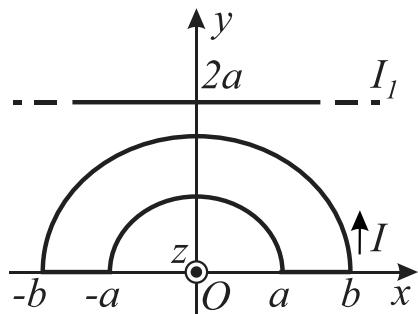


## ZADACI

**Zadatak 1.** Tanak žičani provodnik, sa vremenski konstantnom strujom jačine  $I$ , savijen je tako da oblikuje konturu, prikazanu na slici 1. Provodnik se sastoji od dva pravolinijska i dva lučna segmenta, pri čemu cela kontura leži u  $x$ - $y$  ravni Dekartovog pravouglog koordinatnog sistema. Paralelno sa  $x$  osom, na rastojanju  $2a$  postavljen je veoma dugačak provodnik, sa strujom nepoznatog intenziteta  $I_1$ . Sredina je vazduh.

- Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku, u tački  $O$ , koji potiče od žičane konture.
- Odrediti intenzitet i smer struje  $I_1$ , tako, da ukupan vektor magnetske indukcije u koordinatnom početku bude jednak nuli.

Brojne vrednosti:  $I = 2 \text{ A}$ ,  $a = 3 \text{ cm}$ ,  $b = 4 \text{ cm}$ ,  $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ .

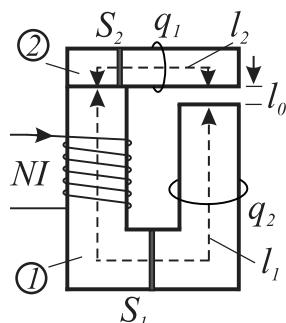


Slika 1.

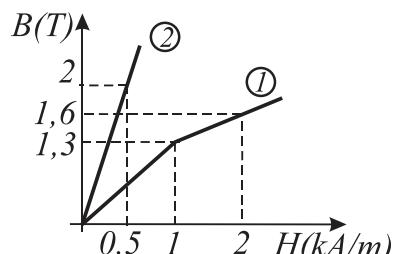
**Zadatak 2.** Na slici 2a je prikazano tanko magnetsko kolo načinjeno od dva različita materijala. Idealizovane krive prvobitnog magnetisanja oba materijala date su na slici 2b. Ako energija sadržana u procepu iznosi  $W_{m0}=18,33 \text{ mJ}$ , odrediti

- Vrednost magnetske indukcije u procepu.
- Broj amperzavojaka potrebnih da se postigne željena energija u procepu.
- Smer i količinu naielktrisanja,  $q_1$  i  $q_2$ , kroz probne zavojke otpornosti  $R_Z = 20 \Omega$ , prilikom uspostavljanja polja.

Brojni podaci:  $S_1 = 2 \text{ cm}^2$ ,  $S_2 = 1,8 \text{ cm}^2$ ,  $S_0 = 2,5 \text{ cm}^2$ ,  $\ell_1 = 30 \text{ cm}$ ,  $\ell_2 = 20 \text{ cm}$ ,  $\ell_0 = 0,2 \text{ mm}$ ,  $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ .



Slika 2a.



Slika 2b.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

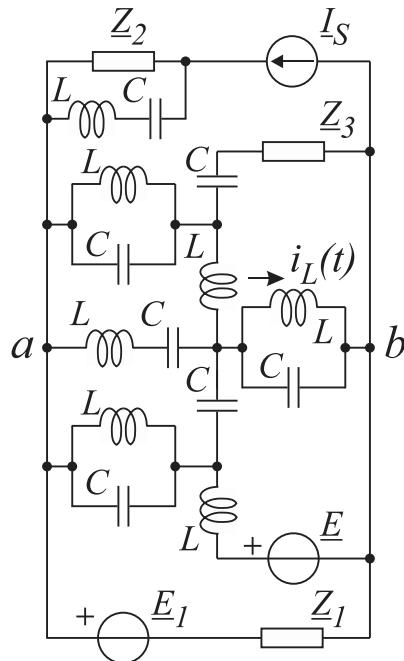
## ZADACI

**Zadatak 1.** U mreži prostoperiodične struje sa slike 1, učestanost generatora ispunjava uslov  $\omega^2 LC = 1$ . Izračunati:

- Kompleksne snage svih generatora.
- Ukupnu snagu Džulovih gubitaka u kolu,
- Vremenski oblik struje kroz kalem  $i_L(t)$ .

Brojni podaci su:

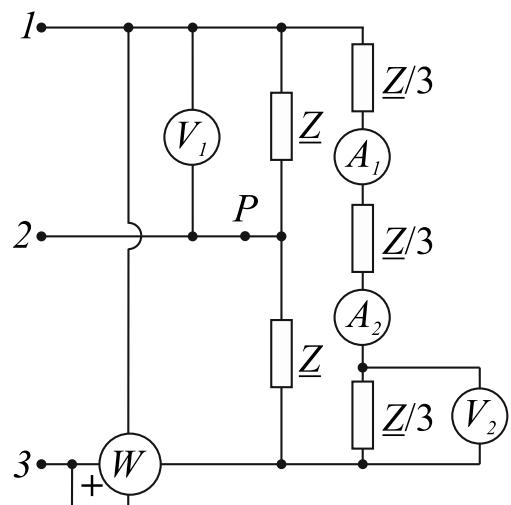
$$E_1 = (50+j20) \text{ V}, E = 100 \text{ V}, I_s = 15 \text{ mA}, X_L = 0,5 \text{ k}\Omega, Z_1 = (5-j2) \text{ k}\Omega, Z_2 = (2+j2) \text{ k}\Omega, Z_3 = (8+j4) \text{ k}\Omega.$$



Slika 1.

**Zadatak 2.** Na slici 2 je prikazan simetričan trofazni prijemnik impedanse  $\underline{Z} = (30+j60) \Omega$ , priključen na mrežu faznog napona  $\underline{U}_1 = 230 \text{ V}$ . U tački P u jednom momentu dolazi do havarije i prekida voda.

- Odrediti pokazivanja idealnih mernih instrumenata pre prekida,
- Odrediti pokazivanja idealnih mernih instrumenata posle prekida,
- Nacrtati fazorske dijagrame veličina od kojih zavise pokazivanja vatmetra pre i posle prekida.

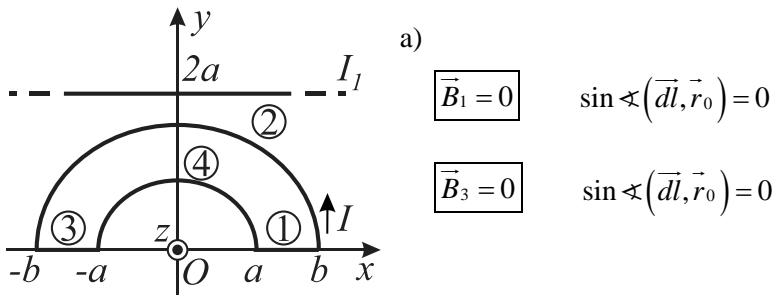


Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se bodože sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1



$$\overrightarrow{dB}_2 = \frac{\mu_0}{4\pi} \frac{I \overrightarrow{dl} \times \vec{r}_0}{r^2}$$

$$dB_2 = \frac{\mu_0}{4\pi} \frac{I dl}{b^2}$$

$$\alpha(\vec{dl}, \vec{r}_0) = \frac{\pi}{2}$$

$$B_2 = \int dB_2 = \frac{\mu_0 I}{4\pi b^2} \int_0^{1/2b\pi} dl = \frac{\mu_0 I}{4\pi b^2} b\pi = \frac{\mu_0 I}{4b}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{4b} \cdot \vec{i}_z}$$

$$\overrightarrow{dB}_4 = \frac{\mu_0}{4\pi} \frac{I \overrightarrow{dl} \times \vec{r}_0}{r^2}$$

$$dB_4 = \frac{\mu_0}{4\pi} \frac{I dl}{a^2}$$

$$\alpha(\vec{dl}, \vec{r}_0) = \frac{\pi}{2}$$

$$B_4 = \int dB_4 = \frac{\mu_0 I}{4\pi a^2} \int_0^{1/2a\pi} dl = \frac{\mu_0 I}{4\pi a^2} a\pi = \frac{\mu_0 I}{4a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I}{4a} \cdot (-\vec{i}_z)}$$

$$\vec{B}_K = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \left( \frac{\mu_0 I}{4b} - \frac{\mu_0 I}{4a} \right) \cdot \vec{i}_z = \frac{\mu_0 I}{4} \left( \frac{1}{b} - \frac{1}{a} \right) \cdot \vec{i}_z$$

$$\boxed{\vec{B}_K = 0,52 \mu T \cdot (-\vec{i}_z)}$$

b)

$$\left| \vec{B}_P \right| = \frac{\mu_0 I_1}{2\pi(2a)} = \frac{\mu_0 I_1}{4\pi a}$$

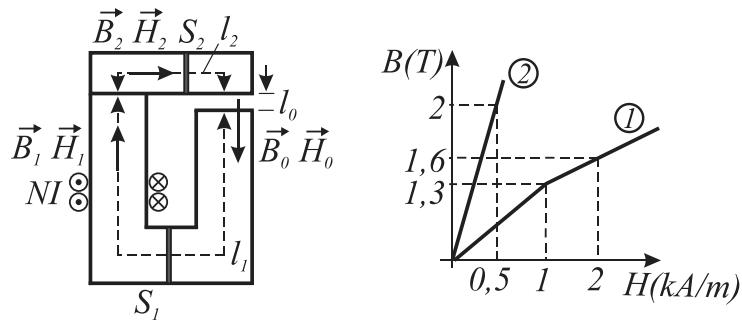
$$\vec{B}_O = \vec{B}_K + \vec{B}_P = 0 \quad \Rightarrow \quad \vec{B}_P = \frac{\mu_0 I_1}{4\pi a} \cdot \vec{i}_z$$

$$\frac{\mu_0 I_1}{4\pi a} = \frac{\mu_0 I}{4} \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$\cancel{\mu_0} \cancel{I_1} = \cancel{\mu_0} \cancel{I} \frac{b-a}{\cancel{4} \cancel{\pi} \cancel{a}}$$

$$\boxed{I_1 = \frac{I \cdot \pi(b-a)}{b} = 1,57 \text{ A}}$$

## K1 Z2



a)

$$\oint_c \vec{H} \cdot d\vec{l} = NI$$

$$H_1 l_1 + H_2 l_2 + H_0 l_0 = NI \quad (1)$$

$$\Phi_1 = \Phi_2 = \Phi_0$$

$$B_1 S_1 = B_2 S_2 = B_0 S_0 \quad (2)$$

$$\text{Iz (2)} \quad B_2 S_2 = B_0 S_0$$

$$B_2 = B_0 \frac{S_0}{S_2} = 0,98 \cdot \frac{2,5}{1,8} = 1,36 \text{ T}$$

$$H_2 = \frac{B_2}{\mu_2} = \frac{1,36}{\frac{2}{500}} = 340 \frac{\text{A}}{\text{m}}$$

$$W_{m_0} = \frac{1}{2} B_0 H_0 V_0 = \frac{1}{2} \frac{B_0^2}{\mu_0} l_0 S_0 \rightarrow B_0 = \sqrt{\frac{2 \mu_0 W_{m_0}}{l_0 S_0}}$$

$$B_0 = 0,98 \text{ T}$$

$$H_0 = \frac{B_0}{\mu_0} = 779,86 \frac{\text{kA}}{\text{m}}$$

$$\text{Iz (2)} \quad B_1 S_1 = B_0 S_0$$

$$B_1 = B_2 \frac{S_2}{S_1} = 1,36 \cdot \frac{1,8}{2} = 1,22 \text{ T}$$

$$H_1 = \frac{B_1}{\mu_1} = \frac{1,22}{\frac{1,3}{1000}} = 938,46 \frac{\text{A}}{\text{m}}$$

$$\text{Iz (1)} \quad NI = 938,46 \cdot 0,3 + 340 \cdot 0,2 + 779,86 \cdot 0,2$$

$NI = 505,51 \text{ Azav}$
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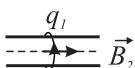
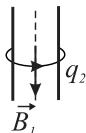
c)

$$q_1 = -\frac{1}{R_Z} (\Phi^{KR} - \Phi^0)$$

$$q_1 = -\frac{1}{20} B_2 S_2 = -\frac{1}{20} 1,36 \cdot 1,8 \cdot 10^{-4}$$

$q_1 = 12,24 \mu\text{C}$
---------------------------

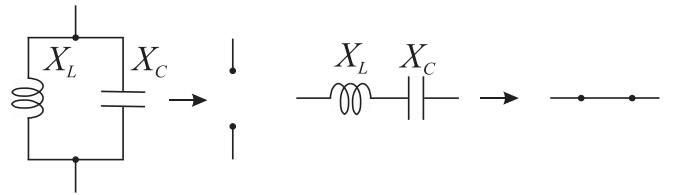
$q_1 = q_2$	na osnovu uslova (2)
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K2 Z1

a)

$$\omega^2 LC = 1 \rightarrow \omega L = \frac{1}{\omega C} \rightarrow X_L = X_C$$



$$\underline{I}_{E1} = \frac{0 - Va + \underline{E}_1}{\underline{Z}_1} = \frac{-\underline{E} + \underline{E}_1}{\underline{Z}_1} = \frac{-100 + (50 + j20)}{(5 - j2) \cdot 10^3} = -10 \text{ mA}$$

$$\underline{I}_{E1} = 10e^{-j90^\circ} \text{ mA}$$

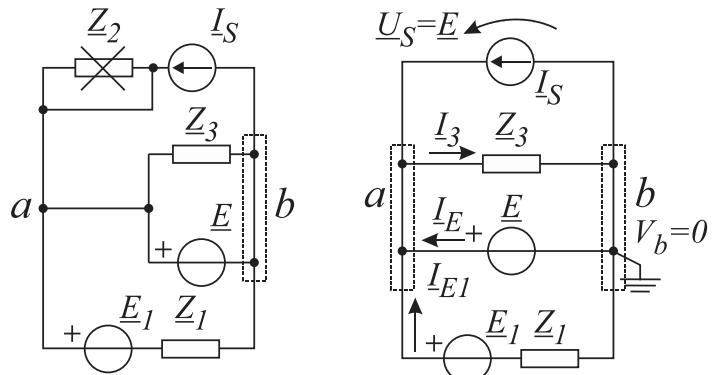
$$\underline{I}_3 = \frac{Va - 0}{\underline{Z}_3} = \frac{\underline{E}_3}{\underline{Z}_3} = \frac{100}{(8 + j4) \cdot 10^3} = 5(2 - j) \text{ mA}$$

$$\underline{I}_3 = 5\sqrt{5}e^{-j26,56^\circ} \text{ mA}$$

$$-\underline{I}_{E1} - \underline{I}_E + \underline{I}_3 - \underline{I}_S = 0$$

$$\underline{I}_E = -\underline{I}_{E1} + \underline{I}_3 - \underline{I}_S = -(-10) + (10 - j5) - 15$$

$$\underline{I}_E = 5(1 - j) \text{ mA}$$



$$\underline{S}_{E1} = \underline{E}_1 \underline{I}_{E1}^* = (50 + j20) \cdot (-10) \cdot 10^{-3} = -(0,5 + j0,2) \text{ VA}$$

$$\underline{S}_E = \underline{E} \underline{I}_E^* = 100 \cdot 5(1 + j) \cdot 10^{-3} = 0,5(1 + j) \text{ VA}$$

$$\underline{S}_{IS} = \underline{U}_S \underline{I}_S^* = \underline{E} \underline{I}_S^* = 100 \cdot 15 \cdot 10^{-3} = 1,5 \text{ VA}$$

b)

$$\sum P_J = \operatorname{Re}\{\underline{S}_{E1} + \underline{S}_E + \underline{S}_{IS}\} = -0,5 + 0,5 + 1,5 = \boxed{1,5 \text{ W}}$$

ili

$$\sum P_J = \operatorname{Re}\{\underline{Z}_1 |\underline{I}_{E1}|^2 + \underline{Z}_3 |\underline{I}_3|^2\} = \boxed{1,5 \text{ W}}$$

c)

$$\underline{I}_L = \frac{V_a}{jX_L \cdot 10^3} = \frac{\underline{E}}{jX_L \cdot 10^3} = \frac{100}{500e^{j90^\circ}} = 0,2e^{-j90^\circ}$$

$$i_L(t) = 0,2\sqrt{2} \cos\left(\underbrace{\frac{1}{\sqrt{LC}}}_{\omega} t - 90^\circ\right) \text{ mA}$$

## K2 Z2

a)

$$\underline{Z} = (30 + j60) \Omega = 67,08 e^{j63,43^\circ} \Omega$$

$$I_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{\underline{U}_1 e^{j0^\circ}}{\frac{1}{3} Z e^{j\varphi}} = \frac{230}{\frac{1}{3} \cdot 67,08 e^{j63,43^\circ}} = 10,29 e^{-j63,43^\circ} \text{ A}$$

$$I_{A1}^{(a)} = I_{A2}^{(a)} = |I_1| = \frac{|I_1|}{\sqrt{3}} = \frac{10,29}{\sqrt{3}} = 5,94 \text{ A}$$

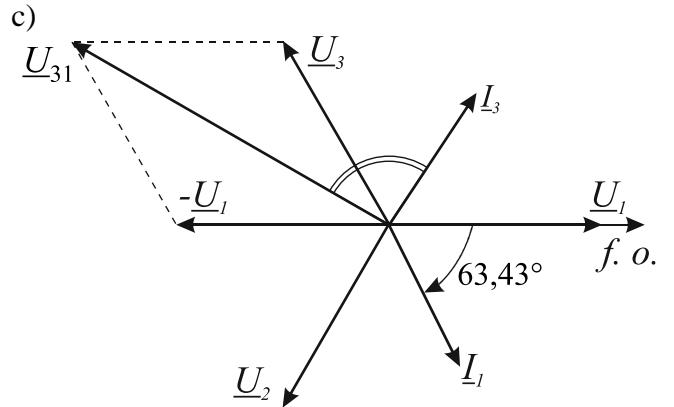
$$U_{V1}^{(a)} = |\underline{U}_{12}| = \sqrt{3} |\underline{U}_1| = \sqrt{3} \cdot 230 = 398,37 \text{ V}$$

$$U_{V2}^{(a)} = \frac{Z}{3} \cdot I_{A1}^{(a)} = \frac{67,08}{3} \cdot 5,94 = 132,82 \text{ V}$$

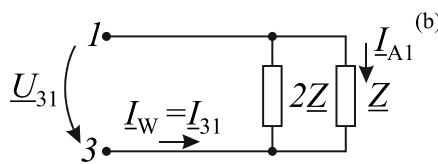
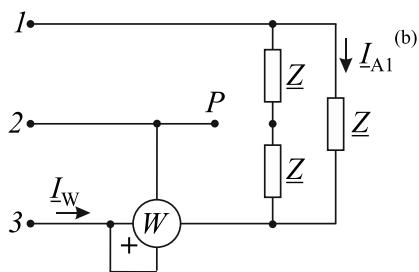
$$P_W^{(a)} = \operatorname{Re} \{ \underline{U}_{31} \underline{I}_3^* \} = U_{31} I_3 \cos \angle(\underline{U}_{31}, \underline{I}_3)$$

$$P_W^{(a)} = \sqrt{3} U_1 I_1 \cos \underbrace{(30^\circ + 63,43^\circ)}_{93,43^\circ} = \sqrt{3} \cdot 230 \cdot 10,29 \cdot \cos 93,43^\circ$$

$$P_W^{(a)} = -245,49 \text{ W}$$



b) Posle prekida



$$\underline{I}_{31} = \frac{\underline{U}_{31}}{\frac{2}{3} \underline{Z}} = \frac{\sqrt{3} \underline{U}_1 e^{j120^\circ}}{\frac{2}{3} \cdot 67,08 e^{j63,43^\circ}} = \frac{398,37 \cdot e^{j120^\circ}}{\frac{2}{3} \cdot 67,08 \cdot e^{j63,43^\circ}}$$

$$I_{31} = 8,9 \cdot e^{j56,57^\circ} \text{ A} = \underline{I}_W$$

$$\underline{I}_{A1}^{(b)} = \frac{\underline{U}_{13}}{\underline{Z}} = \frac{\sqrt{3} \underline{U}_1 e^{-j30^\circ}}{67,08 \cdot e^{j63,43^\circ}} = \frac{398,37 \cdot e^{-j30^\circ}}{67,08 \cdot e^{j63,43^\circ}}$$

$$I_{A1}^{(b)} = 5,93 \cdot e^{-j93,43^\circ} \text{ A}$$

$$I_{A1}^{(b)} = I_{A2}^{(b)} = |I_{A1}^{(b)}| = 5,93 \text{ A}$$

$$U_{V1}^{(b)} = U_{V1}^{(a)}$$

$$U_{V2}^{(b)} = \frac{Z}{3} \cdot I_{A1}^{(b)} = \frac{67,08}{3} \cdot 5,93$$

$$U_{V2}^{(b)} = 132,59 \text{ V}$$

$$P_W^{(b)} = \operatorname{Re} \{ \underline{U}_{32} \underline{I}_W^* \} = \sqrt{3} \cdot U_1 \cdot I_W \cdot \cos \angle(\underline{U}_{32}, \underline{I}_W)$$

$$P_W^{(b)} = 398,37 \cdot 8,9 \cdot \cos(30^\circ + 3,43^\circ)$$

$$P_W^{(b)} = -2958,92 \text{ W}$$

