STUDIJSKI PROGRAM ZA PRIMENJENO SOFTVERSKO INŽENJERSTVO		
K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike (I kolokvijum)	**K1** **18.06.2019.**

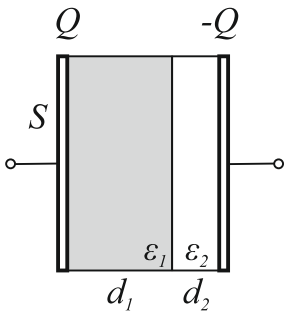
ZADACI

Zadatak 2. Na slici 1 je prikazan pločasti kondenzator ispunjen sa dva sloja dielektrika, relativnih permitivnosti $\varepsilon_{r1} = 2$ i $\varepsilon_{r2} = 4$, debljine $d_2 = 0{,}25\ mm$ i površine ploča $S = 4\ mm^2$.

a) Izvesti u opštim brojevima izraz za kapacitivnost kondenzatora.

b) Odrediti debljinu d_1, tako da kapacitivnost prvog sloja bude šest puta manja od kapacitivnosti drugog sloja.

c) Izračunati maksimalni napon na koji sme da se priključi kondenzator, ako su dielektrične čvrstine $E_{Č1} = 45\ kV/cm$ i $E_{Č2} = 70\ kV/cm$.



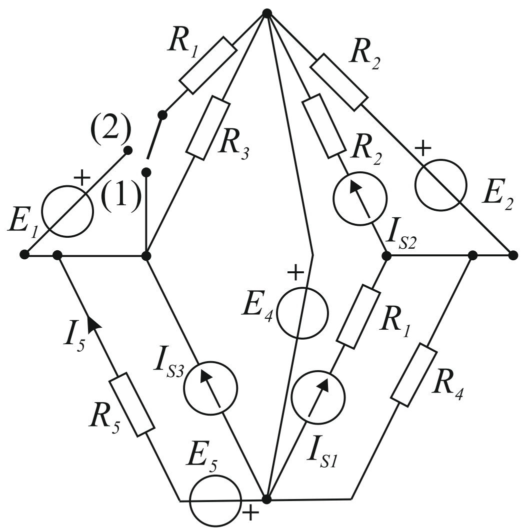
Slika 1.

Zadatak 2. Kada se u kolu vremenski konstantne struje sa slike 2 preklopnik prebaci iz položaja (1) u položaj (2), intenzitet struje I_5, sa naznačenim referentnim smerom na slici, poraste za $0{,}4\ mA$.

a) Primenjujući teoremu superpozicije, odrediti vrednost *ems* naponskog generatora E_1.

b) Odrediti snage strujnih generatora I_{S1} i I_{S3} kada je preklopnik u položaju (2).

Brojni podaci su: $R_1 = R_5 = 10\ k\Omega$, $R_2 = 6\ k\Omega$, $R_3 = 8\ k\Omega$, $R_4 = 2\ k\Omega$, $E_2 = 3\ V$, $E_4 = 1\ V$, $E_5 = 2\ V$, $I_{S1} = 1\ mA$, $I_{S2} = 2\ mA$, $I_{S3} = 3\ mA$.



Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum je neophodno sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike (II kolokvijum)

K2

18.06.2019.

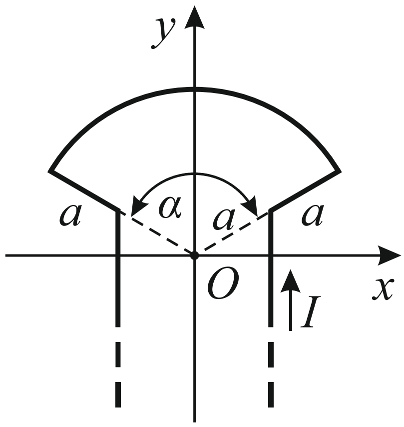
ZADACI

Zadatak 1. Veoma dugačak žičani provodnik sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Kontura se nalazi u x-y ravni Dekartovog pravougaonog koordinatnog sistema:

a) Odrediti u opštim brojevima izraz za ukupan vektor magnetske indukcije u koordinatnom početku.

b) Izračunati intenzitet vektora magnetske indukcije u koordinatnom početku.

Brojne vrednosti su: $I = 3\ A$, $a = 2\ cm$, $\mu_0 = 4\pi \cdot 10^{-7} H/m$, $\alpha = 120°$.



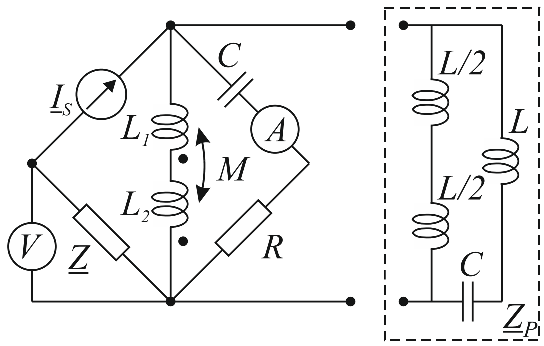
Slika 1.

Zadatak 2. U mreži prostoperiodičnih struja prikazanoj na slici 2:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Odrediti pokazivanja idealnih mernih instrumenata, kada je prijemnik priključen na ulazni deo kola, pri čemu strujni generator $\underline{I}_S$ radi na većoj rezonantnoj učestanosti.

Brojni podaci: $L = 10\ mH$, $C = 40\ nF$, $R = 1\ k\Omega$,
$\underline{Z} = (5 + j3)\ k\Omega$, $\underline{I}_S = (1 - j)\ mA$, $L_1 = 2L$, $L_2 = 3L$, $k = 0{,}82$.



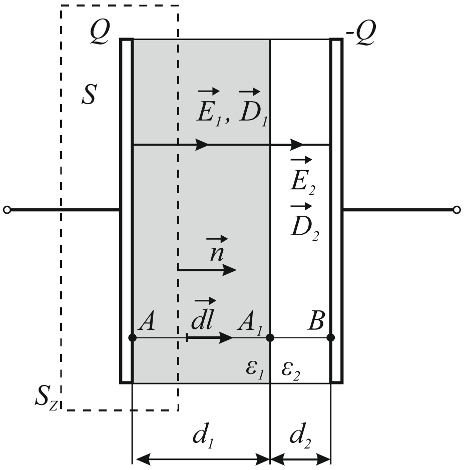
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

PSI ISPIT 18.06.2019.

K1 Z1



a)

Granični uslov:

$D_{n1} = D_{n2}$ $\qquad D_1 = D_2 = D$

$E_{t1} \neq E_{t2}$

$$\oint_S \vec{D} \cdot \vec{dS} = Q_{slobodno\ u\ S}$$

$$\int_S D\, dS = Q$$

$$DS = Q \qquad \boxed{D = \frac{Q}{S}}$$

$$\boxed{E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{\varepsilon_1 S}} \qquad \boxed{E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{\varepsilon_2 S}}$$

$$C = \frac{Q}{U}$$

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_A^{A_1} \vec{E_1} \cdot \vec{dl} + \int_{A_1}^B \vec{E_2} \cdot \vec{dl} = E_1 d_1 + E_2 d_2 = \frac{Q}{S}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$\boxed{C = \frac{Q}{\frac{Q}{S}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)} = \frac{1}{\frac{d_1}{S\varepsilon_1} + \frac{d_2}{S\varepsilon_2}}}$$

b)

$$C_1 = \varepsilon_1 \frac{S}{d_1}, \qquad C_2 = \varepsilon_2 \frac{S}{d_2}$$

$$C_1 = \frac{C_2}{6}$$

$$\varepsilon_1 \frac{S}{d_1} = \frac{1}{6} \varepsilon_2 \frac{S}{d_2}$$

$$d_1 = \frac{6 \cdot \varepsilon_1}{\varepsilon_2} d_2 = \frac{6 \cdot 2\varepsilon_0}{4\varepsilon_0} \cdot 0,25m$$

$$\boxed{d_1 = 0,75\ mm}$$

c)

$$U_{\max} = \frac{Q_{\max}}{C}$$

$$C = 81,27\ nF$$

$$Q_{\max 1} = E_{1\max} \cdot \varepsilon_1 \cdot S = E_{\check{c}1} \cdot \varepsilon_1 \cdot S = 3,186 \cdot 10^{-10} C$$

$$Q_{\max 2} = E_{2\max} \cdot \varepsilon_2 \cdot S = E_{\check{c}2} \cdot \varepsilon_2 \cdot S = 9,912 \cdot 10^{-10} C$$

$$Q_{\max} = \min\{Q_{\max 1}, Q_{\max 2}\} = Q_{\max 1} = 3,186 \cdot 10^{-10} C$$

$$\boxed{U_{\max} = \frac{Q_{\max}}{C} = 3,92\ kV}$$

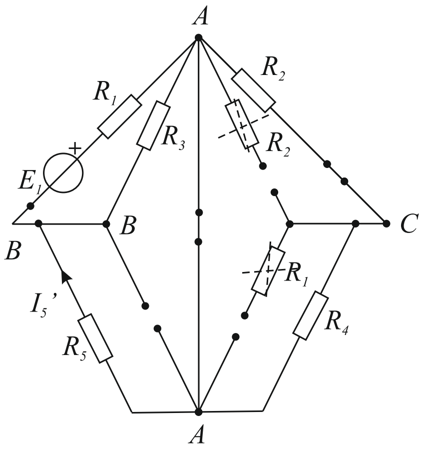
K1 Z2

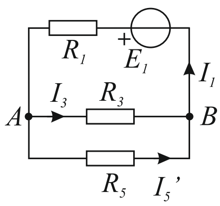
a)

$$\boxed{\begin{matrix}Svi\\ generatori\end{matrix}} = \boxed{\begin{matrix}Svi\\ sem\ E_1\end{matrix}} + \boxed{\begin{matrix}Samo\\ E_1\end{matrix}}$$

(2) (1)

$$I_5^{(2)} = I_5^{(1)} + I_5{}' \qquad \Rightarrow \qquad \Delta I_5 = I_5^{(2)} - I_5^{(1)} = I_5{}'$$

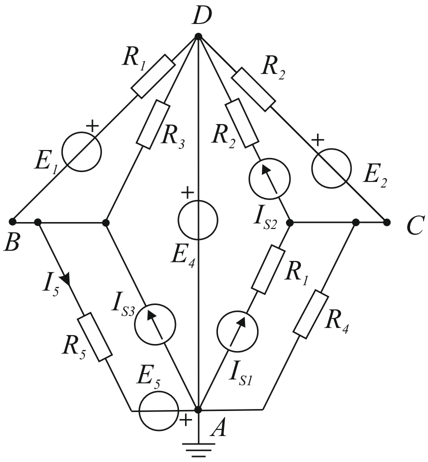




$$\Delta I_5 = I_5{}' = 0,4\,mA$$
$$U_{AB} = R_5 I_5{}' = 4V$$
$$I_3 = \frac{U_{AB}}{R_3} = \frac{4}{8k} = 0,5\,mA$$

$$I_1 = I_5{}' + I_3 = 0,9\,mA$$
$$E_1 = I_1 R_1 + I_3 R_3 = 0,9\,m \cdot 10k + 0,5\,m \cdot 8k$$
$$\boxed{E_1 = 13\,V}$$

b)



$$V_A = 0\,V, \qquad V_D = E_4 = 1V$$

$$V_B\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{\infty}\right) - V_D\left(\frac{1}{R_1}+\frac{1}{R_3}\right) = I_{S3} - \frac{E_1}{R_1} - \frac{E_5}{R_5}$$

$$V_C\left(\frac{1}{R_2+\infty}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_1+\infty}\right) - V_D\left(\frac{1}{R_2+\infty}+\frac{1}{R_2}\right) = I_{S1} - I_{S2} - \frac{E_2}{R_2}$$

$$V_B\left(\frac{1}{10k}+\frac{1}{8k}+\frac{1}{10k}\right) - 1\cdot\left(\frac{1}{10k}+\frac{1}{8k}\right) = 3\,m - \frac{13}{10k} - \frac{2}{10k} \qquad / \cdot 40k$$

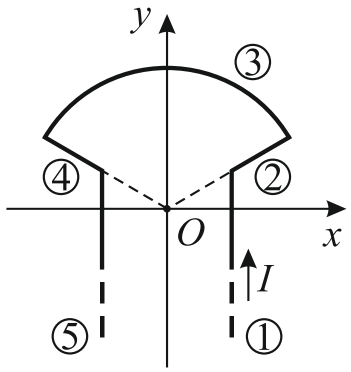
$$V_C\left(\frac{1}{6k}+\frac{1}{2k}\right) - 1\cdot\left(\frac{1}{6k}\right) = 1\,m - 2\,m - \frac{3}{6k} \qquad / \cdot 6k$$

$$13V_B = 69 \qquad \Rightarrow \qquad V_B = 5,31\,V$$
$$4V_C = -8 \qquad \Rightarrow \qquad V_C = -2\,V$$

$$P_{IS1} = U_{IS1} I_{S1} = (R_1 I_{S1} + V_C - V_A) I_{S1}$$
$$P_{IS1} = (10k \cdot 1m - 2)\cdot 1m$$
$$\boxed{P_{IS1} = 8\,mW}$$

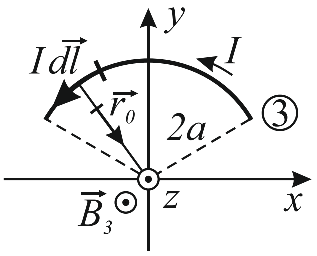
$$P_{IS3} = U_{IS3} I_{S3} = (V_B - V_A) I_{S3}$$
$$P_{IS3} = 5,31 \cdot 3m$$
$$\boxed{P_{IS3} = 15,93\,mW}$$

K2 Z1



$$\vec{B}_{uk} = \sum_{i=1}^{5} \vec{B}_i$$

$$\vec{B}_2 = \vec{B}_4 = 0 \qquad \left(\vec{dl} \times \vec{r}_0 = 0\right)$$



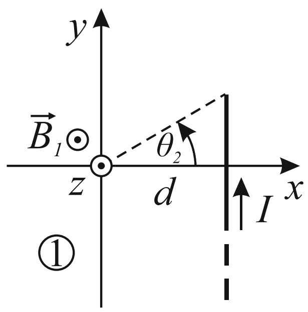
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \vec{r}_0}{r^2}$$

$$dB_3 = \frac{\mu_0}{4\pi} \frac{Idl}{(2a)^2} \sin \measuredangle\left(\vec{dl}, \vec{r}_0\right) \qquad \measuredangle\left(\vec{dl}, \vec{r}_0\right) = \frac{\pi}{2}$$

$$dB_3 = \frac{\mu_0}{4\pi} \frac{Idl}{(2a)^2}$$

$$B_3 = \frac{\mu_0 I}{4\pi (2a)^2} \int_0^{2a\frac{2\pi}{3}} dl = \frac{\mu_0 I}{4\pi (2a)^2} \cdot 2a \frac{2\pi}{3}$$

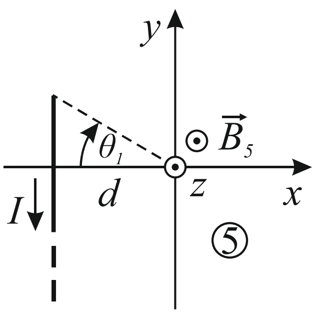
$$\boxed{B_3 = \frac{\mu_0 I}{12a}} \qquad \boxed{\vec{B}_3 = B_3 \cdot (\vec{i}_z)}$$



$$\theta_1 = -\frac{\pi}{2}, \qquad \theta_2 = \frac{\pi}{6}, \qquad d = a\cos\frac{\pi}{6} = \frac{a\sqrt{3}}{2}$$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi \frac{a\sqrt{3}}{2}}\left(\sin\frac{\pi}{6} - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 I}{2\pi a\sqrt{3}} \cdot \left(\frac{1}{2} + 1\right)$$

$$\boxed{B_1 = \frac{3\mu_0 I}{4\pi a\sqrt{3}}} \qquad \boxed{\vec{B}_1 = B_1 \cdot \left(\vec{i}_z\right)}$$



$$\theta_1 = -\frac{\pi}{2}, \qquad \theta_2 = \frac{\pi}{6}, \qquad d = a\cos\frac{\pi}{6} = \frac{a\sqrt{3}}{2}$$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi \frac{a\sqrt{3}}{2}}\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{6}\right)\right) = \frac{\mu_0 I}{2\pi a\sqrt{3}} \cdot \left(1 + \frac{1}{2}\right)$$

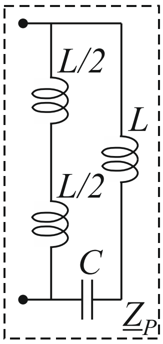
$$\boxed{B_5 = \frac{3\mu_0 I}{4\pi a\sqrt{3}}} \qquad \boxed{\vec{B}_5 = B_5 \cdot \left(\vec{i}_z\right)}$$

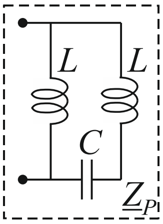
$$\boxed{\vec{B}_{uk} = \left[\frac{3\mu_0 I}{4\pi a\sqrt{3}} + 0 + \frac{\mu_0 I}{12a} + 0 + \frac{3\mu_0 I}{4\pi a\sqrt{3}}\right] \cdot \left(\vec{i}_z\right)}$$

$$\boxed{\vec{B}_{uk} = 67\,\mu T\left(\vec{i}_z\right)}$$

K2 Z2

a)





$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_P = \underline{Z}_1 \parallel j\omega L = \frac{\frac{1-\omega^2 LC}{j\omega C}\, j\omega L}{\frac{1-\omega^2 LC}{j\omega C} + j\omega L} = \frac{\frac{j\omega L\left(1-\omega^2 LC\right)}{j\omega C}}{\frac{1-\omega^2 LC-\omega^2 LC}{j\omega C}}$$

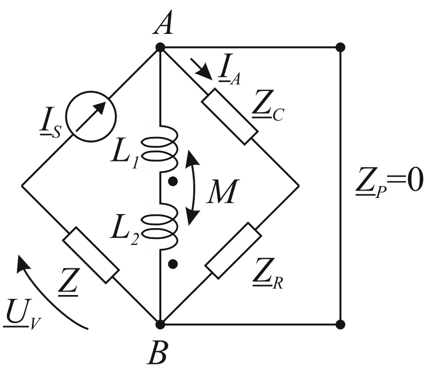
$$\underline{Z}_P = \frac{j\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC} \qquad \boxed{\underline{Z}_P = j\frac{\omega L\left(1-\omega^2 LC\right)}{1-2\omega^2 LC}}$$

$$\mathrm{Im}\{\underline{Z}_P\}=0 \quad \Rightarrow \quad \omega L = 0 \qquad \boxed{\omega_{r1}=0}$$

$$1-\omega^2 LC = 0 \qquad \boxed{\omega_{r2} = \frac{1}{\sqrt{LC}}}$$

$$\mathrm{Im}\{\underline{Y}_P\}=0 \quad \Rightarrow \quad 1-2\omega^2 LC = 0 \qquad \boxed{\omega_{ar1} = \frac{1}{\sqrt{2LC}}} \qquad \boxed{\omega_{ar2} \to \infty}$$

b)



$$\omega_{r2} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 40\cdot 10^{-9}}} = 5\cdot 10^4\,\frac{rad}{s} = \omega$$

$$\underline{Z}_P = 0$$

$$M = k\sqrt{L_1 L_2} = k\sqrt{2L\cdot 3L} = 0{,}82\cdot\sqrt{20\,mH\cdot 30\,mH}$$

$$M = 20\,mH$$

$$\underline{U}_{AB} = 0 \quad \Rightarrow \quad \underline{I}_A = 0$$

$$\boxed{I_A = |\underline{I}_A| = 0\,A}$$

$$\underline{U}_V = \underline{Z}\,\underline{I}_S = (5+j3)k\cdot(1-j)m = 5+3+j3-j5 = (8-j2)V$$

$$U_V = |\underline{U}_V| = \sqrt{8^2+(-2)^2} \qquad \boxed{U_V = 8{,}25\,V}$$