STUDIJSKI PROGRAM MERENJE I REGULACIJA

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

31.08.2019.

ZADACI

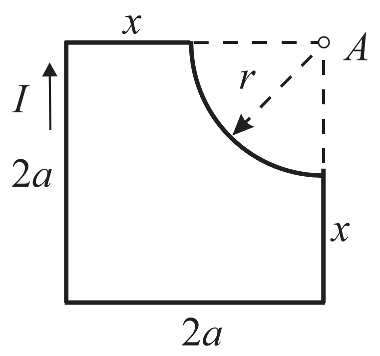
Zadatak 1. Napisati iskaz Bio-Savarovog zakona i primeniti ga pri izračunavanju vektora magnetske indukcije koji u tački A stvaraju:

a) krivolinijski deo ravne strujne konture prikazane na slici 1, B_K;

b) pravolinijski delovi iste ravne strujne konture B_P.

c) Odrediti poluprečnik r lučnog dela konture, tako da u tački A bude ispunjen uslov $B_K = 4B_P$, pri čemu je $x + r = 2a$.

Sredina je vazduh.
Brojne vrednosti su:
$I = 5$ A; $a = 5$ cm, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.



Slika 1.

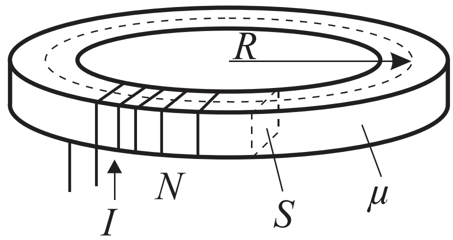
Zadatak 1. Na slici su prikazani tanko torusno jezgro, permeabilnosti $\mu = 1{,}4/1000$ *H/m*, sa N gusto i ravnomerno namotanih zavojaka tanke žice (slika 1a), kao i histerezisna petlja materijala od kog je jezgro načinjeno (Slika 1b). Koordinate vrha histerezisne petlje su H_m=1000 *A/m*, B_m=1,4 *T*. Struja u namotaju je prostoperiodična funkcija vremena $i(t) = I_m cos(2\pi f t)$. Odrediti:

a) u opštim brojevima, izraz za vektor magnetske indukcije,

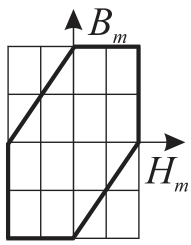
b) izraz za samoinduktivnost namotaja,

c) gubitke usled histerezisa nastale u jezgru u intervalu vremena $\Delta t = 5$ *minuta*.

Ostali brojni podaci su: R =3 *cm*, S =6 *cm*^{2}, N = 628 *zav*, I_m =0,3 *A*, f = 50 *Hz*.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 2 (II kolokvijum)

K2

31.08.2019.

ZADACI

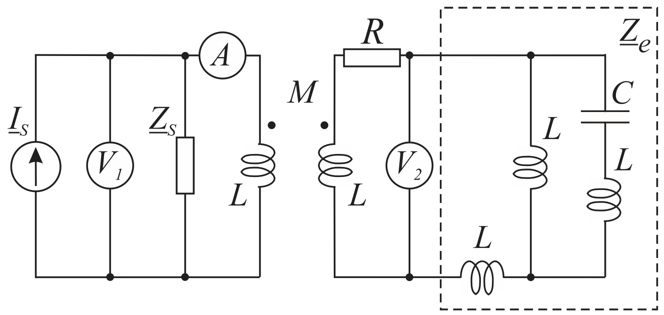
Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_e$.

b) Ako u kolu strujni generator, $\underline{I}_S$, radi na manjoj antirezonantnoj učestanosti, odrediti pokazivanje idealnih mernih instrumenata.

c) Odrediti kompleksnu, aktivnu, reaktivnu i prividnu snagu strujnog generatora $\underline{I}_S$.

Brojni podaci su: $\underline{I}_S$ = (1+j) A, L = 20 mH, C = 250 nF, M = 10 mH, R = 50 Ω, $\underline{Z}_S$ = (50-j100) Ω.



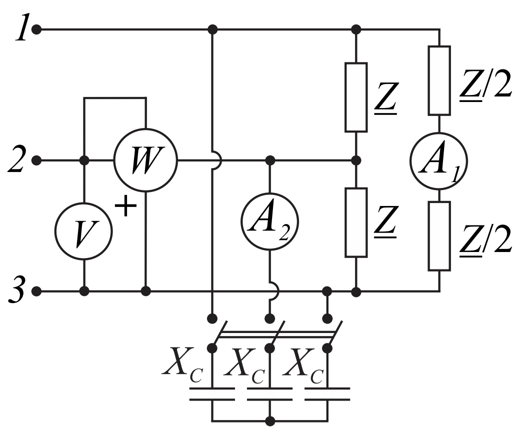
Slika 1.

Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik impedanse $\underline{Z}$ = (60+j180) Ω, priključen na mrežu faznog napona $\underline{U}_1$ = 230 V.

a) Odrediti pokazivanja idealnih mernih instrumenata pre priključivanja kondenzatora.

b) Odrediti reaktanse kondenzatora koje je potrebno povezati sa prijemnikom da bi mu se faktor snage popravio na jedinicu.

c) Odrediti pokazivanja idealnih mernih instrumenata posle priključivanja kondenzatora.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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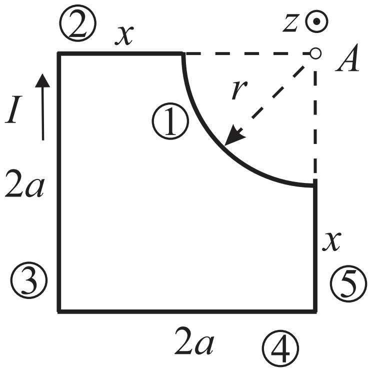
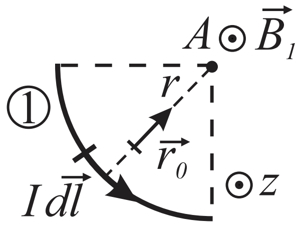
K1 Z1



a)



$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}_0}{r^2}$$

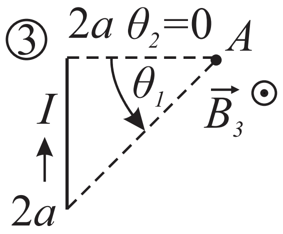
$$dB_1 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_1 = \int dB_1 = \frac{\mu_0 I}{4\pi r^2}\int_0^{\frac{1}{4}2r\pi} dl = \frac{\mu_0 I}{4\pi r^2}\frac{1}{2}r\pi = \frac{\mu_0 I}{8r}$$

$$\boxed{\vec{B}_K = \frac{\mu_0 I}{8r}\cdot\vec{i}_z}$$

b)

$\boxed{\vec{B}_2 = 0}$ $\boxed{\vec{B}_5 = 0}$ $\sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$

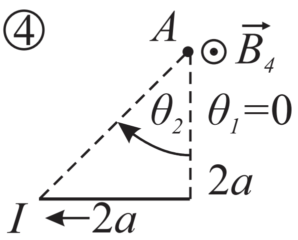


$$d = 2a, \qquad \theta_1 \to -\frac{\pi}{4}, \qquad \theta_2 = 0$$

$$B_3 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi\cdot 2a}\left[0 - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\mu_0 I\sqrt{2}}{16\pi a}$$

$$\boxed{\vec{B}_3 = \frac{\mu_0 I\sqrt{2}}{16\pi a}\cdot\left(-\vec{i}_z\right)}$$

$$d = a, \qquad \theta_1 = 0, \qquad \theta_2 = \frac{\pi}{4}$$



$$B_4 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$B_4 = \frac{\mu_0 I_1}{4\pi\cdot 2a}\left[\frac{\sqrt{2}}{2} - 0\right] = \frac{\mu_0 I\sqrt{2}}{16\pi a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I\sqrt{2}}{16\pi a}\cdot\left(-\vec{i}_z\right) = \vec{B}_3} \qquad \boxed{\vec{B}_P = \frac{\mu_0 I\sqrt{2}}{8\pi a}\cdot\left(-\vec{i}_z\right)}$$

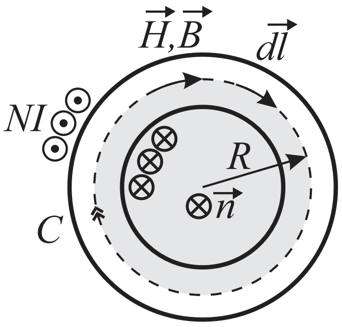
c)

$$B_K = 4B_P$$

$$\frac{\mu_0 I}{8r} = 4\frac{\mu_0 I\sqrt{2}}{8\pi a} \quad \Rightarrow \quad \boxed{r = \frac{\pi a}{4\sqrt{2}} = 2{,}78\text{ cm}}$$

K1 Z2

a)



$$\oint_c \vec{H}\cdot\vec{dl} = NI$$

$$H\cdot 2\pi R = NI$$

$$H = \frac{NI}{2\pi R}$$

$$B = \mu H \quad \Rightarrow \quad \boxed{B = \mu\frac{NI}{2\pi R}}$$

b)

$$I = I_m = 0{,}3\ \text{A}$$

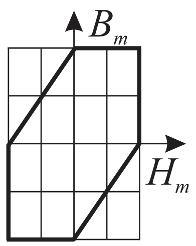
$$\Phi = N\cdot B\cdot S = N\cdot\mu\cdot\frac{NI}{2\pi R}\cdot S = \mu\cdot\frac{N^2 I}{2\pi R}\cdot S$$

$$L = \frac{\Phi}{I} = \frac{\mu\cdot\frac{N^2\cancel{I}}{2\pi R}\cdot S}{\cancel{I}} = \mu\cdot\frac{N^2 S}{2\pi R}$$

$$L = \frac{1{,}4}{1000}\cdot\frac{628^2\cdot 6\cdot 10^{-4}}{2\pi\cdot 3\cdot 10^{-2}}$$

$$\boxed{L = 1{,}75\,H}$$

c)



$$w_h = 12\cdot\frac{B_m}{2}\cdot\frac{H_m}{2} = \frac{12}{4}\cdot B_m\cdot H_m = 3\mu H_m^2 = 3\mu\left(\frac{NI_m}{2\pi R}\right)^2$$

$$W_h = f\cdot t\cdot w_h\cdot V$$

$$W_h = f\cdot t\cdot 3\mu\left(\frac{NI_m}{2\pi R}\right)^2\cdot 2\pi R\cdot S$$

$$W_h = f\cdot t\cdot 3\mu\frac{(NI_m)^2}{2\pi R}\cdot S$$

$$W_h = 50\cdot 5\cdot 60\cdot 3\cdot\frac{1{,}4}{1000}\frac{(628\cdot 0{,}3)^2}{2\pi\cdot 3\cdot 10^{-2}}\cdot 6\cdot 10^{-4}$$

$$\boxed{W_h = 7117{,}92\ J}$$

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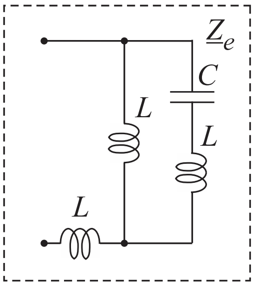
K2 Z1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C}\, j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC-\omega^2 LC}{j\omega C}} = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}$$

$$\underline{Z}_e = \underline{Z}_2 + j\omega L = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC} + j\omega L = \frac{j\omega L\left(1-\omega^2 LC\right) + j\omega L\left(1-2\omega^2 LC\right)}{1-2\omega^2 LC}$$

$$\boxed{\underline{Z}_e = j\frac{\omega L\left(2-3\omega^2 LC\right)}{1-2\omega^2 LC}}$$

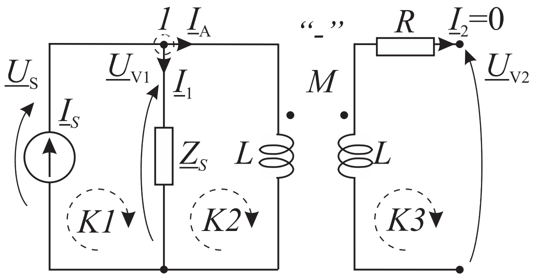
$$\mathrm{Im}\{\underline{Z}_P\} = 0 \;\Rightarrow\; \omega L = 0 \quad \boxed{\omega_{r1} = 0}$$

$$2-3\omega^2 LC = 0 \quad \boxed{\omega_{r2} = \sqrt{\frac{2}{3LC}}}$$

$$\mathrm{Im}\{\underline{Y}_P\} = 0 \;\Rightarrow\; 1-2\omega^2 LC = 0 \quad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}}$$

$$\boxed{\omega_{ar2} \to \infty}$$

b)



$$\omega_{ar1} = \frac{1}{\sqrt{2LC}} = \frac{1}{\sqrt{2\cdot 20\cdot 10^{-3}\cdot 25\cdot 10^{-9}}} = 10^4\,\frac{rad}{s} = \omega$$

$$\underline{Z}_e = \infty$$

$$\underline{Z}_L = j\omega L = j10^4\cdot 20\cdot 10^{-3} = j200\ \Omega$$

$$\underline{Z}_M = j\omega M = j10^4\cdot 10\cdot 10^{-3} = j100\ \Omega$$

Č1: $\underline{I}_S = \underline{I}_A + \underline{I}_1$

K1: $-\underline{U}_S + \underline{U}_{V1} = 0$

K2: $-\underline{Z}_S\underline{I}_1 + \underline{Z}_L\underline{I}_A - \cancel{\underline{Z}_M\underline{I}_2}^{\,0} = 0$

K3: $\cancel{\underline{Z}_L\underline{I}_2}^{\,0} - \underline{Z}_M\underline{I}_1 + \underline{U}_{V2} = 0$

$$\underline{I}_A + \underline{I}_1 = 1 + j \quad /\cdot(50-j100)$$

$$\underline{j200\underline{I}_A - (50-j100)\underline{I}_1 = 0}$$

$$50\underline{I}_A - j100\underline{I}_A + (50-j100)\underline{I}_1 = 150-j50$$

$$\underline{j200\underline{I}_A - (50-j100)\underline{I}_1 = 0}$$

$$(50+j100)\underline{I}_A = 150-j50 \;\Rightarrow\; \underline{I}_A = \frac{150-j50}{50+j100} = (0{,}2-j1{,}4)\ \mathrm{A}$$

$$\underline{I}_1 = 1+j-0{,}2+j1{,}4 = (0{,}8+j2{,}4)\ \mathrm{A}$$

$$\boxed{I_A = |\underline{I}_A| = 1{,}41\ A}$$

$$\underline{U}_{V2} = \underline{Z}_M\underline{I}_A = j100\cdot(0{,}2-j1{,}4) = (140+j20)\,V \quad\Rightarrow\quad \boxed{U_{V2} = |\underline{U}_{V2}| = 141{,}42\,V}$$

$$\underline{U}_{V1} = \underline{Z}_S\underline{I}_1 = (50-j100)\cdot(0{,}8+j2{,}4) = (280+j40)\,V \quad\Rightarrow\quad \boxed{U_{V1} = |\underline{U}_{V1}| = 282{,}84\,V}$$

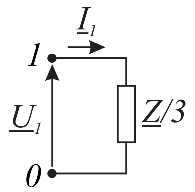
c)

$$\underline{S}_{IS} = \underline{U}_S\underline{I}_S^* = \underline{U}_{V1}\underline{I}_S^* = (280+j40)\cdot(1-j) = (320-j240)\ \mathrm{VA} \quad \boxed{\underline{S}_{IS} = (320-j240)\ \mathrm{VA}}$$

$$\boxed{P_{IS} = 320\ \mathrm{W}} \quad \boxed{Q_{IS} = -240\ \mathrm{VAr}} \quad \boxed{S_{IS} = 400\ \mathrm{VA}}$$

K2 Z2

a)



$$\underline{Z} = (60 + j180)\,\Omega = 189{,}7\, e^{j71{,}56^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\frac{\underline{Z}}{3}} = \frac{U_1\, e^{j0^\circ}}{\frac{1}{3} Z\, e^{j\varphi}} = \frac{230}{\frac{1}{3}\cdot 189{,}7\, e^{j71{,}56^\circ}} = 3{,}64\, e^{-j71{,}56^\circ}\ A$$

$$\boxed{U_V{}^{(a)} = |\underline{U}_{23}| = \sqrt{3}\,|\underline{U}_1| = \sqrt{3}\cdot 230 = 398{,}37\,V}$$

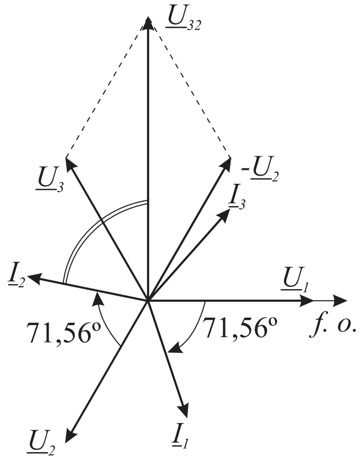
$$\boxed{I_{A1}{}^{(a)} = |\underline{I}_{31}| = \frac{|\underline{I}_1|}{\sqrt{3}} = \frac{3{,}64}{\sqrt{3}} = 2{,}1\,A}$$

$$\boxed{I_{A2}{}^{(a)} = 0}$$

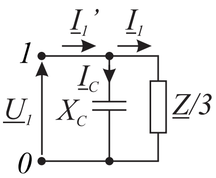
$$P_W{}^{(a)} = \mathrm{Re}\left\{\underline{U}_{32}\,\underline{I}_2{}^*\right\} = U_{32}\, I_2 \cos\sphericalangle\left(\underline{U}_{32}, \underline{I}_2\right)$$

$$P_W{}^{(a)} = \sqrt{3}U_1\, I_1 \cos\underbrace{(120 - 71{,}56^\circ + 30^\circ)}_{78{,}44^\circ} = \sqrt{3}\cdot 230\cdot 3{,}64\cdot \cos 78{,}44^\circ$$

$$\boxed{P_W{}^{(a)} = 290{,}6\,W}$$



b)

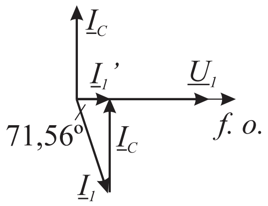


$$\underline{I}_C = \frac{U_1\, e^{j0}}{-jX_C} = \frac{U_1\, e^{j0}}{X_C\, e^{-j\frac{\pi}{2}}} = \frac{U_1}{X_C}\, e^{j\frac{\pi}{2}}\ A$$

I KZ: $\underline{I}_1{}' = \underline{I}_C + \underline{I}_1$

$$I_C = I_1 \sin\varphi$$

$$\frac{U_1}{X_C} = \frac{U_1}{Z/3}\sin\varphi \quad\Rightarrow\quad \frac{1}{X_C} = \frac{3\sin\varphi}{Z} \quad\Rightarrow\quad X_C = \frac{Z}{3\sin\varphi} = \frac{189{,}7}{3\cdot\sin 71{,}56^\circ}$$



$$\boxed{X_C = 66{,}66\,\Omega}$$

$$I_1{}' = I_1 \cos\varphi = 3{,}64\cdot\cos 71{,}56^\circ = 1{,}15\ \mathrm{A} \qquad \underline{I}_1{}' = 1{,}15\, e^{j0^\circ}\ \mathrm{A}$$

$$I_C = I_1 \sin\varphi = 3{,}64\cdot\sin 71{,}56^\circ = 3{,}45\,\mathrm{A}$$

c)

$$\boxed{U_V{}^{(c)} = U_V{}^{(a)} = |\underline{U}_{23}| = 398{,}37\,V}$$

$$\boxed{I_{A1}{}^{(c)} = I_{A1}{}^{(a)} = |\underline{I}_{31}| = 2{,}1\,A}$$

$$\boxed{I_{A2}{}^{(c)} = |\underline{I}_{C2}| = 3{,}45\,\mathrm{V}}$$

$$P_W{}^{(c)} = \mathrm{Re}\left\{\underline{U}_{32}\,\underline{I}_2{}'^*\right\} = U_{23}\, I_2{}'\cos\sphericalangle\left(\underline{U}_{32}, \underline{I}_2{}'\right)$$

$$P_W{}^{(c)} = \sqrt{3}U_1\, I_1{}'\cos\underbrace{(120^\circ + 30^\circ)}_{150^\circ} = \sqrt{3}\cdot 230\cdot 1{,}15\cdot\left(-\frac{\sqrt{3}}{2}\right)$$

$$\boxed{P_W{}^{(c)} = -396{,}75\,W}$$