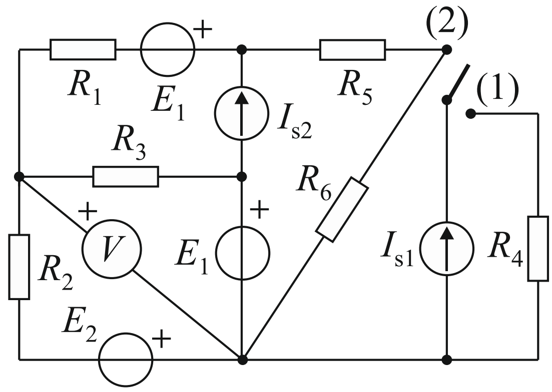
Slika 1.

Zadatak 2. Kada se u kolu vremenski konstantne struje sa slike 2 preklopnik prebaci iz položaja (1) u položaj (2), pokazivanje idealnog voltmetra se poveća za 4 V.

a) Primenjujući teoremu superpozicije, odrediti jačinu struje strujnog generatora I_{S1}.

a) Odrediti snagu strujnog generatora I_{S1} kada je preklopnik u položaju (2). (*Uputsvo*: kolo rešiti primenom metoda konturnih struja).

Brojni podaci su: $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 1\ \Omega$, $R_6 = 7\ \Omega$, $E_1 = 5$ V, $E_2 = 2{,}2$ V, $I_{S2} = 1$ A.



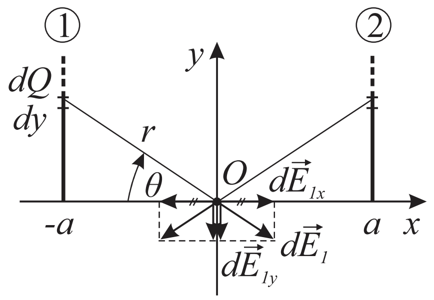
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1

a)



$$d\vec{E}_{1x} + d\vec{E}_{2x} = 0 \quad zbog\ simetrije$$

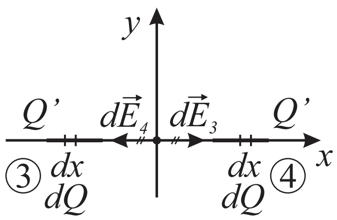
$$dE_{1y} = dE_1 \sin\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q' dy}{4\pi\varepsilon_0 r^2}\sin\theta$$

$$dE_{1y} = \frac{Q' \frac{\cancel{r}\, d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^{\cancel{2}}}\sin\theta = \frac{Q' d\theta}{4\pi\varepsilon_0 \cancel{\cos\theta}\, \frac{a}{\cancel{\cos\theta}}}\sin\theta$$

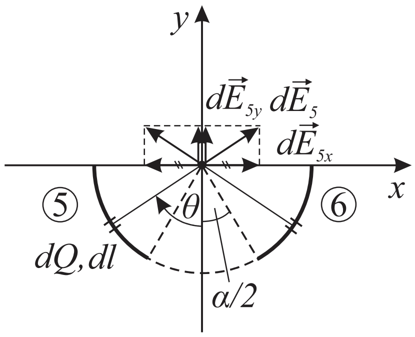
$$E_{12y} = 2\cdot\int\limits_{po\,štapu} dE_{1y} = 2\frac{Q'}{4\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}}\sin\theta d\theta = \frac{Q'}{2\pi\varepsilon_0 a}(-\cos\theta)\Big|_0^{\frac{\pi}{2}}$$

$$E_{12y} = \frac{Q'}{2\pi\varepsilon_0 a}\left(\cos 0 - \cos\frac{\pi}{2}\right) = \frac{Q'}{2\pi\varepsilon_0 a}$$

$$\boxed{\vec{E}_{12y} = \frac{Q'}{2\pi\varepsilon_0 a}\left(-\vec{i}_y\right)}$$



$$d\vec{E}_3 + d\vec{E}_4 = 0 \quad zbog\ simetrije$$



$$d\vec{E}_{5x} + d\vec{E}_{6x} = 0 \quad zbog\ simetrije$$

$$dE_5 = \frac{dQ}{4\pi\varepsilon_0 R^2} = \frac{Q' \cdot dl}{4\pi\varepsilon_0 R^2} = \frac{Q' \cdot \cancel{R} \cdot d\theta}{4\pi\varepsilon_0 R^{\cancel{2}}}$$

$$dE_{5y} = dE_5 \cdot \cos\theta$$

$$E_{56y} = 2\cdot\int\limits_{po\,luku} dE_{5y} = 2\int_{\alpha/2}^{\pi/2}\frac{Q' \cdot \cos\theta \cdot d\theta}{4\pi\varepsilon_0 R} = \frac{Q'}{2\pi\varepsilon_0 R}\cdot\sin\theta\Big|_{\alpha/2}^{\pi/2} = \frac{Q'}{2\pi\varepsilon_0 R}\left(1-\sin\frac{\alpha}{2}\right)$$

$$(R = \frac{2a}{3}) \qquad \Rightarrow \qquad E_{56y} = \frac{3Q'}{4\pi\varepsilon_0 a}\left(1-\sin\frac{\alpha}{2}\right)$$

$$\boxed{\vec{E}_{56y} = \frac{3Q'}{4\pi\varepsilon_0 a}\left(1-\sin\frac{\alpha}{2}\right)\cdot\left(\vec{i}_y\right)}$$

$$\boxed{\vec{E}_0 = \vec{E}_{12y} + \vec{E}_{56y} = \left(-\frac{Q'}{2\pi\varepsilon_0 a} + \frac{3Q'}{4\pi\varepsilon_0 a}\left(1-\sin\frac{\alpha}{2}\right)\right)\left(\vec{i}_y\right) = \left(\frac{Q'}{4\pi\varepsilon_0 a}\left(1-3\sin\frac{\alpha}{2}\right)\right)\left(\vec{i}_y\right)}$$

b)

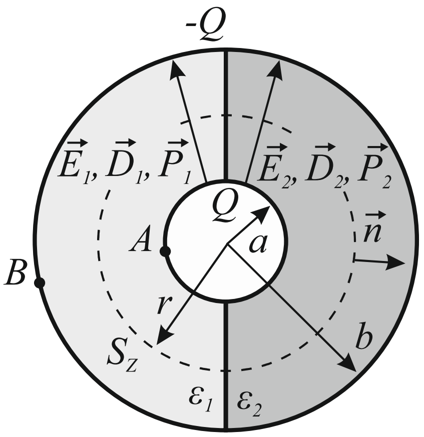
$$\frac{Q'}{4\pi\varepsilon_0 a}\left(1-3\sin\frac{\alpha}{2}\right) = 0$$

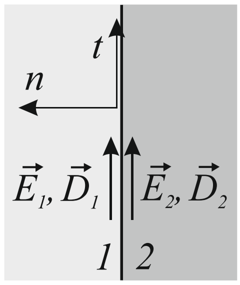
$$3\sin\frac{\alpha}{2} = 1$$

$$\frac{\alpha}{2} \approx 20^0 \rightarrow \boxed{\alpha \approx 40^0}$$

K1 Z2

a)





Granični uslov:

$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$

$D_{n1} \neq D_{n2}$

$$\oint_{S_Z} \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S_Z}$$

$$\int_{S_1} D_1\, ds + \int_{S_2} D_2\, ds = Q$$

$$D_1\, 2r^2\pi + D_2\, 2r^2\pi = Q$$

$$\varepsilon_1 E\, 2r^2\pi + \varepsilon_2 E\, 2r^2\pi = Q$$

$$\boxed{E = \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2r^2\pi}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2r^2\pi} dr = \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

b)

Nakon ispustanja tečnog dielektrika: $\varepsilon_2 \quad \rightarrow \quad \varepsilon_0 \qquad \Rightarrow \qquad C^{NOVO} = \dfrac{(\varepsilon_{r1} + 1) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$

$$C^{NOVO} = \frac{C}{2}, \qquad C = 2C^{NOVO} \qquad \Rightarrow \qquad \frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = 2\frac{(\varepsilon_{r1} + 1) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

$$\varepsilon_{r2} = \varepsilon_{r1} + 2 = 7 \qquad \boxed{\varepsilon_2 = 7\,\varepsilon_0}$$

c)

$$\Delta W_e = W_e^{NOVO} - W_e = \frac{1}{2}\frac{Q^2}{C^{NOVO}} - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\frac{C}{2}} - \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{C}$$

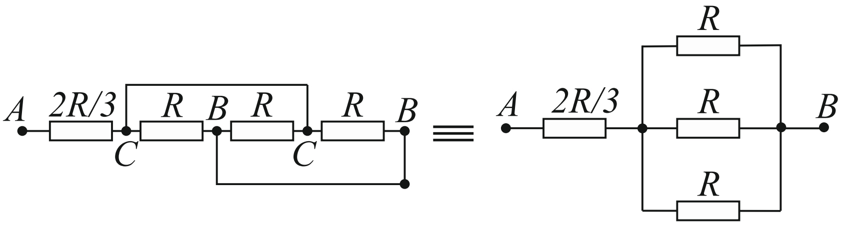
$$\Delta W_e = \frac{1}{2}\frac{Q^2}{\frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}} = \frac{Q^2\left(\frac{1}{a} - \frac{1}{b}\right)}{(\varepsilon_{r1} + \varepsilon_{r2}) 4\pi\varepsilon_0} = \frac{\left(1{,}2 \cdot 10^{-6}\right)^2 \cdot \left(\frac{1}{2 \cdot 10^{-3}} - \frac{1}{5 \cdot 10^{-3}}\right)}{(5+7) \cdot 4\pi \cdot 8{,}85 \cdot 10^{-12}}$$

$$\boxed{\Delta W_e = 0{,}32\ J}$$

MH ISPIT, K2, 12.10.2019.

K2 Z1

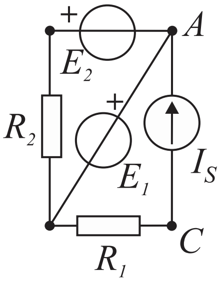
a)

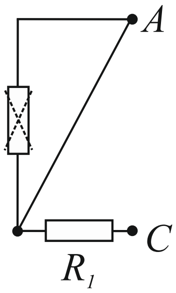


$$R_{AB} = \frac{2R}{3} + R \parallel R \parallel R = \frac{2R}{3} + \frac{R}{3} = R$$

$$\boxed{R_{AB} = R}$$

b)

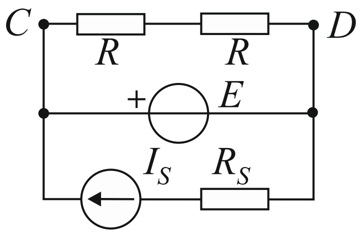




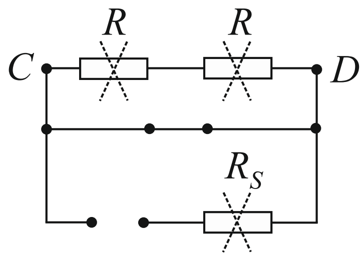
$$R_{T1} = R_1 = 2\Omega$$

$$E_{T1} = U_{AC} = E_1 + R_1 I_S = 6V$$

c)

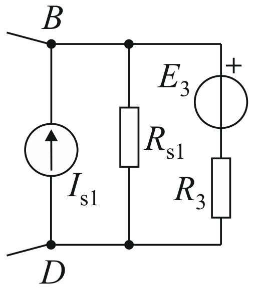


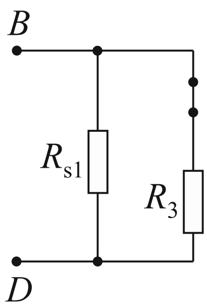
$$E_{T2} = E = 2V$$



$$R_{T2} = R_{CD} = 0$$

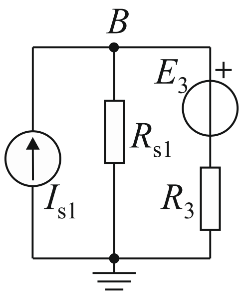
d)





$$R_{T3} = R_{BD} = R_{S1} \parallel R_3$$

$$R_{T3} = \frac{\cancel{3} \cdot 6}{\cancel{9}} = 2\Omega$$

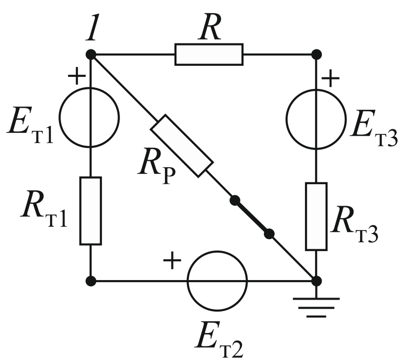


$$\left(\frac{1}{\infty} + \frac{1}{R_{S1}} + \frac{1}{R_3}\right) V_B = I_{S1} + \frac{E_3}{R_3}$$

$$E_{T3} = U_{BD} = V_B = \left(\frac{1}{3} + \frac{1}{6}\right) V_B = \frac{3}{6} + 2$$

$$E_{T3} = 5V$$

e)



$$\left(\frac{1}{R_{T1}} + \frac{1}{R_P} + \frac{1}{R + R_{T3}}\right) V_1 = \frac{E_{T1} + E_{T2}}{R_{T1}} + \frac{E_{T3}}{R + R_{T3}}$$

$$(3 + 2 + 1) V_1 = 24 + 5$$

$$6 V_1 = 29$$

$V_1 = \frac{29}{6} = 4{,}83V > U_{R_{P\max}} \quad \rightarrow \quad$ R_P će pregoreti nakon zatvaranja prekidača.

$$P_{R_{P\max}} = \frac{U_{R_{P\max}}^{\;2}}{R_P} \quad \rightarrow \quad U_{R_{P\max}} = \sqrt{R_P \cdot P_{R_{P\max}}} = \sqrt{3 \cdot 3} = 3V$$

K2 Z2

a)

Iz teksta zadatka: $U_{V1}^{(2)} = U_{V1}^{(1)} + \Delta U_{V1} = U_{V1}^{(1)} + 4V$

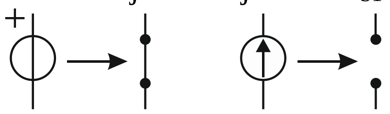
Po teoremi superpozicije:

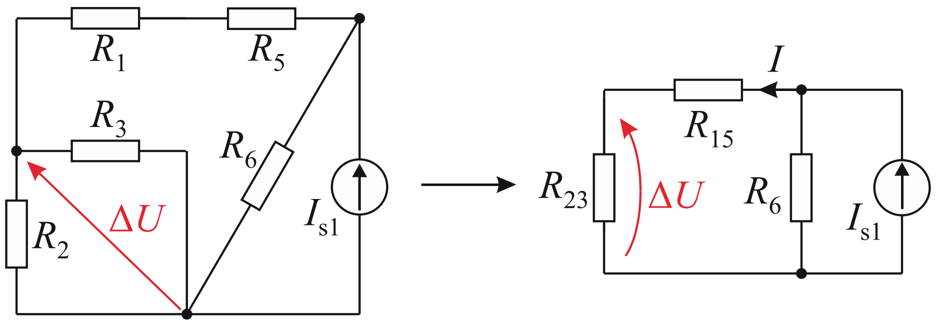
$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_{S1}\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_{S1}\end{matrix}} \quad \longrightarrow \quad \Delta U_V = U_V' = 4V$$

(2) (1)

$$U_{V1}^{(2)} = U_{V1}^{(1)} + U_V'$$

Kolo u kojem deluje samo I_{S1}:





$$R_{23} = R_2 \parallel R_3 = 3 \parallel 6 = 2\,\Omega$$

$$R_{15} = R_1 + R_5 = 4 + 1 = 5\,\Omega$$

$$I = \frac{R_6}{R_6 + R_{15} + R_{23}} I_{S1}$$

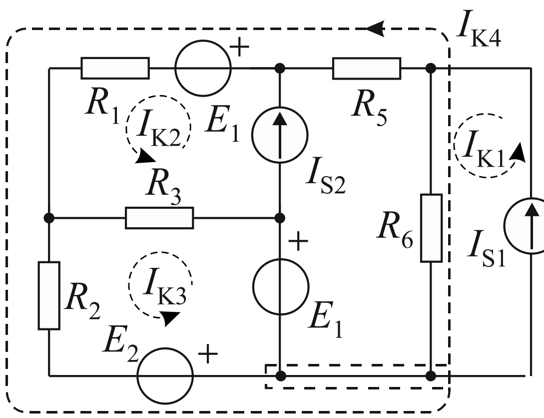
$$I = \frac{\Delta U}{R_{23}} = \frac{4}{2} = 2A$$

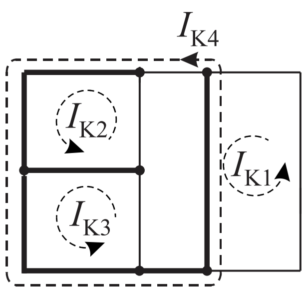
$$I_{S1} = \frac{R_6 + R_{15} + R_{23}}{R_6} I$$

$$I_{S1} = \frac{7+5+2}{7} \cdot 2 = 4A$$

$$\boxed{I_{S1} = 4A}$$

b)





$$MKS \quad n_g - (n_{\check{C}} - 1) - n_S$$

$$\underbrace{8-(5-1)}_{4konture} - 2 = 2\ jedna\check{c}ine$$

$K1: \quad I_{K1} = I_{S1} = 4A$

$K2: \quad I_{K2} = I_{S2} = 1A$

$K3: \quad (R_2 + R_3) I_{K3} - R_3 I_{K2} + R_2 I_{K4} = E_1 + E_2$

$K4: \quad (R_1 + R_2 + R_6 + R_5) I_{K4} - R_6 I_{K1} + R_1 I_{K3} = -E_1 + E_2$

$K3: \quad 9I_{K3} + 9I_{K4} = 6 + 5{,}5 + 2{,}2 = 13{,}2$

$K4: \quad 3I_{K3} + 15I_{K4} = 28 - 4 - 5 + 2{,}2 = 21{,}2$

$$-42 I_{K4} = -50{,}4$$

$$I_{K4} = 1{,}2A$$

$$I_{K3} = \frac{13{,}2 - 3I_{K4}}{9} = 1{,}07A$$

$$P_{I_{S1}} = U_{S1} I_{S1} = 19{,}6 \cdot 4 = 78{,}4W$$

$$U_{S1} = R_6 I_6 = R_6 (I_{S1} - I_{K4}) = 7 \cdot (4 - 1{,}2) 19{,}6V$$

$$\boxed{P_{I_{S1}} = 78{,}4W}$$