**STUDIJSKI PROGRAM MERENJE I REGULACIJA**

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

# Osnovi elektrotehnike 2 (I kolokvijum)

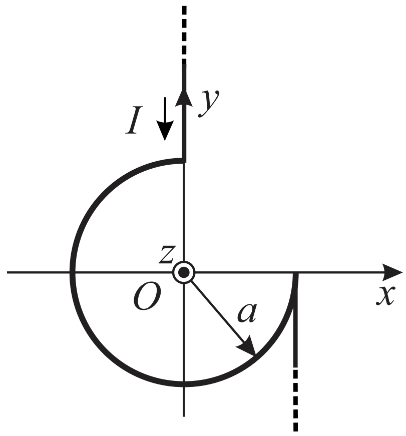
**K1**

**30.09.2018.**

## ZADACI

**Zadatak 1**. Veoma dugačak provodnik je savijen kao što je prikazano na slici 1. Celokupan sistem leži u *x*-*y* ravni pravouglog Dekartovog koordinatnog sistema. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku. Sredina je vazduh.

Brojne vrednosti su: $I$ =10 A, $a$ = 10 cm, $\mu_0 = 4\pi\cdot10^{-7}$ H/m.



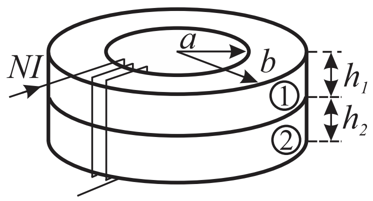

Slika 1.

**Zadatak** Na slici 2 je prikazano debelo torusno jezgro, načinjeno od dva različita materijala i njihove krive prvobitnog magnetisanja. Na jezgro je ravnomerno namotano $N$ = 170 zavojaka u kojima se uspostavila struja jačine $I$. Odrediti:

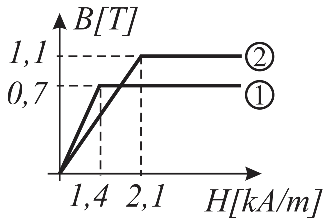
a) jačinu struje, $I$, pri kojoj će tri četvrtine dela jezgra načinjenog od materijala 2 biti u zasićenju, i

b) fluks vektora magnetske indukcije kroz poprečni presek jezgra, pri struji određenoj pod a).

Dimenzije jezgra su: $a$ = 3 *cm*, $b$ = 5 *cm*, $h_1 = h_2$ = 1 *cm*.




Slika 2a




Slika 2b

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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| K TET<br>Katedra za teorijsku elektrotehniku<br>www.ktet.ftn.uns.ac.rs | Osnovi elektrotehnike 2<br>(II kolokvijum) | **K2**<br>**30.09.2018.** |
|---|---|---|

## ZADACI

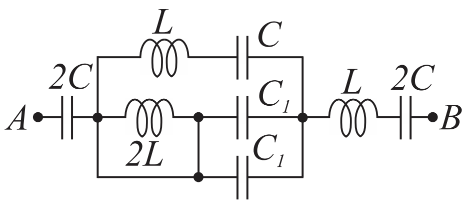
**Zadatak 1.** Na slici 1a je prikazana mreža reaktivnih komponenti, a na slici 1b "fizička" šema dva magnetski spregnuta kalema.

a) Izračunati ekvivalentnu impedansu $\underline{Z}_{AB}$ mreže sa slike 1a, i odrediti sve rezonantne i antirezonantne učestanosti.

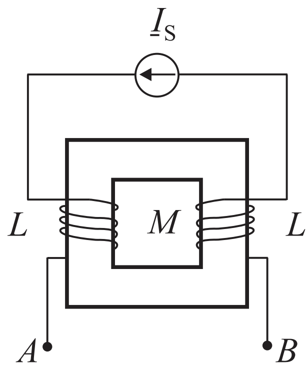
b) Ako se kolo na slici 1a priključi između tačaka A i B na slici 1b i ako generator radi na manjoj rezonantnoj učestanosti, skicirati električnu šemu novog kola i postaviti odgovarajuće tačke, kojima se na ovoj šemi prikazuje smer namotavanja kalemova.

c) Izračunati kompleksnu snagu strujnog generatora.

Brojne vrednosti su:
$L$ = 20 mH, $M$ = 12 mH $C$ = 5 nF, $C_1$ = 2,5 nF,
$\underline{I}_S$ = (1-j1) mA.




Slika 1a.




Slika 1b.

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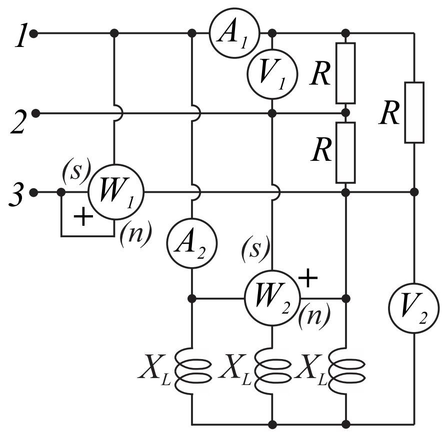
**Zadatak 2.** Simetričan trofazni prijemnik, sa slike 2, priključen je na mrežu faznog napona prve faze $U_1$ = 230 V.

a) Odrediti pokazivanja idealnih mernih instrumenata.

b) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.

Brojni podaci: $R$ = 30 Ω, $X_L$ = 20 Ω.




Slika 2.

## PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% od ukupnog broja poena na zadacima. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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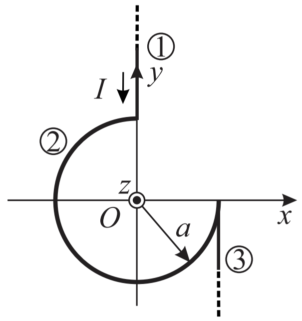
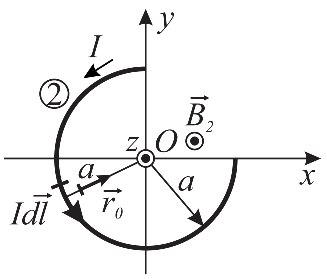
K1 Z1



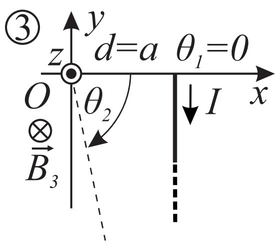

$$\boxed{\vec{B}_1 = 0} \qquad \left( I\,\vec{dl} \times \vec{r}_0 = 0 \right)$$




$$d\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \vec{r}_0}{r^2}$$

$$dB_2 = \frac{\mu_0}{4\pi} \frac{I\,dl}{a} \sin\sphericalangle\left(\vec{dl}, \vec{r}_0\right) = \frac{\mu_0}{4\pi} \frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl}, \vec{r}_0\right) = \frac{\pi}{2}$$

$$B_2 = \frac{\mu_0}{4\pi} \frac{I}{a^2} \int_0^{\frac{3}{4}2a\cdot\pi} dl = \frac{\mu_0}{4\pi} \frac{I}{a^2} \frac{3}{2} a\pi \qquad \boxed{B_2 = \frac{3\mu_0 I}{8a}} \qquad \boxed{\vec{B}_2 = B_2 \cdot \vec{i}_z}$$



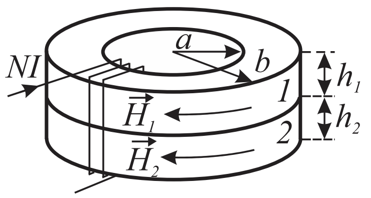

$$\theta_1 = 0, \qquad \theta_2 \to \frac{\pi}{2}, \qquad d = a$$

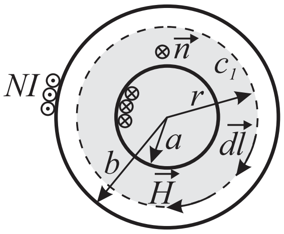
$$B_3 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{\mu_0 I}{4\pi a}(1-0)$$

$$\boxed{B_3 = \frac{\mu_0 I}{4\pi a}} \qquad \boxed{\vec{B}_3 = B_3 \cdot \left(-\vec{i}_z\right)}$$

$$\boxed{\vec{B}_0 = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \left(B_2 - B_3\right)\cdot\vec{i}_z = \left(\frac{3\mu_0 I}{8a} - \frac{\mu_0 I}{4\pi a}\right)\cdot\vec{i}_z = \frac{\mu_0 I}{4a}\left(\frac{3}{2} - \frac{1}{\pi}\right)\cdot\vec{i}_z} \qquad \boxed{\vec{B}_0 = 37{,}12\ \mu T \cdot \vec{i}_z}$$

## K1 Z2




Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_C \vec{H} \cdot \vec{dl} = NI$$

$$H\, 2\pi r = NI$$

$$H = \frac{NI}{2\pi r} \qquad a \le r \le b$$

a)

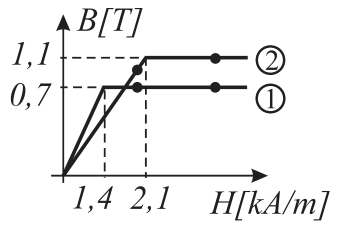
$$r_z = a + \frac{3}{4}(b-a) = 4{,}5\,cm$$

$$\frac{NI}{2\pi r_z} = H_{c2} = 2100\ A/m \quad \rightarrow \quad I = \frac{H_{c2}\, 2\pi r_z}{N} = \frac{2100 \cdot 2\pi \cdot 0{,}045}{170} \qquad \boxed{I = 3{,}49\ A}$$

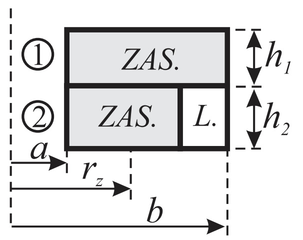
b)

$$H_{1\max}(r=a) = \frac{NI}{2\pi a} = 3150\ A/m$$

$$H_{1\min}(r=b) = \frac{NI}{2\pi b} = 1890\ A/m$$




Sledi da je materijal 1 u potpunom zasićenju.




$$\Phi = \Phi_1 + \Phi_2$$

$$\Phi_1 = B_{c1}(b-a)\,h_1 = 0{,}7 \cdot (0{,}05 - 0{,}03) \cdot 0{,}01$$

$$\boxed{\Phi_1 = 140\ \mu Wb}$$

$$\Phi_2 = B_{c2}(r_z - a)\,h_2 + \int_{r_z}^{b} \mu_2 \frac{NI}{2\pi r} dr\, h_2 = B_{c2}(r_z - a)\,h_2 + \frac{1{,}1}{2100}\frac{NI}{2\pi} h_2 \ln\frac{b}{r_z}$$

$$\Phi_2 = 1{,}1 \cdot (0{,}045 - 0{,}03) \cdot 0{,}01 + \frac{1{,}1}{2100}\,\frac{170 \cdot 3{,}49}{2\pi} \cdot 0{,}01 \cdot \ln\frac{0{,}05}{0{,}045}$$

$$\Phi_2 = 165\ \mu Wb + 52{,}14\ \mu Wb$$

$$\boxed{\Phi_2 = 217{,}14\ \mu Wb}$$

$$\boxed{\Phi = 357{,}14\ \mu Wb}$$

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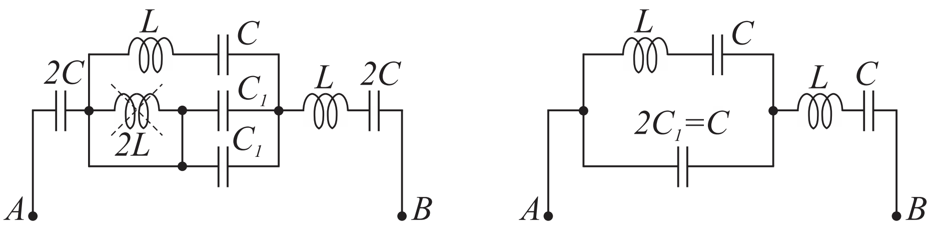
## K2 Z1

a)




$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \| \frac{1}{j\omega C} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C}+\frac{1}{j\omega C}} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{2-\omega^2 LC}{j\omega C}} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

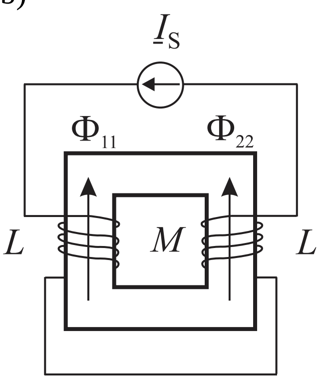
$$\underline{Z}_{AB} = \underline{Z}_2 + j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{1-\omega^2 LC}{j\omega C}$$

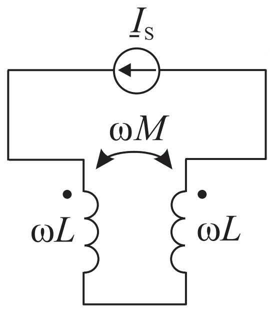
$$\underline{Z}_{AB} = \frac{\left(1-\omega^2 LC\right)+\left(1-\omega^2 LC\right)\left(2-\omega^2 LC\right)}{j\omega C\left(2-\omega^2 LC\right)} = \frac{\left(1-\omega^2 LC\right)\left(3-\omega^2 LC\right)}{j\omega C\left(2-\omega^2 LC\right)} \qquad \boxed{\underline{Z}_{AB} = -j\frac{\left(1-\omega^2 LC\right)\left(3-\omega^2 LC\right)}{\omega C\left(2-\omega^2 LC\right)}}$$

$$\operatorname{Im}\{\underline{Z}_{AB}\}=0 \quad \Rightarrow \quad \boxed{\omega_{r1} = \sqrt{\frac{1}{LC}}} \quad \boxed{\omega_{r2} = \sqrt{\frac{3}{LC}}}$$

$$\operatorname{Im}\{\underline{Y}_{AB}\}=0 \quad \Rightarrow \quad \boxed{\omega_{ar1} = 0} \quad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}} \quad \boxed{\omega_{ar3} \to \infty}$$

b)




Sprega je negativna.

$$\omega = \omega_{r1} = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{20\cdot 10^{-3}\cdot 5\cdot 10^{-9}}} = 10^5 \ rad/s$$

$$\underline{Z}_{AB} = 0$$

$$\underline{Z}_L = j\omega L = j10^5 \cdot 20\cdot 10^{-3} = j2000\,\Omega$$

$$\underline{Z}_M = j\omega M = j10^5 \cdot 12\cdot 10^{-3} = j1200\,\Omega$$

b)

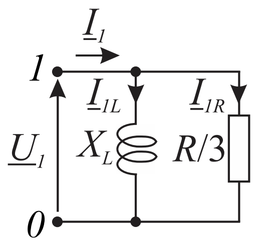
$$\underline{U}_S = \left(\underline{Z}_L \underline{I}_S - \underline{Z}_M \underline{I}_S\right) + \left(\underline{Z}_L \underline{I}_S - \underline{Z}_M \underline{I}_S\right) = \left(2\underline{Z}_L - 2\underline{Z}_M\right)\underline{I}_S$$

$$\underline{U}_S = 1{,}6(1+j1)\ \text{V}$$

$$\boxed{\underline{S} = \underline{U}_S \underline{I}_S^* = j32\ \text{VA}}$$

K2 Z2

a)




$$\underline{I}_{1R}=\frac{\underline{U}_1}{R/3}=\frac{230\,e^{j0°}}{10}=23\,e^{j0°}\ \text{A}=23\ \text{A}$$

$$\underline{I}_{1L}=\frac{\underline{U}_1}{jX_L}=\frac{230\,e^{j0°}}{j20}=\frac{230\,e^{j0°}}{20\,e^{j90°}}=11{,}5\,e^{-j90°}\ \text{A}=-j11{,}5\ \text{A}$$

$$\underline{I}_1=\underline{I}_{1R}+\underline{I}_{1L}=(23-j11{,}5)\ \text{A}=25{,}71\,e^{j-26{,}57}\ \text{A}$$

$$\underline{I}_2=\underline{I}_1\,e^{-j120°}=25{,}71\,e^{-j146{,}57°}\ \text{A}$$

$$\underline{I}_3=\underline{I}_1\,e^{-j240°}=25{,}71\,e^{-j266{,}57°}\ \text{A}$$

$$\boxed{I_{A1}=|\underline{I}_{1R}|=23\ \text{A}}$$

$$\boxed{I_{A2}=|\underline{I}_{1L}|=11{,}5\ \text{A}}$$

$$\boxed{U_{V1}=|\underline{U}_{12}|=\sqrt{3}\,|\underline{U}_1|=\sqrt{3}\cdot 230=398{,}37\ \text{V}}$$

$$\boxed{U_{V2}=|\underline{U}_3|=230\ \text{V}}$$

$$P_{W1}=\text{Re}\left\{\underline{U}_{31}\,\underline{I}_3^{\,*}\right\}=U_{31}\,I_3\cos\sphericalangle\left(\underline{U}_{31},\underline{I}_3\right)=\sqrt{3}U_1\,I_1\cos\left(30°+26{,}57°\right)=\sqrt{3}\cdot 230\cdot 25{,}71\cdot\cos 56{,}57°$$

$$\boxed{P_{W1}=5642{,}57\ \text{W}}$$

$$\underline{I}_{2L}=\underline{I}_{1L}\,e^{-j120°}=11{,}5\,e^{-j210°}\ \text{A}$$

$$P_{W2}=\text{Re}\left\{\underline{U}_{31}\,\underline{I}_{2L}^{\,*}\right\}=U_{31}\,I_{2L}\cos\sphericalangle\left(\underline{U}_{31},\underline{I}_{2L}\right)=\sqrt{3}U_1\,I_{1L}\cos 0°=\sqrt{3}\cdot 230\cdot 11{,}5\cdot 1$$

$$\boxed{P_{W2}=4581{,}27\ \text{W}}$$

b)