STUDIJSKI PROGRAM MERENJE I REGULACIJA

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1
(I kolokvijum)

A

01.12.2018.

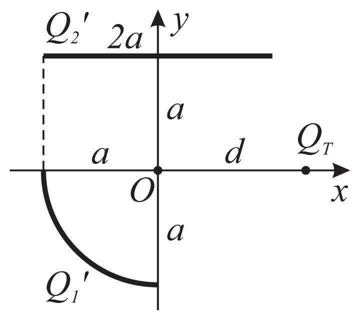
ZADACI

Zadatak 1. Na slici 1 je prikazana složena, ravnomerno naelektrisana tanka struktura, sastavljena od štapa savijenog u obliku luka poluprečnika a, i pravolinijskog štapa, dužine $2a$, postavljenog paralelno sa x osom. Celokupna struktura se nalazi u x-y ravni Dekartovog pravouglog koordinatnog sistema. Na x osi, na rastojanju d od koordinatnog početka, postavljeno je tačkasto naelektrisanje Q_T.

a) Izračunati podužnu količinu naelektrisanja Q_1' tako, da rezultantni vektor jačine elekričnog polja u tački O ima samo x komponentu.

b) Izračunati udaljenost tačkastog naelektrisanja od koordinatnog početka, $d > a$, tako da potencijal tačke O, koji potiče od štapa savijenog u obliku luka i tačkastog naelektrisanja (bez pravolinijskog štapa), u odnosu na referentnu tačku u beskonačnosti, bude jednak nuli.

Sistem se nalazi u vazduhu. Brojni podaci su:
$Q_2' = 2$ nC/m, $Q_T = -0{,}1$ nC, $a = 2$ cm, $\varepsilon_0 = 8{,}85\cdot10^{-12}$ F/m.



Slika 1.

Zadatak 2. Čvrst dielektrik u koaksijalnom kablu, dužine $L = 1$ m, ne naleže savršeno na unutrašnju elektrodu. Zbog toga, oko unutrašnje elektrode poluprečnika $a = 2$ mm, postoji sloj vazduha, debljine $\delta = 0{,}5$ mm, kao što je prikazano na slici 2. Poluprečnik spoljašnje elektrode je $b = 5$ mm. Permitivnost dielektrika iznosi $8\varepsilon_0$, električna čvrstina vazduha je $E_{č0} = 30$ kV/cm, a dielektrika, $E_{čd} = 150$ kV/cm.

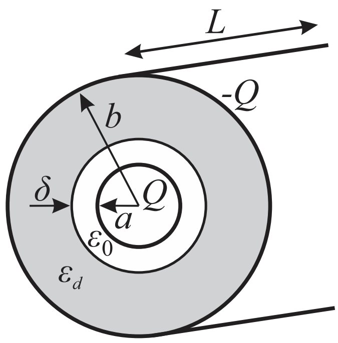
a) Izvesti, u oštim brojevima, izraz za kapacitivnost kabla.

b) Odrediti brojnu vrednost kapacitivnosti kabla.

c) Koristeći izraz pod a), izračunati kapacitivnost kabla za slučaj da dielektrik savršeno naleže na unutrašnju elektrodu ($\delta = 0$).

d) Izračunati koliko puta je veći maksimalan napon na koji kondenzator sme da se priključi pri idealnom naleganju dielektrika na unutrašnju elektrodu, u odnosu na kondenzator sa vazdušnim slojem, $U_{max(\delta=0)}/U_{max(\delta\neq0)}$.

e) Odrediti energiju sadržanu u oba kondenzatora, kada se oni priključe na napon $U = 1$kV.



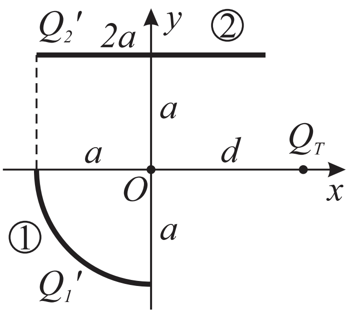
Slika 2.

PRAVILA POLAGANJA

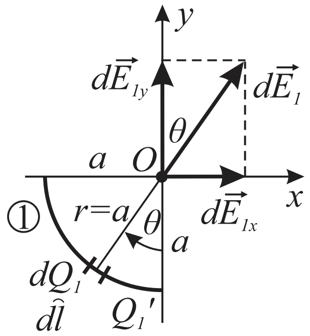
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

MiR I KOLOKVIJUM 01.12.2018. GRUPA A

Z1



a)



Pretpostavka: $Q_1'>0$

$$dE_1=\frac{dQ_1}{4\pi\varepsilon_0 r^2}=\frac{Q_1'\,dl}{4\pi\varepsilon_0 r^2}$$

$$dE_{1x}=dE_1\sin\theta$$

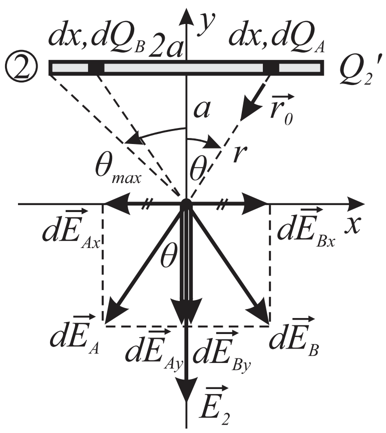
$$dE_{1y}=dE_1\cos\theta$$

$$E_{1x}=\int\limits_{\text{duž luka}} dE_{1x}=\int\limits_{\text{duž luka}}\frac{Q_1'\,dl}{4\pi\varepsilon_0 r^2}\sin\theta=\int\limits_{\text{duž luka}}\frac{Q_1'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\sin\theta=\frac{Q_1'}{4\pi\varepsilon_0 a}\int_0^{\pi/2}\sin\theta\,d\theta=\frac{Q_1'}{4\pi\varepsilon_0 a}\left(\cos 0-\cos\pi/2\right)$$

$$\boxed{E_{1x}=\frac{Q_1'}{4\pi\varepsilon_0 a}}\qquad\boxed{\vec{E}_{1x}=E_{1x}\cdot\vec{i}_x}$$

$$E_{1y}=\int\limits_{\text{duž luka}} dE_{1y}=\int\limits_{\text{duž luka}}\frac{Q_1'\,dl}{4\pi\varepsilon_0 r^2}\cos\theta=\int\limits_{\text{duž luka}}\frac{Q_1'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta=\frac{Q_1'}{4\pi\varepsilon_0 a}\int_0^{\pi/2}\cos\theta\,d\theta=\frac{Q_1'}{4\pi\varepsilon_0 a}\left(\sin\pi/2-\sin 0\right)$$

$$\boxed{E_{1y}=\frac{Q_1'}{4\pi\varepsilon_0 a}}\qquad\boxed{\vec{E}_{1y}=E_{1y}\cdot\vec{i}_y}$$



$$d\vec{E}_{Ax}+d\vec{E}_{Bx}=0\ \rightarrow\ \vec{E}_{2x}=0$$

$$dE_{2y}=2dE_{Ay}=2dE_A\cos\theta=2\frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta=2\frac{Q_2'dx}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$dE_{2y}=2\frac{Q_2'\dfrac{\cancel{r}\,d\theta}{\cancel{\cos\theta}}}{4\pi\varepsilon_0 r^{\cancel{2}}}\cancel{\cos\theta}=2\frac{Q_2'd\theta}{4\pi\varepsilon_0\dfrac{a}{\cos\theta}}=2\frac{Q_2'd\theta}{4\pi\varepsilon_0 a}\cos\theta$$

$$E_{2y}=2\frac{Q_2'}{4\pi\varepsilon_0 a}\int_0^{\theta_{\max}}\cos\theta\,d\theta=\frac{Q_2'}{2\pi\varepsilon_0 a}\sin\theta_{\max}=\frac{Q_2'}{2\pi\varepsilon_0 a}\frac{\sqrt{2}}{2}$$

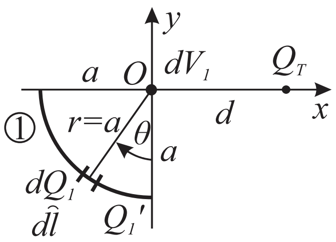
$$\boxed{\vec{E}_{2y}=\frac{Q_2'\sqrt{2}}{4\pi\varepsilon_0 a}\cdot\left(-\vec{i}_y\right)}$$

$$\vec{E}_{Oy}=\vec{E}_{1y}+\vec{E}_{2y}=E_{1y}\cdot\vec{i}_y+E_{2y}\cdot\left(-\vec{i}_y\right)=0\qquad\Rightarrow\qquad E_{1y}=E_{2y}$$

$$\frac{Q_1'}{4\pi\varepsilon_0 a}=\frac{Q_2'\sqrt{2}}{4\pi\varepsilon_0 a}\qquad\Rightarrow\qquad Q_1'=\sqrt{2}Q_2'$$

$$\boxed{Q_1'=2{,}83\text{nC/m}}$$

b)



$R\rightarrow\infty$

$$dV_1=\frac{dQ}{4\pi\varepsilon_0 r}=\frac{Q_1'\cdot dl}{4\pi\varepsilon_0 a}$$

$$V_1=\int\limits_{\text{duž luka}} dV_1=\int_0^{\frac{1}{4}2\pi a}\frac{Q_1'\cdot dl}{4\pi\varepsilon_0 a}=\frac{Q_1'}{4\pi\varepsilon_0 a}\cdot\frac{a\pi}{2}$$

$$V_1=\frac{Q_1'}{8\varepsilon_0}$$

$$V_T=\frac{Q_T}{4\pi\varepsilon_0 d}$$

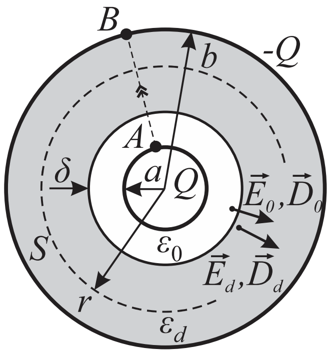
$$V_O=V_1+V_T=\frac{Q_1'}{8\varepsilon_0}+\frac{Q_T}{4\pi\varepsilon_0 d}=0$$

$$\frac{Q_1'}{8\varepsilon_0}=-\frac{Q_T}{4\pi\varepsilon_0 d}\qquad\Rightarrow\qquad d=-\frac{8Q_T\varepsilon_0}{4Q_1'\pi\varepsilon_0}=-\frac{2Q_T}{Q_1'\pi}$$

$$\boxed{d=2{,}25\text{cm}}$$

Z2

a)



Granični uslov:

$$D_{n0} = D_{nd} \qquad D_0 = D_d = D$$

$$E_o \neq E_d$$

$$\oint_S \vec{D}\cdot\vec{ds} = Q_{slobodno\ u\ S}$$

$$\int_{S_{OM}} D\,ds = Q$$

$$D\,2r\pi\,L = Q \qquad \boxed{D = \frac{Q}{2\pi r\,L}}, \qquad a \le r \le b$$

$$\boxed{E_0 = \frac{D}{\varepsilon_0} = \frac{Q}{2\pi\varepsilon_0 r\,L}}, \qquad a \le r \le a+\delta$$

$$\boxed{E_d = \frac{D}{\varepsilon_d} = \frac{Q}{2\pi\varepsilon_d r\,L}}, \qquad a+\delta \le r \le b$$

$$U_{AB} = \int_A^B \vec{E}\cdot\vec{dl} = \int_a^b E\,dr = \int_a^{c=a+\delta} E_0\,dr + \int_c^b E_d\,dr = \int_a^c \frac{Q}{2\pi\varepsilon_0 r\,L}\,dr + \int_c^b \frac{Q}{2\pi\varepsilon_d r\,L}\,dr = \frac{Q}{2\pi\varepsilon_0\,L}\ln\frac{c}{a} + \frac{Q}{2\pi\varepsilon_d\,L}\ln\frac{b}{c}$$

$$U_{AB} = \frac{Q}{2\pi\varepsilon_0\,L}\ln\frac{c}{a} + \frac{Q}{2\pi\varepsilon_d\,L}\ln\frac{b}{c}$$

$$\boxed{C = \frac{Q}{U_{AB}} = \frac{2\pi\varepsilon_0\,L}{\ln\frac{c}{a} + \frac{1}{\varepsilon_{rd}}\ln\frac{b}{c}}}$$

b) $\boxed{C = 179{,}5\,\mathrm{pF}}$

c) $\delta = 0\ (c = a) \quad \Rightarrow \quad C_1(\delta = 0) = \dfrac{2\pi\varepsilon_0\,L}{\ln 1 + \frac{1}{\varepsilon_{rd}}\ln\frac{b}{a}} = \dfrac{2\pi\varepsilon_0\,L}{\frac{1}{\varepsilon_{rd}}\ln\frac{b}{a}} \quad \Rightarrow \quad \boxed{C_1 = 485{,}5\,\mathrm{pF}}$

d)

$$U_{\max} = \frac{Q_{\max}}{C}$$

$\delta \neq 0$

$$E_{0\max} = \frac{Q_{\max 0}}{2\pi\varepsilon_0 aL} = \frac{Q_{\max 0}}{2\pi\,\varepsilon_0 aL} \le E_{\check{c}0} \quad \Rightarrow \quad Q_{\max 0} \le E_{\check{c}0}\,2\pi\,\varepsilon_0 aL = 0{,}334\,\mu\mathrm{C}$$

$$E_{d\max} = \frac{Q_{\max d}}{2\pi\varepsilon_d cL} = \frac{Q_{\max d}}{2\pi\,8\,\varepsilon_0 cL} = \frac{Q_{\max d}}{16\pi\,\varepsilon_0 cL} \le E_{\check{c}d} \quad \Rightarrow \quad Q_{\max d} \le E_{\check{c}d}16\pi\,\varepsilon_0 cL = 16{,}7\,\mu\mathrm{C}$$

$$Q_{\max} = \min\{Q_{\max 0}, Q_{\max d}\} = Q_{\max 0} = 0{,}334\,\mu\mathrm{C}$$

$$U_{\max}(\delta \neq 0) = \frac{Q_{\max 0}}{C} = \frac{0{,}334\,\mu}{179{,}5\,\mathrm{p}} = 1{,}858\,\mathrm{kV}$$

$\delta = 0$

$$E_{d\max} = \frac{Q_{\max d}}{2\pi\varepsilon_d aL} = \frac{Q_{\max d}}{2\pi\,8\,\varepsilon_0 aL} = \frac{Q_{\max d}}{16\pi\,\varepsilon_0 aL} \le E_{\check{c}d} \quad \Rightarrow \quad Q_{\max d} \le E_{\check{c}d}16\pi\,\varepsilon_0 aL = 13{,}35\,\mu\mathrm{C}$$

$$U_{\max}(\delta = 0) = \frac{Q_{\max d}}{C_1} = \frac{13{,}35\,\mu}{485{,}5\,\mathrm{p}} = 27{,}5\,\mathrm{kV}$$

$$\boxed{\frac{U_{\max}(\delta=0)}{U_{\max}(\delta\neq 0)}=\frac{27{,}5\,\mathrm{k}}{1{,}858\,\mathrm{k}}=14{,}8}$$

e)

$$\boxed{W_{\delta\neq 0}=\frac{1}{2}CU^2=89{,}75\,\mu\mathrm{J}} \qquad \boxed{W_{\delta=0}=\frac{1}{2}C_1U^2=242{,}75\,\mu\mathrm{J}}$$