STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (I kolokvijum)	**K1** **26.06.2019.**

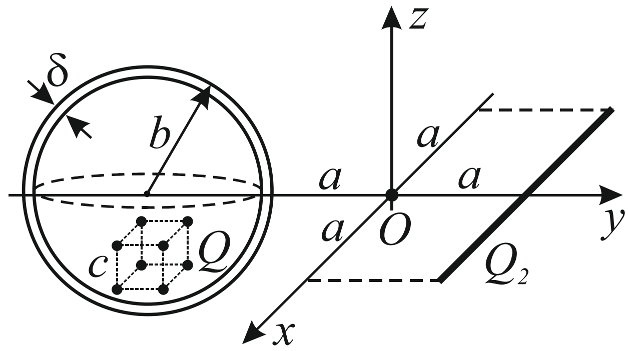
ZADACI

Zadatak 1. U svakom od temena zamišljene kocke, stranice c, nalazi sa po jedno tačkasto naelektrisanje, naelektrisano količinom naelektrisanja Q. Tačkasta naelektrisanja se nalaze unutar nenaelektrisane provodne sferne ljuske, debljine δ, spoljašnjeg poluprečnika b, kao što je prikazano na slici 1. Centar sferne ljuske se nalazi na y osi, pri čemu je udaljenost spoljašnje průvrši ljuske od centra Dekartovog koordinatnog sistema jednaka a. Tanak štap, dužine $2a$, naelektrisan je ravnomerno količinom naelektrisanja Q_2 i postavljen je u x-y ravni, paralelno sa x osom, kao što je prikazano na slici 1. Odrediti:

a) Količine naelektrisanja na površima provodne ljuske.

b) Količinu naelektrisanja štapa, Q_2, tako da ukupan vektor jačine električnog polja u koordinatnom početku bude jednak nuli.

Brojni podaci su: $a = 2\ cm$, $b = 2{,}3\ cm$, $c = 1\ cm$, $Q = 15\ nC$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.



Slika 1.

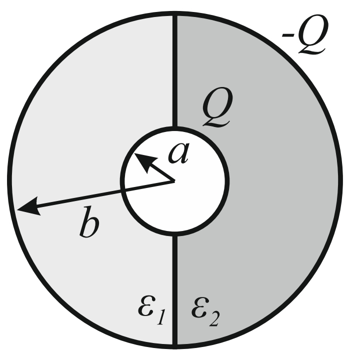
Zadatak 2. Na slici 2 je prikazan sferni kondenzator, ispunjen sa dva sloja dielektrika, čvrstim permitivnosti ε_1 i tečnim permitivnosti ε_2. Poluprečnici elektroda kondenzatora su a i b.

a) Odrediti, u opštim brojevima, izraz za ekvivalentnu kapacitivnost kondenzatora.

b) Izračunati permitivnost tečnog dielektrika, ε_2, ako se nakon njegovog potpunog ispuštanja kapacitivnost kondenzatora smanji dva puta.

c) Izračunati promenu energije sadržane u kondenzatoru koja nastaje nakon ispuštanja tečnog dielektrika.

Brojni podaci su: $a = 2\ mm$, $b = 5\ mm$, $\varepsilon_1 = 5\varepsilon_0$, $Q = 1{,}2\ \mu C$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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Osnovi elektrotehnike 1 (II kolokvijum)

K2

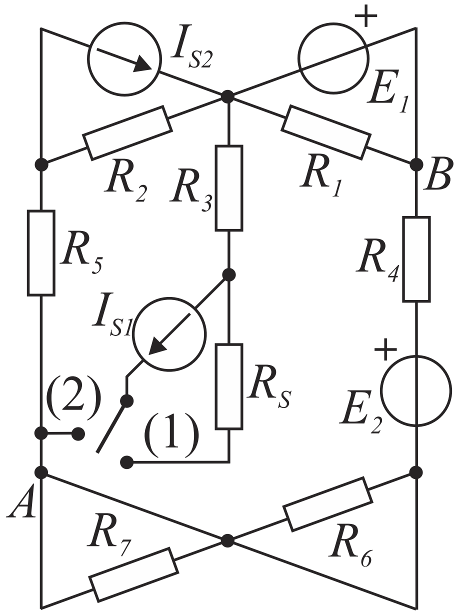
26.06.2019.

ZADACI

Zadatak 1. Kada se u mreži sa slike 1, preklopnik prebaci iz položaja (1) u položaj (2), napon između tačaka A i B se poveća za $\Delta U_{AB} = 5\ V$.

a) Primenjujući teoremu superpozicije, izračunati jačinu struje strujnog generatora, I_{S1}.
b) Izračunati snagu strujnog generatora I_{S1}, kada je preklopnik u položaju (2).
c) Izračunati snagu na otporniku otpornosti R_5, kada je preklopnik u položaju (2).

Brojni podaci su: $R_1 = 20\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = R_6 = R_7 = 15\ \Omega$, $R_S = 100\ \Omega$, $E_1 = 2\ V$, $E_2 = 6\ V$, $I_{S2} = 5\ A$.

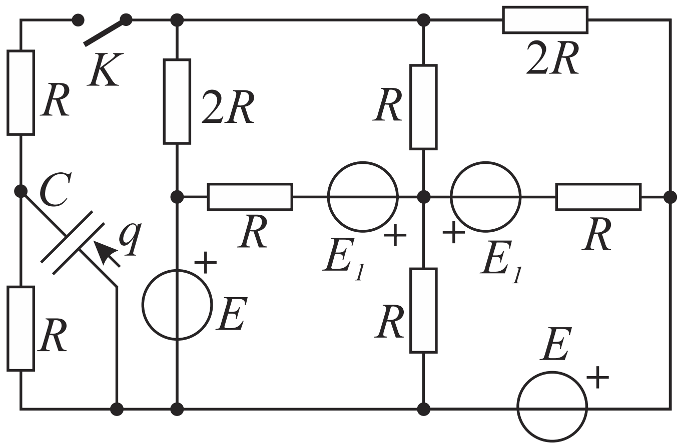


Slika 1.

Zadatak 2. U mreži vremenski konstantnih struja sa slike 2, izračunati količinu naelektrisanja q, koja će proteći kroz kondenzator C nakon zatvaranja prekidača K, u naznačenom referentnom smeru.

Prilikom rešavanja zadatka, primeniti Tevenenovu teoremu i metodu konturnih struja.

Brojni podaci su: $R = 28\ \Omega$, $E = 6\ V$, $E_1 = 3\ V$, $C = 3\ \mu F$.



Slika 2.

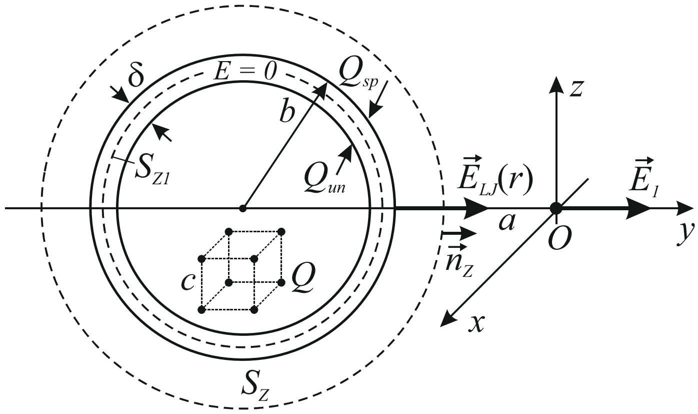
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET1, 26.06.2019.

I-1

a)



$$\oint_{S_{Z1}} \vec{E} \cdot d\vec{S} = \frac{Q_{unutar\,S_{Z1}}}{\varepsilon_0} \qquad E = 0, \quad \text{u provodniku}$$

$$0 = \frac{8Q + Q_{un}}{\varepsilon_0}$$

$$\boxed{Q_{un} = -8Q = -120\,nC}$$

$$Q_{LJ} = Q_{un} + Q_{sp} = 0$$

$$\boxed{Q_{sp} = -Q_{un} = 120\,nC}$$

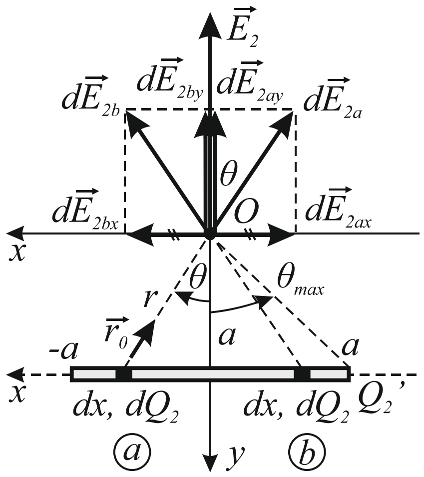
b)

$$\oint_{S_Z} \vec{E} \cdot d\vec{S} = \frac{Q_{unutar\,S_Z}}{\varepsilon_0} \qquad \sphericalangle\left(\vec{E}_{LJ}, \vec{n}_Z\right) = 0$$

$$E_{LJ}(r)\,4\pi r^2 = \frac{8Q + Q_{un} + Q_{sp}}{\varepsilon_0} = \frac{8Q}{\varepsilon_0}$$

$$E_{LJ}(r) = \frac{8Q}{4\pi\varepsilon_0 r^2}\,, \qquad r > b$$

$$\boxed{\vec{E}_1 = \vec{E}_{LJ\,u\,O}(r = a + b) = \frac{8Q}{4\pi\varepsilon_0 (a+b)^2} \cdot \vec{i}_y}$$



$$dE_{2a} = dE_{2b} = \frac{dQ_2}{4\pi\varepsilon_0 r^2} = \frac{Q_2'\,dx}{4\pi\varepsilon_0 r^2} \qquad Q_2' = \frac{Q_2}{2a}$$

$$d\vec{E}_{2ax} + d\vec{E}_{2bx} = 0 \quad \Rightarrow \quad \boxed{\vec{E}_{2x} = 0}$$

$$dE_{2ay} = dE_{2a}\cos\theta = \frac{Q_2'\,dx}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q_2'\dfrac{r\,d\theta}{\cos\theta}}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q_2'}{4\pi\varepsilon_0 \dfrac{a}{\cos\theta}}d\theta$$

$$E_2 = 2\int\limits_{\substack{po\ pola\\ štapa}} dE_{2ay} = 2\frac{Q_2'}{4\pi\varepsilon_0 a}\int_0^{\theta_{\max}} \cos\theta\,d\theta = \frac{Q_2'}{2\pi\varepsilon_0 a}\left(\sin\theta_{\max} - \sin 0\right)$$

$$\boxed{E_2 = \frac{Q_2'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}} \qquad \boxed{\vec{E}_2 = E_2 \cdot \left(-\vec{i}_y\right)}$$

$$\vec{E}_O = \vec{E}_1 + \vec{E}_2 = \frac{8Q}{4\pi\varepsilon_0 (a+b)^2} \cdot \vec{i}_y + \frac{Q_2'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2} \cdot \left(-\vec{i}_y\right) = 0$$

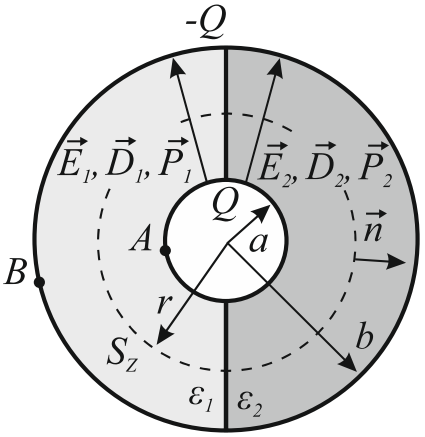
$$\Rightarrow \quad \frac{8Q}{4\pi\varepsilon_0 (a+b)^2} = \frac{Q_2'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}$$

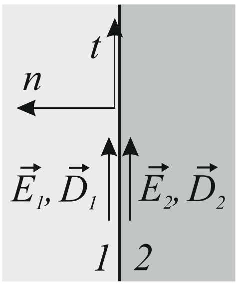
$$Q_2' = \frac{Q_2}{2a} = \frac{Q \cdot 8a}{\sqrt{2}(a+b)^2} \quad \Rightarrow \quad Q_2 = \frac{Q \cdot 16a^2}{\sqrt{2}(a+b)^2} \qquad \boxed{Q_2 = 36{,}7\,nC}$$

<u>Ispit iz OET1, 26.06.2019.</u>

I-2

a)





Granični uslov:

$$E_{t1} = E_{t2} \qquad E_1 = E_2 = E$$

$$D_{n1} \neq D_{n2}$$

$$\oint_{S_Z} \vec{D} \cdot \vec{ds} = Q_{slobodno\ u\ S_Z}$$

$$\int_{S_1} D_1\, ds + \int_{S_2} D_2\, ds = Q$$

$$D_1\, 2r^2\pi + D_2\, 2r^2\pi = Q$$

$$\varepsilon_1\, E\, 2r^2\pi + \varepsilon_2\, E\, 2r^2\pi = Q$$

$$\boxed{E = \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2r^2\pi}}, \qquad a \le r \le b$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2r^2\pi} dr = \frac{Q}{(\varepsilon_1 + \varepsilon_2) 2\pi} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}}$$

b)

Nakon ispustanja tečnog dielektrika: $\varepsilon_2 \quad \rightarrow \quad \varepsilon_0 \quad \Rightarrow \quad C^{NOVO} = \dfrac{(\varepsilon_{r1} + 1) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$

$$C^{NOVO} = \frac{C}{2}, \qquad C = 2C^{NOVO} \qquad \Rightarrow \qquad \frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = 2\frac{(\varepsilon_{r1} + 1) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

$$\varepsilon_{r2} = \varepsilon_{r1} + 2 = 7 \qquad \boxed{\varepsilon_2 = 7\,\varepsilon_0}$$

c)

$$\Delta W_e = W_e^{NOVO} - W_e = \frac{1}{2}\frac{Q^2}{C^{NOVO}} - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\frac{C}{2}} - \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{C}$$

$$\Delta W_e = \frac{1}{2}\frac{Q^2}{\frac{(\varepsilon_{r1} + \varepsilon_{r2}) 2\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}} = \frac{Q^2 \left(\frac{1}{a} - \frac{1}{b} \right)}{(\varepsilon_{r1} + \varepsilon_{r2}) 4\pi\varepsilon_0} = \frac{\left(1{,}2 \cdot 10^{-6}\right)^2 \cdot \left(\frac{1}{2 \cdot 10^{-3}} - \frac{1}{5 \cdot 10^{-3}} \right)}{(5+7) \cdot 4\pi \cdot 8{,}85 \cdot 10^{-12}}$$

$$\boxed{\Delta W_e = 0{,}32\ J}$$

Ispit iz OET1, 26.06.2019.

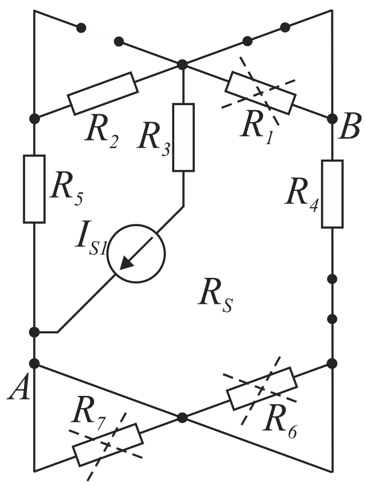
II-1

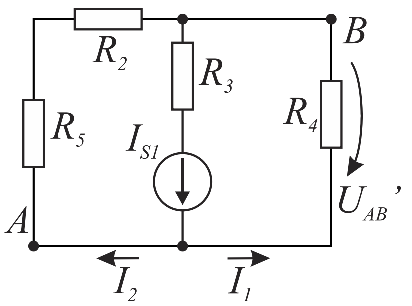
a)

$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ I_{S1}\end{matrix}} + \boxed{\begin{matrix}Samo\\ I_{S1}\end{matrix}}$$

(2) (1)

$$U_{AB}^{(2)} = U_{AB}^{(1)} + U_{AB}' \qquad \Rightarrow \qquad \Delta U_{AB} = U_{AB}^{(2)} - U_{AB}^{(1)} = U_{AB}' = 5\,V$$





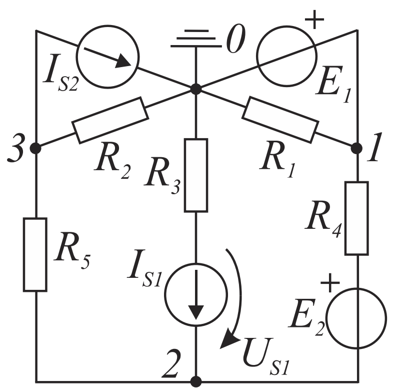
$$I_1 = \frac{U_{AB}}{R_4} = \frac{5}{5} = 1\,A$$

$$I_2 = \frac{U_{AB}}{R_5 + R_2} = \frac{5}{15+5} = 0,25\,A$$

$$I_{S1} = I_1 + I_2$$

$$\boxed{I_{S1} = 1,25\,A}$$

b)



$$V_0 = 0\,V, \qquad V_1 = E_1 = 2\,V$$

$$V_2\left(\frac{1}{R_4} + \frac{1}{R_3+\infty} + \frac{1}{R_5}\right) - V_1\left(\frac{1}{R_4}\right) - V_3\left(\frac{1}{R_5}\right) = I_{S1} - \frac{E_2}{R_4}$$

$$V_3\left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{\infty}\right) - V_2\left(\frac{1}{R_5}\right) = -I_{S2}$$

$$V_2\left(\frac{1}{5} + \frac{1}{15}\right) - 2\cdot\left(\frac{1}{5}\right) - V_3\left(\frac{1}{15}\right) = 1,25 - \frac{6}{5} \qquad /\cdot 15$$

$$V_3\left(\frac{1}{15} + \frac{1}{5}\right) - V_2\left(\frac{1}{15}\right) = -5 \qquad /\cdot 15$$

$$\left.\begin{matrix}4V_2 - V_3 = 6,75\\ 4V_3 - V_2 = -75\end{matrix}\right\} \quad \Rightarrow \quad V_2 = -3,2\,V, \qquad V_3 = -19,55\,V$$

$$U_{S1} = U_{20} + R_3\,I_{S1} = \left(V_2 - V_0\right) + R_3\,I_{S1} = \left(-3,2-0\right) + 10\cdot 1,25 = -3,2 + 12,5 = 9,3\,V$$

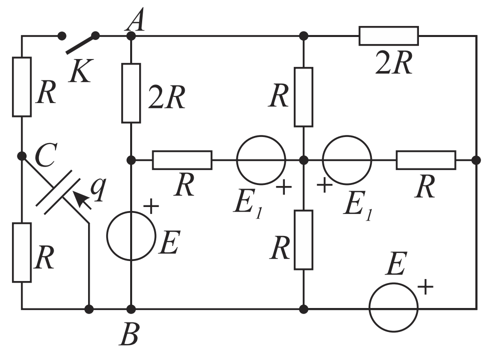
$$P_{S1} = U_{S1}\,I_{S1} = 9,3\cdot 1,25 \qquad \boxed{P_{S1} = 11,625\,W}$$

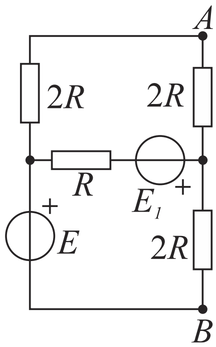
c)

$$U_{R5} = U_{23} = V_2 - V_3 = -3,2 - \left(-19,55\right) = 16,35\,V$$

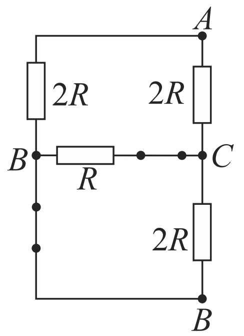
$$P_{R5} = \frac{U_{R5}^{\ 2}}{R_5} = \frac{16,35^2}{15} = \qquad \boxed{P_{R5} = 17,82\,W}$$

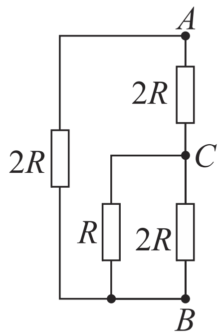
II-2





Kolo je simetrično u odnosu na tačke *A* i *B*.

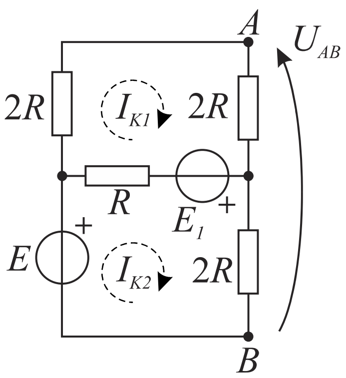




$$2R_T = R_{AB} = \left[\left(2R \parallel R\right) + 2R\right] \parallel 2R = \left(\frac{2}{3}R + 2R\right) \parallel 2R = \frac{8}{3}R \parallel 2R$$

$$2R_T = \frac{\frac{8}{3}R \cdot 2R}{\frac{8}{3}R + 2R} = \frac{\frac{16}{3}R \cdot R}{\frac{14}{3}R} = \frac{16}{14}R = 32\,\Omega$$

$$\boxed{R_T = 16\,\Omega}$$



$$I_{K1}\left(2R + 2R + R\right) - I_{K2}\,R = -E_1$$

$$I_{K2}\left(2R + R\right) - I_{K1}\,R = E + E_1$$

$$140\,I_{K1} - 28\,I_{K2} = -3$$

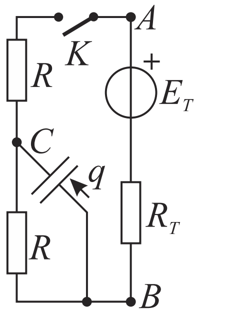
$$-28\,I_{K1} + 84\,I_{K2} = 9$$

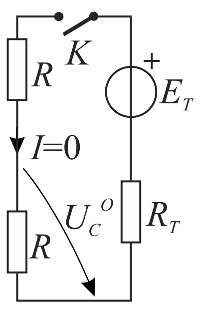
$$I_{K1} = 0\ A$$

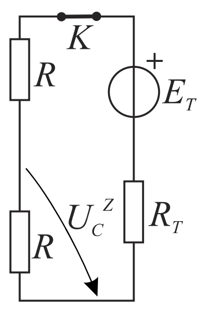
$$I_{K2} = 107{,}14\ mA$$

$$E_T = U_{AB} = 2R\,I_{K1} + 2R\,I_{K2} = 56 \cdot 0 + 56 \cdot 107{,}14 \cdot 10^{-3}$$

$$\boxed{E_T = 6\,V}$$







Prekidač K otvoren:

$$I = 0\ A \quad \Rightarrow \quad U_C^{\ O} = 0\,V$$

Prekidač K zatvoren:

$$U_C^{\ Z} = -\frac{R}{R_T + R + R}E_T = -\frac{28}{16 + 28 + 28} \cdot 6$$

$$U_C^{\ Z} = -2{,}33\,V$$

$$q = C\left(U_C^{\ Z} - U_C^{\ O}\right) = 3 \cdot 10^{-6} \cdot \left(-2{,}33 - 0\right) \qquad \boxed{q = -7\ \mu C}$$