

## ZADACI

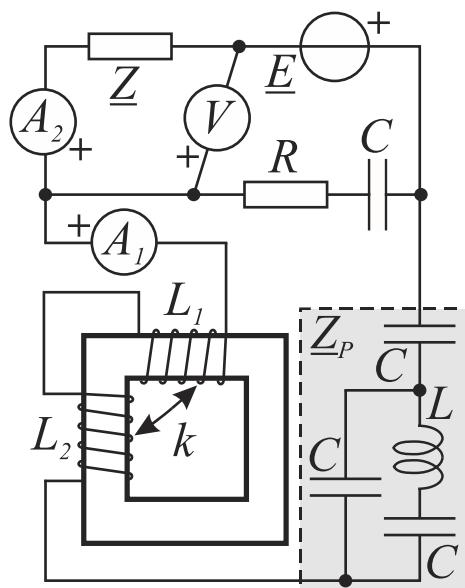
**Zadatak 1.** U mreži prostoperiodične struje prikazanoj na slici 1:

- Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse  $\underline{Z}_P$ .
- Nacrtati pojednostavljenu električnu šemu kola i u njoj označiti tip magnetske sprega koja postoji između namotaja induktivnosti  $L_1$  i  $L_2$ .
- Odrediti pokazivanja idealnih mernih instrumenata, u situaciji kada je ugaona učestanost generatora elektromotorne sile  $\underline{E}$  jednaka većoj antirezonantnoj učestanosti prijemnika  $\underline{Z}_P$ .

Brojni podaci:  $L = 10 \text{ mH}$ ,  $C = 200 \mu\text{F}$ ,  $R = 4 \Omega$ ,

$\underline{Z} = (1 + j5) \Omega$ ,  $\underline{E} = (15 + j5) \text{ V}$ ,

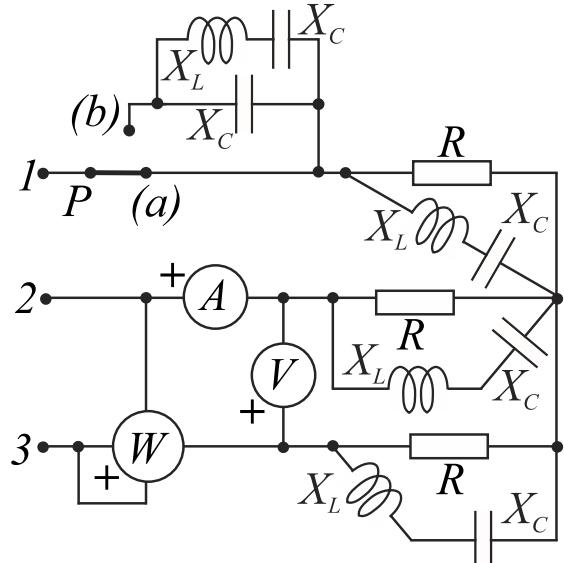
$L_1 = 2L$ ,  $L_2 = 3L$ ,  $k = 0,7$ .



Slika 1.

**Zadatak 2.** Prijemnik sa slike 2 je sastavljen od otpornika otpornosti  $R = 20 \Omega$ , kalemova reaktanse  $X_L = 20 \Omega$  i kondenzatora reaktanse  $X_C = 10 \Omega$ , i priključen je na mrežu faznog napona  $\underline{U}_1 = 230 \text{ V}$ . Kada je prekidač  $P$  u položaju (a), trofazni prijemnik je simetričan.

- Odrediti pokazivanja idealnih mernih instrumenata kada je prekidač  $P$  u inicijalnom položaju (a), kao na slici 2.
- Odrediti pokazivanja idealnih mernih instrumenata nakon promene položaja prekidača  $P$  u položaj (b).
- Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



Slika 2.

## PRAVILA POLAGANJA

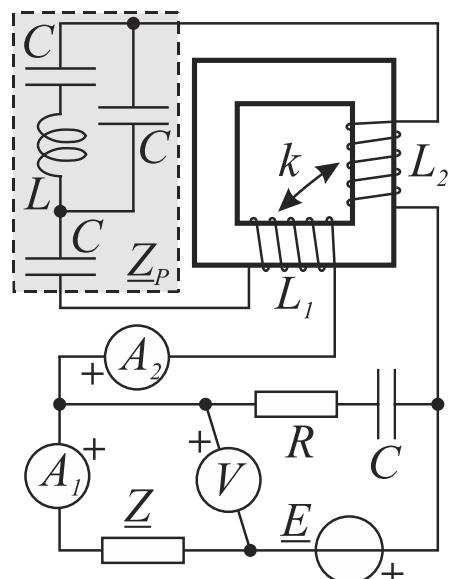
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

## ZADACI

**Zadatak 1.** U mreži prostoperiodične struje prikazanoj na slici 1:

- Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse  $\underline{Z}_P$ .
- Nacrtati pojednostavljenu električnu šemu kola i u njoj označiti tip magnetske sprega koja postoji između namotaja induktivnosti  $L_1$  i  $L_2$ .
- Odrediti pokazivanja idealnih mernih instrumenata, u situaciji kada je ugaona učestanost generatora elektromotorne sile  $\underline{E}$  jednaka većoj antirezonantnoj učestanosti prijemnika  $\underline{Z}_P$ .

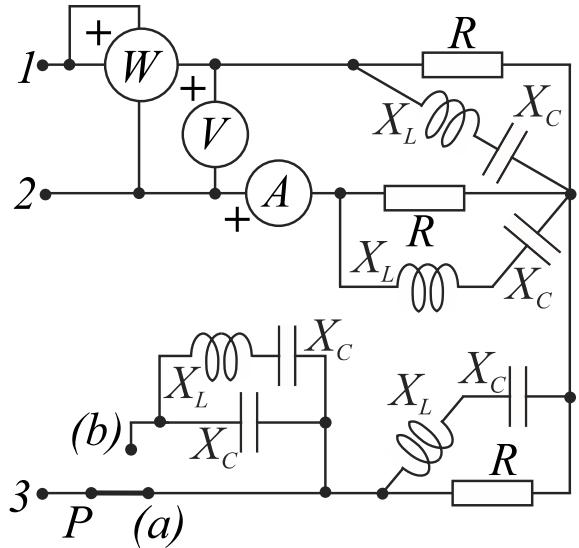
Brojni podaci:  $L = 10 \text{ mH}$ ,  $C = 50 \mu\text{F}$ ,  $R = 4 \Omega$ ,  
 $\underline{Z} = (1 + j10) \Omega$ ,  $\underline{E} = (20 + j10) \text{ V}$ ,  
 $L_1 = 2L$ ,  $L_2 = 4L$ ,  $k = 0,8$ .



Slika 1.

**Zadatak 2.** Prijemnik sa slike 2 je sastavljen od otpornika otpornosti  $R = 40 \Omega$ , kalemova reaktanse  $X_L = 40 \Omega$  i kondenzatora reaktanse  $X_C = 20 \Omega$ , i priključen je na mrežu faznog napona  $\underline{U}_1 = 230 \text{ V}$ . Kada je prekidač  $P$  u položaju (a), trofazni prijemnik je simetričan.

- Odrediti pokazivanja idealnih mernih instrumenata kada je prekidač  $P$  u inicijalnom položaju (a), kao na slici 2.
- Odrediti pokazivanja idealnih mernih instrumenata nakon promene položaja prekidača  $P$  u položaj (b).
- Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

a)

$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel \frac{1}{j\omega C} = \frac{\frac{1 - \omega^2 LC}{j\omega C}}{\frac{1 - \omega^2 LC}{j\omega C} + \frac{1}{j\omega C}} = \frac{\frac{1 - \omega^2 LC}{j\omega C}}{\frac{2 - \omega^2 LC}{j\omega C}} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)}$$

$$\underline{Z}_P = \underline{Z}_2 + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)} + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)} + \frac{2 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)}$$

$$\underline{Z}_P = -j \frac{3 - 2\omega^2 LC}{\omega C(2 - \omega^2 LC)}$$

$$\text{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow$$

$$\omega_{r1} = \sqrt{\frac{3}{2LC}}$$

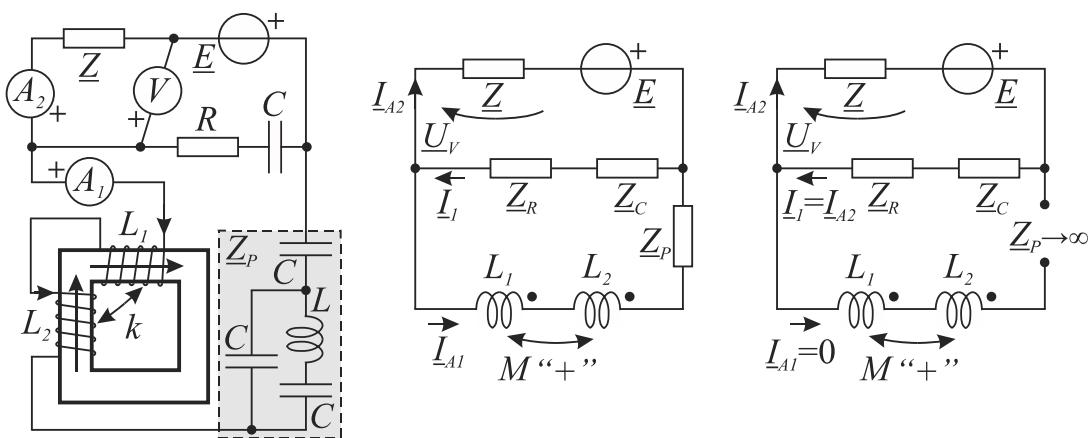
$$\omega_{r2} \rightarrow \infty$$

$$\text{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow$$

$$\omega_{ar1} = 0$$

$$\omega_{ar2} = \sqrt{\frac{2}{LC}}$$

b)



c)

$$\omega = \omega_{ar2} = \sqrt{\frac{2}{LC}} = \sqrt{\frac{2}{10 \cdot 10^{-3} \cdot 200 \cdot 10^{-6}}} = 1000 \frac{\text{rad}}{\text{s}}$$

$$M = k\sqrt{L_1 L_2} = k\sqrt{2L \cdot 3L} = 0,7 \cdot \sqrt{20 \cdot 10^{-3} \cdot 30 \cdot 10^{-3}} = 17,15 \text{ mH}$$

$$\underline{Z}_R = R = 4 \Omega, \quad \underline{Z}_C = -j \frac{1}{\omega C} = -j5 \Omega$$

$$\underline{Z}_P \rightarrow \infty \quad \Rightarrow \quad I_{A1} = 0 \text{ A}$$

$$-\underline{E} + \underline{Z}_C \underline{I}_{A2} + \underline{Z}_R \underline{I}_{A2} + \underline{Z} \underline{I}_{A2} = 0 \quad \Rightarrow \quad \underline{I}_{A2} = \frac{\underline{E}}{\underline{Z}_C + \underline{Z}_R + \underline{Z}} = \frac{15 + j5}{-j5 + 4 + (1 + j5)} = (3 + j) \text{ A}$$

$$I_{A2} = |\underline{I}_{A2}| = \sqrt{3^2 + 1^2} \quad I_{A2} = 3,16 \text{ A}$$

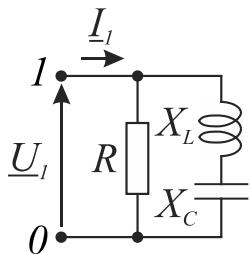
$$\underline{U}_V = \underline{Z} \underline{I}_{A2} = (1 + j5) \cdot (3 + j) = 3 - 5 + j + j15 = (-2 + j16) \text{ V}$$

$$U_V = |\underline{U}_V| = \sqrt{(-2)^2 + 16^2} \quad U_V = 16,12 \text{ V}$$

II-2

A grupa

a)



$$\underline{Z} = \frac{R(jX_L - jX_C)}{R + (jX_L - jX_C)} = \frac{20 \cdot (j20 - j10)}{20 + (j20 - j10)}$$

$$\underline{Z} = (4 + j8) \Omega = 8,94 e^{j63,43^\circ} \Omega$$

$$I_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{\underline{U}_1 e^{j0^\circ}}{\underline{Z} e^{j\varphi}} = \frac{230}{8,94 e^{j63,43^\circ}} = 25,73 e^{-j63,43^\circ} A$$

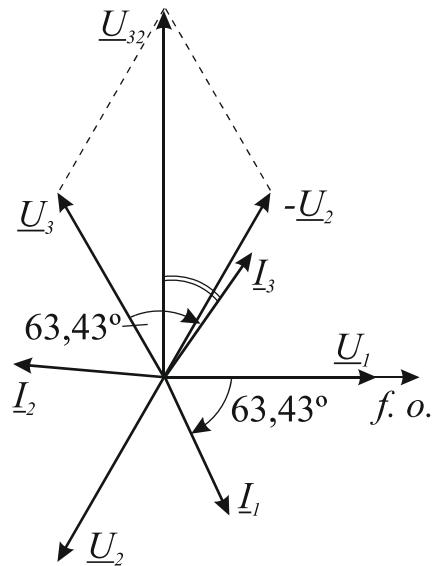
$$I_A^{(a)} = |I_2| = |I_1| = 25,73 A$$

$$U_V^{(a)} = |\underline{U}_{32}| = \sqrt{3} |\underline{U}_1| = 398,4 V$$

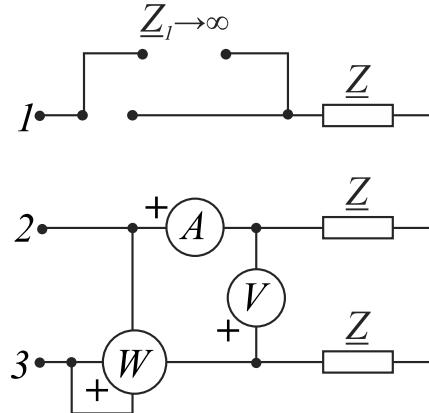
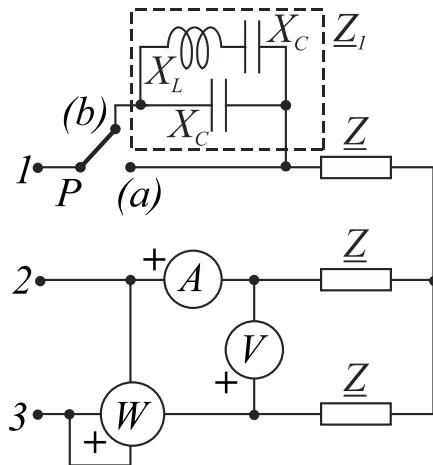
$$P_W^{(a)} = \operatorname{Re}\{\underline{U}_{32} I_3^*\} = \underline{U}_{32} I_3 \cos \angle(\underline{U}_{32}, I_3)$$

$$P_W^{(a)} = \sqrt{3} \underline{U}_1 I_1 \cos(63,43^\circ - 30^\circ) = \sqrt{3} \cdot 230 \cdot 25,73 \cdot \cos 33,43^\circ$$

$$P_W^{(a)} = 8554,32 W$$



b)



$$\underline{Z}_l = \frac{(jX_L - jX_C) \cdot (-jX_C)}{(jX_L - jX_C) + (-jX_C)} = \frac{(j20 - j10) \cdot (-j10)}{(j20 - j10) + (-j10)} \quad \underline{Z}_l \rightarrow \infty$$

$$I_W = \frac{\underline{U}_{32}}{2\underline{Z}} = \frac{\sqrt{3} \cdot 230 e^{j90^\circ}}{2 \cdot 8,94 e^{j63,43^\circ}} = 22,28 e^{j26,57^\circ} A$$

$$I_A^{(b)} = -I_W = -22,28 e^{j26,57^\circ} = 22,28 e^{-j153,43^\circ} A$$

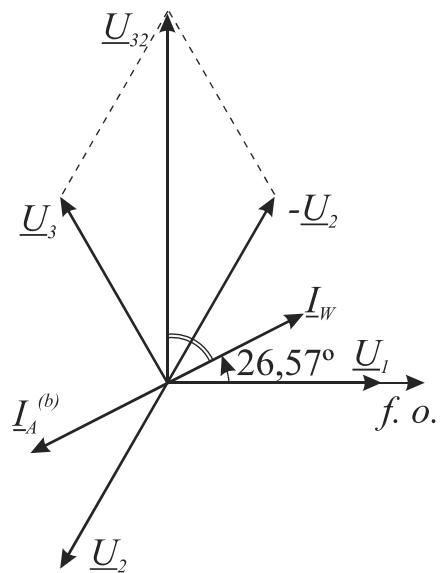
$$I_A^{(b)} = |I_A^{(b)}| = 22,28 A$$

$$U_V^{(b)} = |\underline{U}_{32}| = \sqrt{3} |\underline{U}_1| = 398,4 V$$

$$P_W^{(b)} = \operatorname{Re}\{\underline{U}_{32} I_W^*\} = \underline{U}_{32} I_W \cos \angle(\underline{U}_{32}, I_W)$$

$$P_W^{(b)} = \sqrt{3} \underline{U}_1 I_W \cos(90^\circ - 26,57^\circ) = \sqrt{3} \cdot 230 \cdot 22,28 \cdot \cos 63,43^\circ$$

$$P_W^{(b)} = 3970,03 W$$



a)

$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel \frac{1}{j\omega C} = \frac{\frac{1 - \omega^2 LC}{j\omega C}}{\frac{1 - \omega^2 LC}{j\omega C} + \frac{1}{j\omega C}} = \frac{\frac{1 - \omega^2 LC}{j\omega C}}{\frac{2 - \omega^2 LC}{j\omega C}} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)}$$

$$\underline{Z}_p = \underline{Z}_2 + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)} + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)} + \frac{2 - \omega^2 LC}{j\omega C(2 - \omega^2 LC)}$$

$$\underline{Z}_p = -j \frac{3 - 2\omega^2 LC}{\omega C(2 - \omega^2 LC)}$$

$$\text{Im}\{\underline{Z}_p\} = 0 \quad \Rightarrow$$

$$\omega_{r1} = \sqrt{\frac{3}{2LC}}$$

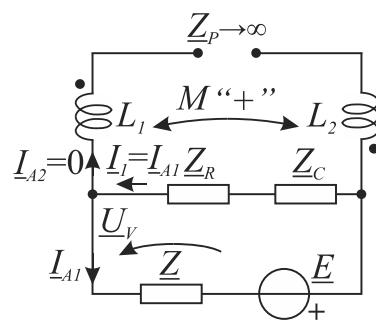
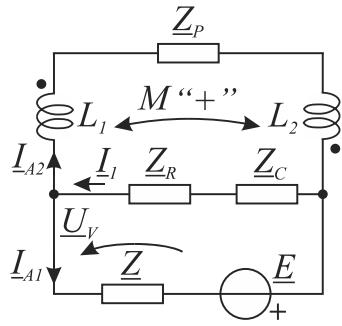
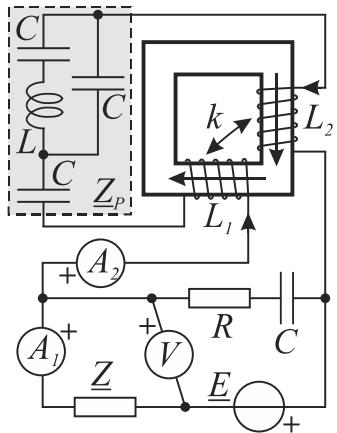
$$\omega_{r2} \rightarrow \infty$$

$$\text{Im}\{\underline{Y}_p\} = 0 \quad \Rightarrow$$

$$\omega_{ar1} = 0$$

$$\omega_{ar2} = \sqrt{\frac{2}{LC}}$$

b)



c)

$$\omega = \omega_{ar2} = \sqrt{\frac{2}{LC}} = \sqrt{\frac{2}{10 \cdot 10^{-3} \cdot 50 \cdot 10^{-6}}} = 2000 \frac{\text{rad}}{\text{s}}$$

$$M = k\sqrt{L_1 L_2} = k\sqrt{2L \cdot 4L} = 0,8 \cdot \sqrt{20 \cdot 10^{-3} \cdot 40 \cdot 10^{-3}} = 22,63 \text{ mH}$$

$$\underline{Z}_R = R = 4 \Omega, \quad \underline{Z}_C = -j \frac{1}{\omega C} = -j10 \Omega$$

$$\underline{Z}_p \rightarrow \infty \quad \Rightarrow \quad I_{A2} = 0 \text{ A}$$

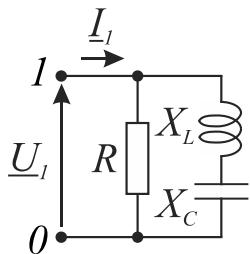
$$-\underline{E} + \underline{Z}_C \underline{I}_{A1} + \underline{Z}_R \underline{I}_{A1} + \underline{Z} \underline{I}_{A1} = 0 \quad \Rightarrow \quad \underline{I}_{A1} = \frac{\underline{E}}{\underline{Z}_C + \underline{Z}_R + \underline{Z}} = \frac{20 + j10}{-j10 + 4 + (1 + j10)} = (4 + j2) \text{ A}$$

$$I_{A1} = |\underline{I}_{A1}| = \sqrt{4^2 + 2^2} \quad |I_{A1}| = 4,47 \text{ A}$$

$$\underline{U}_V = \underline{Z} \underline{I}_{A1} = (1 + j10) \cdot (4 + j2) = 4 - 20 + j2 + j40 = (-16 + j42) \text{ V}$$

$$U_V = |\underline{U}_V| = \sqrt{(-16)^2 + 42^2} \quad |U_V| = 44,94 \text{ V}$$

a)



$$\underline{Z} = \frac{R(jX_L - jX_C)}{R + (jX_L - jX_C)} = \frac{40 \cdot (j40 - j20)}{40 + (j40 - j20)}$$

$$\underline{Z} = (8 + j16) \Omega = 17,89 e^{j63,43^\circ} \Omega$$

$$I_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{\underline{U}_1 e^{j0^\circ}}{Z e^{j\varphi}} = \frac{230}{17,89 e^{j63,43^\circ}} = 12,86 e^{-j63,43^\circ} A$$

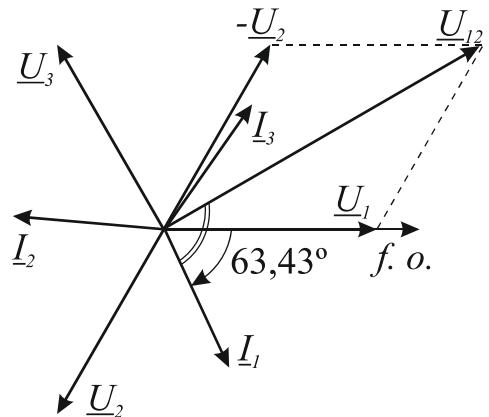
$$I_A^{(a)} = |I_2| = |I_1| = 12,86 A$$

$$U_V^{(a)} = |\underline{U}_{12}| = \sqrt{3} |\underline{U}_1| = 398,4 V$$

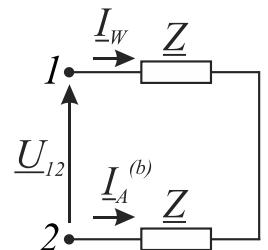
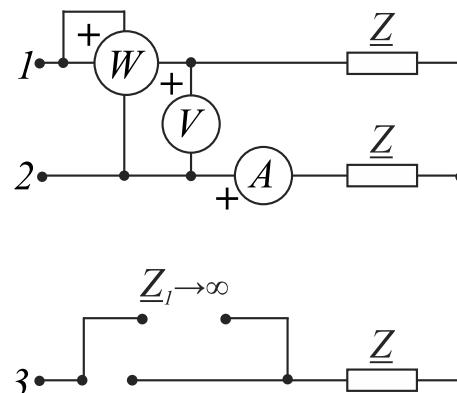
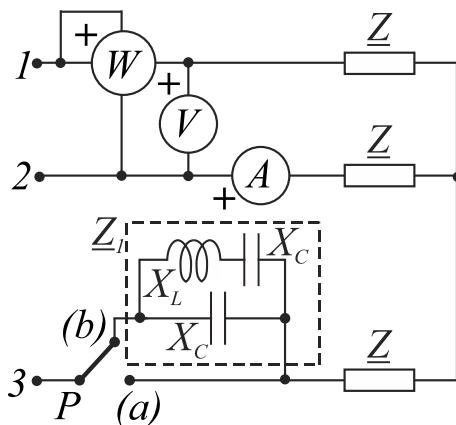
$$P_W^{(a)} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_1^*\} = \underline{U}_{12} \underline{I}_1 \cos \angle(\underline{U}_{12}, \underline{I}_1)$$

$$P_W^{(a)} = \sqrt{3} \underline{U}_1 \underline{I}_1 \cos(63,43^\circ + 30^\circ) = \sqrt{3} \cdot 230 \cdot 12,86 \cdot \cos 93,43^\circ$$

$$P_W^{(a)} = -306,51 W$$



b)



$$\underline{Z}_1 = \frac{(jX_L - jX_C) \cdot (-jX_C)}{(jX_L - jX_C) + (-jX_C)} = \frac{(j40 - j20) \cdot (-j20)}{(j40 - j20) + (-j20)} \quad \underline{Z}_1 \rightarrow \infty$$

$$I_W = \frac{\underline{U}_{12}}{2\underline{Z}} = \frac{\sqrt{3} \cdot 230 e^{j30^\circ}}{2 \cdot 17,89 e^{j63,43^\circ}} = 11,13 e^{-j33,43^\circ} A$$

$$I_A^{(b)} = -I_W = -11,13 e^{-j33,43^\circ} = 11,13 e^{j146,57^\circ} A$$

$$I_A^{(b)} = |I_A^{(b)}| = 11,13 A$$

$$U_V^{(b)} = |\underline{U}_{12}| = \sqrt{3} |\underline{U}_1| = 398,4 V$$

$$P_W^{(b)} = \operatorname{Re}\{\underline{U}_{12} \underline{I}_W^*\} = \underline{U}_{12} \underline{I}_W \cos \angle(\underline{U}_{12}, \underline{I}_W)$$

$$P_W^{(b)} = \sqrt{3} \underline{U}_1 \underline{I}_W \cos(30^\circ + 33,43^\circ) = \sqrt{3} \cdot 230 \cdot 11,13 \cdot \cos 63,43^\circ$$

$$P_W^{(b)} = 1983,23 W$$

