STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K T E T
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

K1

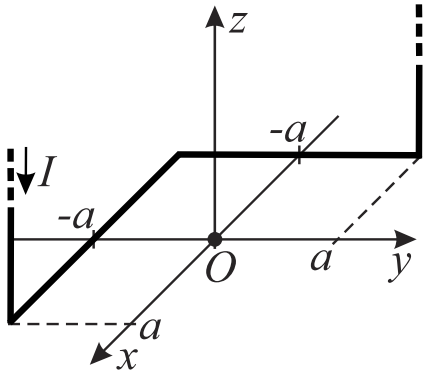
26.09.2018.

ZADACI

Zadatak 1. Na slici 1 je prikazan tanak, veoma dugačak, žičani provodnik sa vremenski konstantnom strujom jačine I. Provodnik se sastoji iz četiri pravolinijska segmenta. Dva segmenta, dužine $2a$, leže u x-y ravni zadatog koordinatnog sistema, dok du druga dva paralelna sa z-osom. Sredina je vazduh.

a) Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije u tački O (koordinatni početak).
b) Izračunati intenzitet vektora magnetske indukcije u tački O.

Brojne vrednosti su: $I = 1\ A$, $a = 1\ cm$, $\mu_0 = 4\pi \cdot 10^{-7}\ H/m$.

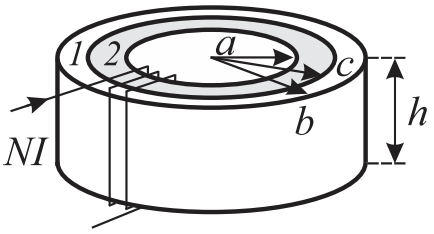


Slika 1.

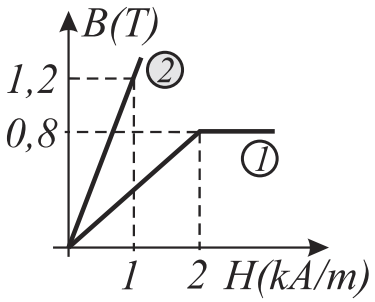
Zadatak 2. Ne debelom torusnom jezgru načinjenom od dva feromagnetska materijala, prikazanom na slici 2a, nalazi se namotaj sa $N = 1256$ zavojaka tanke, provodne žice, namotanih ravnomerno i gusto po celom torusu. Krive prvobitnog magnetisanja oba materijala su prikazane na slici 2b.

a) Izračunati maksimalnu vrednost jačine struje u namotajima, I, pri kojoj su oba sloja jezgra u linearnom režimu rada.
b) Odrediti nove režime rada, oba sloja jezgra, nakon povećanja struje određene pod a) za 50%.
c) Odrediti energiju sadržanu u jezgru nakon povećanja struje određene pod a) za 50%.

Dimenzije jezgra su $a = 3\ cm$, $c = 6\ cm$, $b = 8\ cm$, $h = 7\ cm$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 2 (II kolokvijum)	**K2** **26.09.2018.**

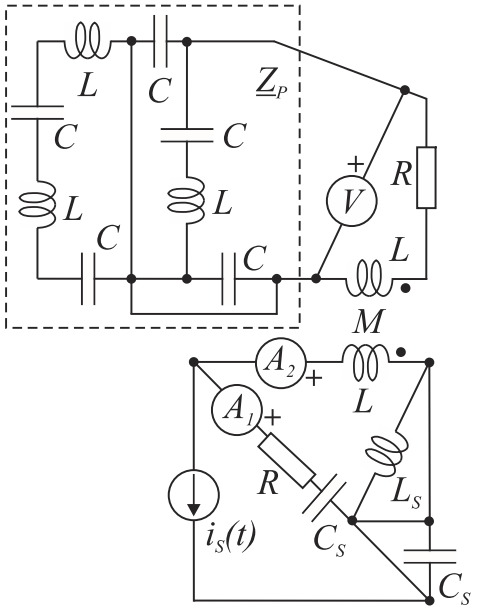
ZADACI

Zadatak 1. U mreži prostoperiodičnih struja prikazanoj na slici 1:

a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}_P$.

b) Ako je ugaona učestanost struje strujnog generatora $\underline{I}_S$ jednaka većoj antirezonantnoj učestanosti prijemnika, odrediti pokazivanja idealnih mernih instrumenata.

Brojni podaci su: $i_S(t) = 2\cdot\cos(\omega t + \pi/4)\ A$, $R = 50\ \Omega$, $C = 500\ nF$, $C_S = 1\ \mu F$, $L = L_S = 40\ mH$, $M = 10\ mH$.



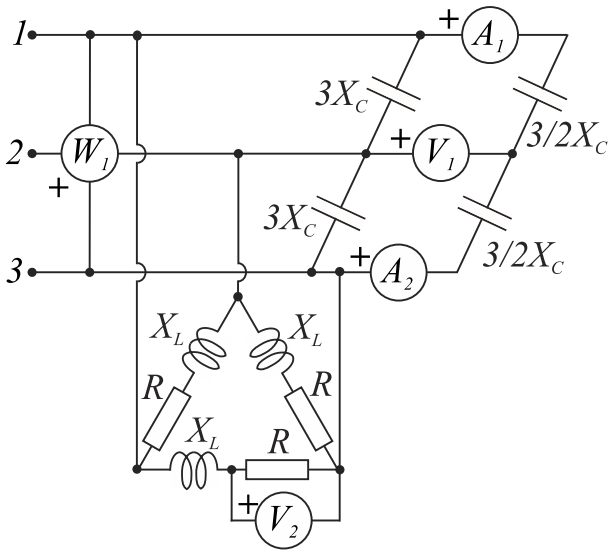
Slika 1.

Zadatak 2. Simetričan trofazni prijemnik priključen je na simetričnu trofaznu mrežu faznog napona prve faze $\underline{U}_1 = 230\ V$, kao što je prikazano na slici 2. Odrediti:

a) Pokazivanje idealnih mernih instrumenata.

b) Skicirati fazorski dijagram svih veličina od kojih zavisi pokazivanje mernih instrumenata.

Brojni podaci su: $R = 20\ \Omega$, $X_L = 20\ \Omega$, $X_C = 50\ \Omega$.



Slika 2.

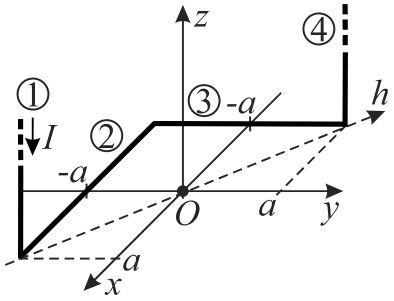
PRAVILA POLAGANJA

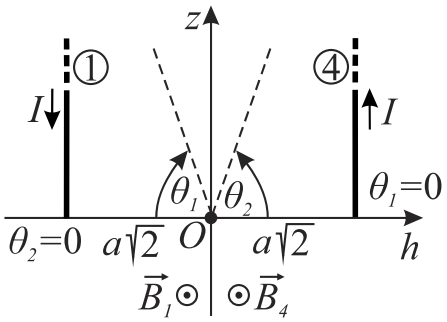
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

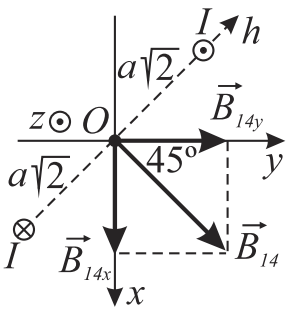
Ispit iz OET2, 26.09.2018.

I-1

a)







1 $d = a\sqrt{2}, \qquad \theta_1 \to -90°, \qquad \theta_2 = 0$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a\sqrt{2}}\left[0-(-1)\right] = \frac{\sqrt{2}\mu_0 I}{8\pi a}$$

4 $d = a\sqrt{2}, \qquad \theta_1 = 0, \qquad \theta_2 \to 90°$

$$B_4 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a\sqrt{2}}(1-0) = \frac{\sqrt{2}\mu_0 I}{8\pi a}$$

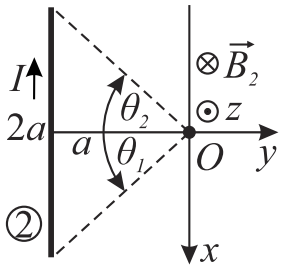
$$B_{14} = B_1 + B_4 = 2\frac{\sqrt{2}\mu_0 I}{8\pi a} = \frac{\sqrt{2}\mu_0 I}{4\pi a}$$

$$B_{14x} = B_{14}\sin 45° = \frac{\sqrt{2}\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2} = \frac{\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_{14x} = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_x}$$

$$B_{14y} = B_{14}\cos 45° = \frac{\sqrt{2}\mu_0 I}{4\pi a}\frac{\sqrt{2}}{2} = \frac{\mu_0 I}{4\pi a}$$

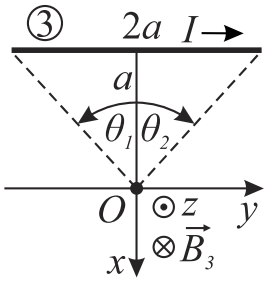
$$\boxed{\vec{B}_{14y} = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_y}$$



2 $d = a, \qquad \theta_1 \to -45°, \qquad \theta_2 = 45°$

$$B_2 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\sqrt{2}\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_2 = \frac{\sqrt{2}\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$



3 $d = a, \qquad \theta_1 \to -45°, \qquad \theta_2 = 45°$

$$B_3 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\sqrt{2}\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_3 = \frac{\sqrt{2}\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

$$\boxed{\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_x + \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_y + \frac{\sqrt{2}\mu_0 I}{2\pi a}\cdot\left(-\vec{i}_z\right)}$$

b)

$$\vec{B}_O = 10\ \mu T\cdot\vec{i}_x + 10\ \mu T\cdot\vec{i}_y + 28{,}28\ \mu T\cdot\left(-\vec{i}_z\right)$$

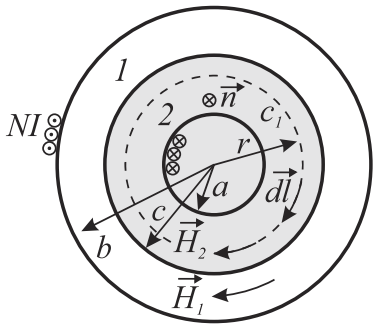
$$\left|\vec{B}_O\right| = \sqrt{(10\ \mu T)^2 + (10\ \mu T)^2 + (28{,}28\ \mu T)^2}$$

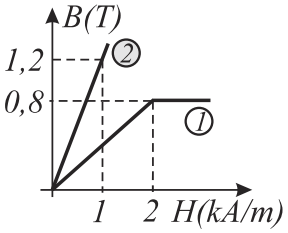
$$\boxed{\left|\vec{B}_O\right| = 31{,}62\ \mu T}$$

Ispit iz OET2, 26.09.2018.

I-2

a)





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_{c1} \vec{H} \cdot \vec{dl} = NI$$

$$H\ 2r\pi = NI$$

$$H = \frac{NI}{2r\pi} \qquad a < r < b$$

Materijal 2 je stalno u linearnom režimu rada.

$$H_{1\max}(r = c) = \frac{NI}{2\pi c} \le H_{K1} = 2000\ A/m$$

$$I \le \frac{H_{K1}\ 2\pi c}{N} = \frac{2000 \cdot 2\pi \cdot 0{,}06}{1256} = 0{,}6\ A$$

$$\boxed{I = 0{,}6\ A}$$

b)

Materijal 2 je stalno u linearnom režimu rada.

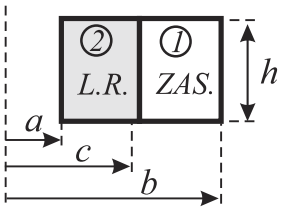
$$I^N = 1{,}5I = 0{,}9\ A$$

$$H_{1\max}(r = c) = \frac{NI^N}{2\pi c} = \frac{1256 \cdot 0{,}9}{2\pi \cdot 0{,}06} = 3000\ A/m \ge H_{K1}$$

$$H_{1\min}(r = b) = \frac{NI^N}{2\pi b} = \frac{1256 \cdot 0{,}9}{2\pi \cdot 0{,}08} = 2250\ A/m \ge H_{K1}$$

$\Rightarrow$ Materijal 1 u zasićenju.

c)



$$W_{m1} = \frac{1}{2} B_{K1} H_{K1} V_1 = \frac{1}{2} B_{K1} H_{K1} \left(b^2 - c^2\right)\pi\ h$$

$$W_{m1} = \frac{1}{2} \cdot 0{,}8 \cdot 2000 \cdot \left(0{,}08^2 - 0{,}06^2\right) \cdot 3{,}14 \cdot 0{,}07$$

$$\boxed{W_{m1} = 0{,}49\ J}$$

$$W_{m2} = \int_{V_2} \omega_{m2}\ dV = \int_{V_2} \frac{1}{2} B_2\ H_2\ dV = \int_{V_2} \frac{1}{2} \mu_2 \left(\frac{NI^N}{2\pi r}\right)^2 2\pi r\ dr\ h = \int_{V_2} \frac{1}{2} \mu_2 \frac{\left(NI^N\right)^2}{2\pi r}\ dr\ h$$

$$W_{m2} = \frac{1}{2} \mu_2 \frac{\left(NI^N\right)^2}{2\pi} h \int_a^c \frac{dr}{r} = \frac{1}{2} \mu_2 \frac{\left(NI^N\right)^2}{2\pi} h \ln\frac{c}{a} = \frac{1}{2} \frac{1{,}2}{1000} \frac{\left(1256 \cdot 0{,}9\right)^2}{6{,}28} \cdot 0{,}07 \cdot \ln\frac{0{,}06}{0{,}03}$$

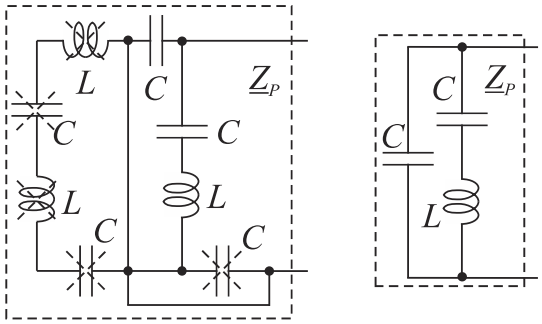
$$\boxed{W_{m2} = 5{,}92\ J}$$

$$\boxed{W_m = W_{m1} + W_{m2} = 6{,}41\ J}$$

Ispit iz OET2, 26.09.2018.

II-1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

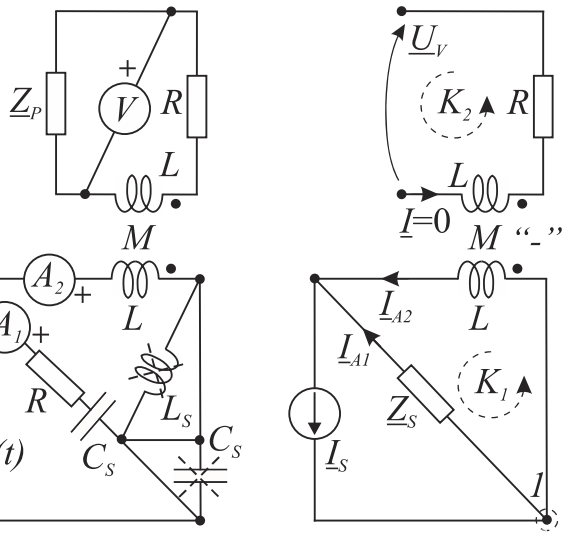
$$\underline{Z}_P = \underline{Z}_1 \,\|\, \frac{1}{j\omega C} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{1-\omega^2 LC}{j\omega C}+\dfrac{1}{j\omega C}} = \frac{\dfrac{1-\omega^2 LC}{j\omega C}\dfrac{1}{j\omega C}}{\dfrac{2-\omega^2 LC}{j\omega C}}$$

$$\boxed{\underline{Z}_P = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}}$$

$\mathrm{Im}\{\underline{Z}_P\} = 0 \quad \Rightarrow \quad 1-\omega^2 LC = 0$ $\qquad \boxed{\omega_{r1} = \sqrt{\frac{1}{LC}}}$ $\qquad \boxed{\omega_{r2} \to \infty}$

$\mathrm{Im}\{\underline{Y}_P\} = 0 \quad \Rightarrow \quad \omega C = 0$ $\qquad \boxed{\omega_{ar1} = 0}$ $\qquad 2-\omega^2 LC = 0$ $\qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$

b)



$$\omega = \omega_{ar1} = \sqrt{\frac{2}{LC}} = \frac{\sqrt{2}}{\sqrt{40\cdot 10^{-3}\cdot 500\cdot 10^{-9}}} = 10^4\ \frac{rad}{s}$$

$$\underline{Z}_L = j\omega L = j10^4 \cdot 40\cdot 10^{-3} = j400\ \Omega$$

$$\underline{Z}_M = j\omega M = j10^4 \cdot 10\cdot 10^{-3} = j100\ \Omega$$

$$\underline{Z}_S = R - j\frac{1}{\omega C_S} = 50 - j\frac{1}{10^4\cdot 10^{-6}} = (50 - j100)\ \Omega$$

$$\underline{I}_S = \frac{2}{\sqrt{2}}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = (1+j)\ A$$

Č$_1$: $\underline{I}_S = \underline{I}_{A1} + \underline{I}_{A2}$

K$_1$: $-\underline{Z}_S\underline{I}_{A1} + \underline{Z}_L\underline{I}_{A2} - \underline{Z}_M \cancel{\underline{I}^0} = 0$

K$_2$: $\underline{Z}_L \cancel{\underline{I}^0} - \underline{Z}_M\underline{I}_{A2} + R\cancel{\underline{I}^0} + \underline{U}_V = 0$

Č$_1$ $\Rightarrow$ $\underline{I}_{A2} = \underline{I}_S - \underline{I}_{A1}$

K$_1$ $\Rightarrow$ $\underline{I}_{A1} = \frac{\underline{Z}_L}{\underline{Z}_S}\underline{I}_{A2} = \frac{\underline{Z}_L}{\underline{Z}_S}(\underline{I}_S - \underline{I}_{A1})$

$$\underline{I}_{A1} = \frac{\underline{Z}_L}{\underline{Z}_S\left(\frac{\underline{Z}_L}{\underline{Z}_S}+1\right)}\underline{I}_S = \frac{\underline{Z}_L}{\underline{Z}_L+\underline{Z}_S}\underline{I}_S = \frac{j400}{j400+(50-j100)}\cdot(1+j) = \frac{-400+j400}{50+j300} = (1{,}08 + j1{,}51)\ A$$

$$I_{A1} = |\underline{I}_{A1}| = \sqrt{1{,}08^2 + 1{,}51^2} \qquad \boxed{I_{A1} = 1{,}86\ A}$$

Č$_1$ $\Rightarrow$ $\underline{I}_{A2} = \underline{I}_S - \underline{I}_{A1} = (1+j) - (1{,}08 + j1{,}51) = (-0{,}08 - j0{,}51)\ A$

$$I_{A2} = |\underline{I}_{A2}| = \sqrt{(-0{,}08)^2 + (-0{,}51)^2} \qquad \boxed{I_{A2} = 0{,}52\ A}$$

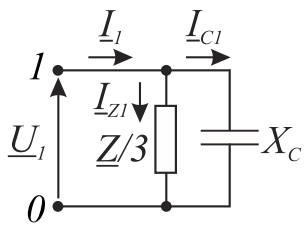
K$_2$ $\Rightarrow$ $\underline{U}_V = \underline{Z}_M\underline{I}_{A2} = j100\cdot(-0{,}08 - j0{,}51) = (51 - j8)\ V$

$$U_V = |\underline{U}_V| = \sqrt{51^2 + (-8)^2} \qquad \boxed{U_V = 51{,}62\ V}$$

Ispit iz OET2, 26.09.2018.

II-2

a)



$$\underline{Z} = R + jX_L = 20 + j20 = 20\sqrt{2}\,e^{j45^\circ}\ \Omega$$

$$\underline{Z}_e = \frac{\underline{Z}}{3} \| \left(-jX_C\right) = \frac{20 + j20}{3} \| \left(-j50\right) = \left(8{,}67 + j6{,}36\right)\Omega = 10{,}75\,e^{j36{,}25^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}_e} = \frac{230\,e^{j0^\circ}}{10{,}75\,e^{j36{,}25^\circ}} = 21{,}4\,e^{-j36{,}25^\circ}\ A$$

$$P_{W1} = \mathrm{Re}\left\{\underline{U}_{31}\,\underline{I}_2^{\,*}\right\} = U_{31}\,I_1 \cos \sphericalangle\left(\underline{U}_{31}, \underline{I}_2\right) = \sqrt{3}\,U_1\,I_1 \cos\left(90^\circ - 36{,}25^\circ\right) = \sqrt{3}\cdot 230 \cdot 21{,}4 \cdot \cos 53{,}75^\circ$$

$$\boxed{P_{W1} = 5041\,W}$$

$$\underline{U}_{V1} = \underline{U}_{23} + \frac{1}{2}\underline{U}_{31} = \sqrt{3}\cdot 230 \cdot e^{-j90^\circ} + \frac{1}{2}\cdot\sqrt{3}\cdot 230\cdot e^{j150^\circ} = -j\sqrt{3}\cdot 230 + \frac{1}{2}\cdot\sqrt{3}\cdot 230\cdot\left(-\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$

$$\underline{U}_{V1} = \left(172{,}5 - j298{,}78\right)V$$

$$U_{V1} = \left|\underline{U}_{V1}\right| = \sqrt{172{,}5^2 + \left(-298{,}78\right)^2} \qquad \boxed{U_{V1} = 345\,V}$$

$$U_{V2} = R\left|\underline{I}_{Z13}\right| = R\frac{\left|\underline{U}_{13}\right|}{\left|\underline{Z}\right|} = R\frac{\sqrt{3}\,U_1}{Z} = 20\cdot\frac{\sqrt{3}\cdot 230}{20\sqrt{2}} \qquad \boxed{U_{V2} = 281{,}7\,V}$$

$$I_{A1} = \left|\underline{I}_{C13}\right| = \frac{\left|\underline{U}_{13}\right|}{3X_C} = \frac{\sqrt{3}\cdot 230}{150} \qquad \boxed{I_{A1} = 2{,}66\ A}$$

$$I_{A2} = I_{A1} \qquad \boxed{I_{A2} = 2{,}66\ A}$$

b)

