STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET

Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 1 (I kolokvijum)

K1

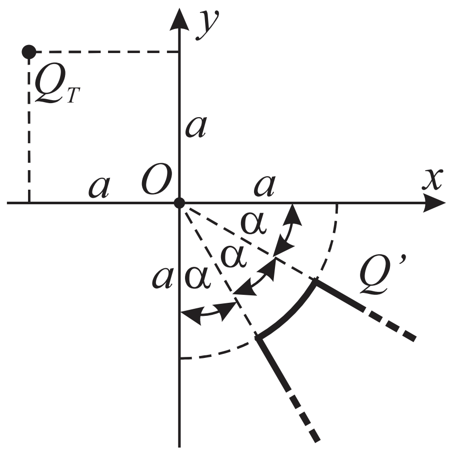
28.08.2018.

ZADACI

Zadatak 1. Veoma dugačak tanak štap, od izolacionog materijala, savijen kao što je prikazano na slici 1, naelektrisan je ravnomerno podužnom gustinom naelektrisanja Q'. Štap se sastoji od dela u obliku luka, poluprečnika a, i dva pravolinijska polubeskonačna segmenta. Struktura se nalazi u x-y ravni zadatog koordinatnog sistema.

a) Izvesti u opštim brojevima izraz za vektor jačine električnog polja u tački O (centar koordinatnog sistema), koji potiče od štapa.
b) Odrediti naelektrisanje tačkastog naelektrisanja Q_T, tako da rezultantni vektor jačine električnog polja u tački O bude jednak nuli.

Brojni podaci su: $a = 1\ cm$, $Q' = 5\ nC/m$, $\alpha = \pi/6$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$.

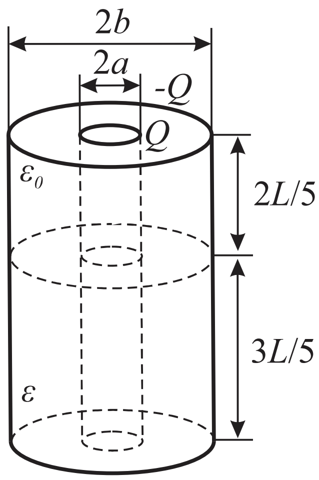


Slika 1.

Zadatak 2. Koaksijalni kabl poluprečnika elektroda a i b, dužine L, ispunjen je do 3/5 svoje zapremine čvrstim dielektrikom relativne permitivnosti $\varepsilon_r = 2$, i postavljen u vertikalni položaj, kao što je prikazano na slici 2. Elektrode kondenzatora su naelektrisane naelektrisanjem Q i $-Q$.

a) Odrediti ukupnu količinu vezanog naelektrisanja, koje se formira uz spoljašnju elektrodu.
b) Izvesti u opštim brojevima izraz za kapacitivnost kondenzatora u vertikalnom položaju.
c) Za koliko će se promeniti vrednost kapacitivnosti kondenzatora kada se on postavi u horizontalni položaj?

Brojni podaci su: $a = 1\ mm$, $b = 2{,}7\ mm$, $L = 5\ m$, $Q = 10\ nC$.



Slika 2

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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K TET Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs	Osnovi elektrotehnike 1 (II kolokvijum)	**K2** **28.08.2018.**

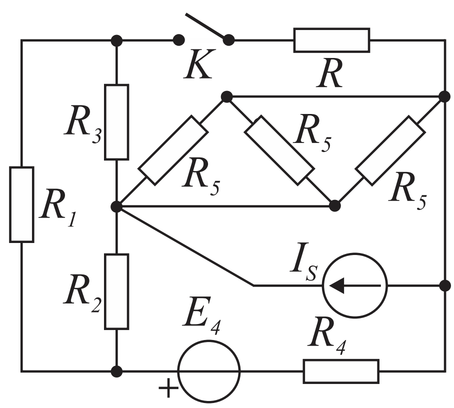
ZADACI

Zadatak 1. U kolu vremenski konstantnih struja, sa slike 1, zatvaranjem prekidača K, otpornik otpornosti $R = 250\ \Omega$, maksimalne snage $P_{max} = 2{,}5\ mW$, priključuje se u kolo.

a) Proveriti da li će otpornik pregoreti nakon zatvaranja prekidača K,

b) Ukoliko hoće, odrediti otpornost zaštitnog otpornika koji treba dodati da bi se pregorevanje sprečilo.

Brojni podaci su: $E_4 = 12\ V$, $I_S = 4\ mA$, $R_1 = 2\ k\Omega$, $R_2 = 3\ k\Omega$, $R_3 = 5\ k\Omega$, $R_4 = 0{,}4\ k\Omega$, $R_5 = 4{,}5\ k\Omega$.



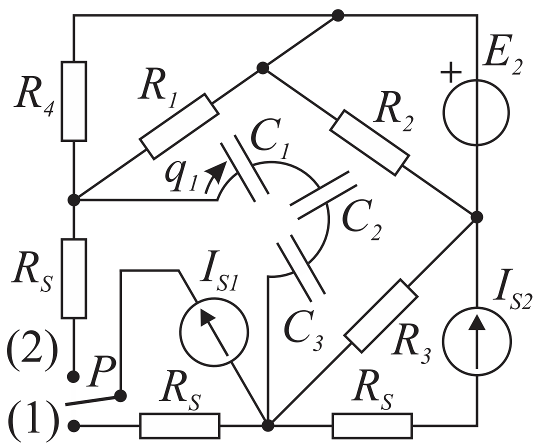
Slika 1.

Zadatak 2. U kolu vremenski konstantnih struja, sa slike 2, posle prebacivanja preklopnika P iz položaja (1) u položaj (2), kroz kondenzator kapacitivnosti C_1 protekne količina naelektrisanja q_1, u naznačenom referentnom smeru.

a) Odrediti jačinu struje strujnog generatora, I_{S1}.

b) Izračunati snage svih generator u stacionarnom stanju mreže, koje nastane kada je preklopnik u položaju (2).

Brojni podaci su: $C_1 = 2C_2 = 3C_3 = 30\ nF$, $q_1 = 120\ nC$, $E_2 = 10\ V$, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 15\ \Omega$, $R_S = 5\ \Omega$, $I_{S2} = 1\ A$.



Slika 2.

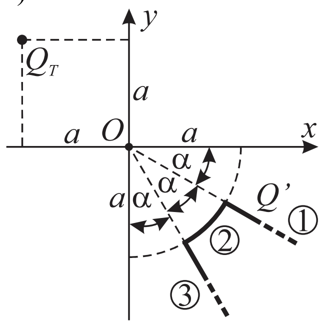
PRAVILA POLAGANJA

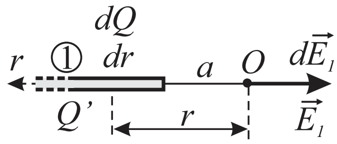
Za položen kolokvijum neophodno je sakupiti više od 50% poena na svakom od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

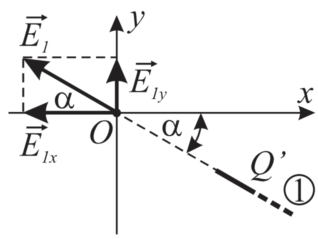
Ispit iz OET1, 28.08.2018.

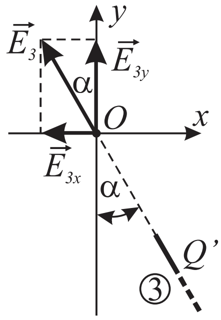
I-1

a)









$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'\,dr}{4\pi\varepsilon_0 r^2}$$

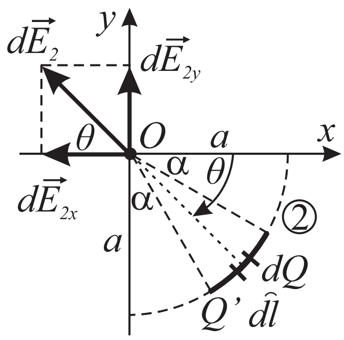
$$E_1 = \int\limits_{\substack{po\\ štapu}} dE_1 = \frac{Q'}{4\pi\varepsilon_0}\int_a^\infty \frac{dr}{r^2} = \frac{Q'}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{\infty}\right) = \frac{Q'}{4\pi\varepsilon_0 a} = E_3$$

$$E_{1x} = E_1\cos\alpha = \frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} \qquad \boxed{E_{1x} = \frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}} \qquad \boxed{\vec{E}_{1x} = E_{1x}\cdot\left(-\vec{i}_x\right)}$$

$$E_{1y} = E_1\sin\alpha = \frac{Q'}{4\pi\varepsilon_0 a}\frac{1}{2} \qquad \boxed{E_{1y} = \frac{Q'}{8\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_{1y} = E_{1y}\cdot\vec{i}_y}$$

$$E_{3x} = E_3\sin\alpha = \frac{Q'}{4\pi\varepsilon_0 a}\frac{1}{2} \qquad \boxed{E_{3x} = \frac{Q'}{8\pi\varepsilon_0 a}} \qquad \boxed{\vec{E}_{3x} = E_{3x}\cdot\left(-\vec{i}_x\right)}$$

$$E_{3y} = E_3\cos\alpha = \frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2} \qquad \boxed{E_{3y} = \frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}} \qquad \boxed{\vec{E}_{3y} = E_{3y}\cdot\vec{i}_y}$$



$$dE_{2x} = dE_2\cos\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\cos\theta = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\cos\theta$$

$$E_{2x} = \int\limits_{\substack{po\\ luku}} dE_{2x} = \frac{Q'}{4\pi\varepsilon_0 a}\int_\alpha^{2\alpha}\cos\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\sin\frac{\pi}{3}-\sin\frac{\pi}{6}\right) = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$\boxed{E_{2x} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)} \qquad \boxed{\vec{E}_{2x} = E_{2x}\cdot\left(-\vec{i}_x\right)}$$

$$dE_{2y} = dE_2\sin\theta = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{Q'\,dl}{4\pi\varepsilon_0 a^2}\sin\theta = \frac{Q'\,a\,d\theta}{4\pi\varepsilon_0 a^2}\sin\theta$$

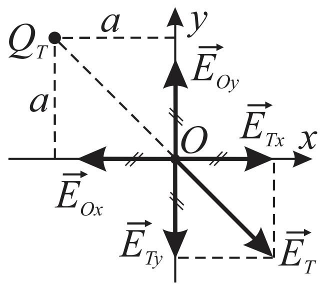
$$E_{2y} = \int\limits_{\substack{po\\ luku}} dE_{2y} = \frac{Q'}{4\pi\varepsilon_0 a}\int_\alpha^{2\alpha}\sin\theta\,d\theta = \frac{Q'}{4\pi\varepsilon_0 a}\left(\cos\frac{\pi}{6}-\cos\frac{\pi}{3}\right) = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$\boxed{E_{2y} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)} \qquad \boxed{\vec{E}_{2y} = E_{2y}\cdot\vec{i}_y}$$

$$\vec{E}_O = \vec{E}_1+\vec{E}_2+\vec{E}_3 = \left(\frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}+\frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)+\frac{Q'}{8\pi\varepsilon_0 a}\right)\cdot\left(-\vec{i}_x\right)+\left(\frac{Q'}{8\pi\varepsilon_0 a}+\frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)+\frac{Q'}{4\pi\varepsilon_0 a}\frac{\sqrt{3}}{2}\right)\cdot\vec{i}_y$$

$$\boxed{\vec{E}_O = \frac{\sqrt{3}\,Q'}{4\pi\varepsilon_0 a}\cdot\left(-\vec{i}_x\right)+\frac{\sqrt{3}\,Q'}{4\pi\varepsilon_0 a}\cdot\vec{i}_y}$$

b)



$$E_T = \frac{Q_T}{4\pi\varepsilon_0\left(a\sqrt{2}\right)^2} = \frac{Q_T}{8\pi\varepsilon_0 a^2}$$

$$E_{Tx} = E_{Ty} = E_T \cos\frac{\pi}{4} = \frac{Q_T}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2}$$

$$\vec{E}_O = 0 \qquad \Rightarrow \qquad E_{Tx} = E_{Ox}, \qquad E_{Ty} = E_{Oy}$$

$$\frac{Q_T}{8\pi\varepsilon_0 a^2}\frac{\sqrt{2}}{2} = \frac{\sqrt{3}\,Q'}{4\pi\varepsilon_0 a}$$

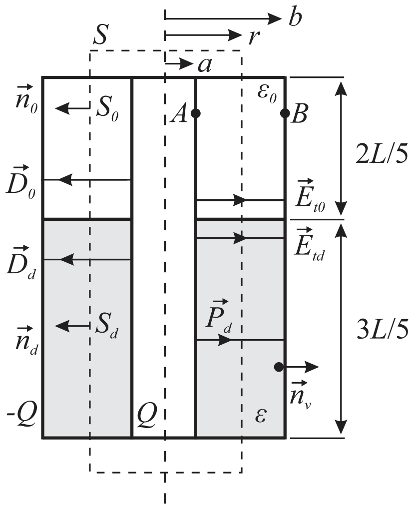
$$Q_T = \frac{4a\sqrt{3}\,Q'}{\sqrt{2}}$$

$$\boxed{Q_T = 0,24\ nC}$$

Ispit iz OET1, 28.08.2018.

I-2

a)



Granični uslov:

$$E_{t0} = E_{td} \qquad E_0 = E_d = E$$

$$D_{n0} \neq D_{nd}$$

$$\oint_S \vec{D} \cdot \vec{ds} = Q_{uS}$$

$$\int_{S_{OM}} D\, ds = Q \qquad \int_{OM_0} D_0\, ds + \int_{OM_d} D_d\, ds = Q$$

$$D_0\, 2\pi r \frac{2L}{5} + D_d\, 2\pi r \frac{3L}{5} = Q \qquad D_0 = \varepsilon_0 E \qquad D_d = \varepsilon E = 2\varepsilon_0 E$$

$$\boxed{E = \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi r}}, \qquad a \le r \le b$$

$$\boxed{D_0 = \varepsilon_0 E = \varepsilon_0 \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi r} \qquad D_d = \varepsilon_d E = 2\varepsilon_0 \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi r}}, \qquad a \le r \le b$$

$$P_d = D_d - \varepsilon_0 E = (\varepsilon - \varepsilon_0) E = \varepsilon_0 E = \varepsilon_0 \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi r} = 5 \frac{Q}{16 L \pi r}$$

$$\sigma_v = \vec{P}_d \cdot \vec{n}_v = P_d (r = b) = 5 \frac{Q}{16 L \pi b}$$

$$Q_v = \sigma_v\, 2\pi b \frac{3L}{5} = 5 \frac{Q}{16 L \pi b} 2\pi b \frac{3L}{5} \qquad \boxed{Q_v = \frac{3}{8} Q} \qquad \boxed{Q_v = 3{,}75\, nC}$$

b)

$$U_{AB} = \int_A^B \vec{E} \cdot \vec{dl} = \int_a^b E\, dr = \int_a^b \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi r} dr = \frac{Q}{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi} \ln \frac{b}{a}$$

$$C = \frac{Q}{U_{AB}} = \frac{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi}{\ln \frac{b}{a}}$$

$$\boxed{C_V = \frac{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi}{\ln \frac{b}{a}}}$$

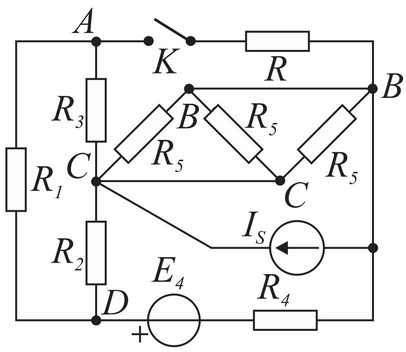
c)

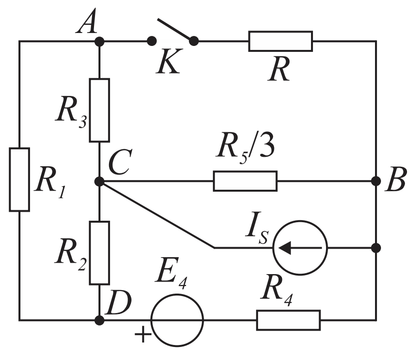
$$C_V = C_H = \frac{\left(\varepsilon_0 \frac{2L}{5} + \varepsilon_0 \frac{6L}{5}\right) 2\pi}{\ln \frac{b}{a}} = 444{,}6\ pF$$

$$\boxed{\Delta C = C_H - C_V = 0}$$

<u>Ispit iz OET1, 28.08.2018.</u>

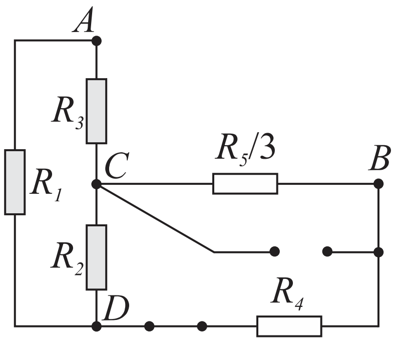
II-1





a)

R_T:



$$R_A = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{2k \cdot 5k}{2k + 3k + 5k} = 1\,k\Omega$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{3k \cdot 5k}{2k + 3k + 5k} = 1{,}5\,k\Omega$$

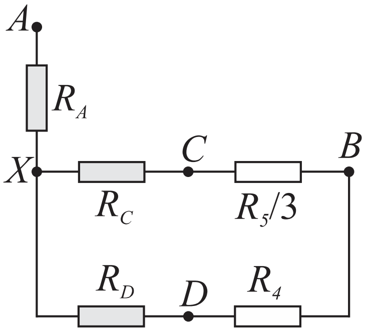
$$R_D = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{2k \cdot 3k}{2k + 3k + 5k} = 0{,}6\,k\Omega$$

$$R_T = R_{AB} = R_A + \left[\left(R_C + \frac{R_5}{3}\right) \| \left(R_D + R_4\right)\right]$$

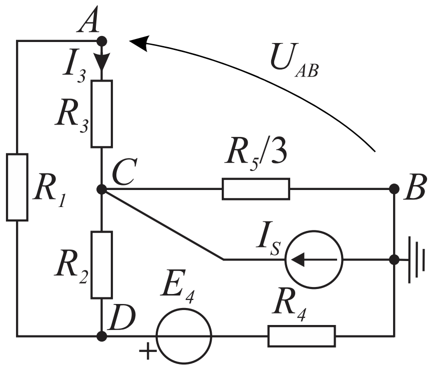
$$R_T = 1k + \left[\left(1{,}5k + 1{,}5k\right) \| \left(0{,}6k + 0{,}4k\right)\right]$$

$$R_T = 1k + \left(3k \| 1k\right) = 1k + 0{,}75k$$

$$\boxed{R_T = 1{,}75\,k\Omega}$$



E_T:



$$V_B = 0\,V$$

$$V_C\left(\frac{1}{R_1 + R_3} + \frac{1}{R_2} + \frac{1}{\infty} + \frac{1}{R_5/3}\right) - V_D\left(\frac{1}{R_1 + R_3} + \frac{1}{R_2}\right) = I_S$$

$$V_D\left(\frac{1}{R_1 + R_3} + \frac{1}{R_2} + \frac{1}{R_4 + 0}\right) - V_C\left(\frac{1}{R_1 + R_3} + \frac{1}{R_2}\right) = \frac{E_4}{R_4}$$

$$V_C\left(\frac{1}{2k + 5k} + \frac{1}{3k} + \frac{1}{1{,}5k}\right) - V_D\left(\frac{1}{2k + 5k} + \frac{1}{3k}\right) = 4m \qquad / \cdot 21k$$

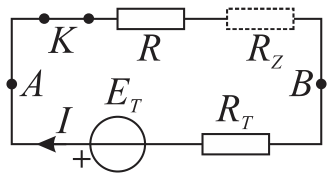
$$V_D\left(\frac{1}{2k + 5k} + \frac{1}{3k} + \frac{1}{0{,}4k}\right) - V_C\left(\frac{1}{2k + 5k} + \frac{1}{3k}\right) = \frac{12}{0{,}4k} \qquad / \cdot 42k$$

$$\left.\begin{array}{l} 24V_C - 10V_D = 84 \\ 125V_D - 20V_C = 1260 \end{array}\right\} \Rightarrow \quad V_C = 8{,}25\,V, \quad V_D = 11{,}4\,V$$

$$I_3 = \frac{V_D - V_C}{R_1 + R_3} = \frac{11{,}4 - 8{,}25}{2k + 5k} = 0{,}45\,mA$$

$$E_T = U_{AB} = R_3 I_3 + \left(V_C - V_B\right) = 5k \cdot 0{,}45m + \left(8{,}25 - 0\right)$$

$$\boxed{E_T = 10{,}5\,V}$$



$$I_{\max} = \sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{2,5\,m}{250}} = 3,16\;mA$$

$$I = \frac{E_T}{R_T + R} = \frac{10,5}{1,75\,k + 0,25\,k} = 5,25\;mA > I_{\max} \quad \Rightarrow \quad \text{Treba dodati } R_Z.$$

b)

$$I' = \frac{E_T}{R_T + R + R_Z} \le I_{\max}$$

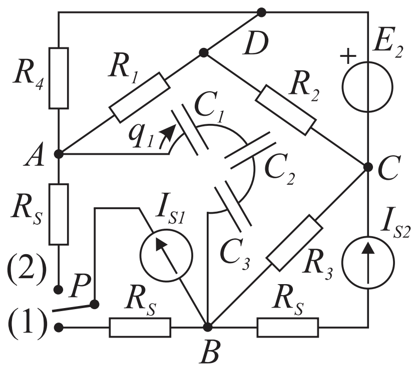
$$R_Z \ge \frac{E_T}{I_{\max}} - R_T - R = \frac{10,5}{3,16\,m} - 1,75\,k - 0,25\,k$$

$$\boxed{R_Z \ge 1,32\;k\Omega}$$

Ispit iz OET1, 28.08.2018.

II-2

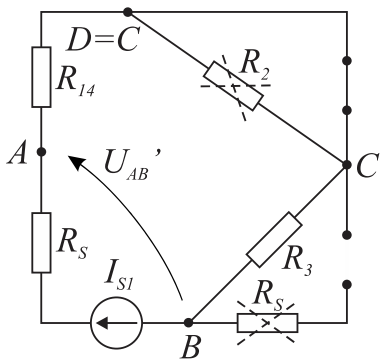
a)

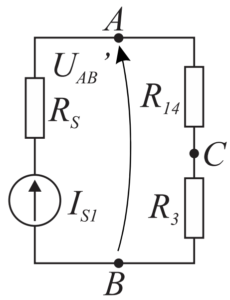


Svi generatori	=	Svi sem I_{S1}	+	Samo I_{S1}
(2)		(1)		

$$U_{AB}^{(2)} = U_{AB}^{(1)} + U_{AB}'$$

$$\Delta U_{AB} = U_{AB}^{(2)} - U_{AB}^{(1)} = U_{AB}'$$





$$U_{AB}' = U_{C1}' + U_{C2}' + U_{C3}' = \frac{q_1}{C_1} + \frac{q_1}{C_2} + \frac{q_1}{C_3}$$

$$U_{AB}' = \frac{120\,nC}{30\,nF} + \frac{120\,nC}{15\,nF} + \frac{120\,nC}{10\,nF} = 24\,V$$

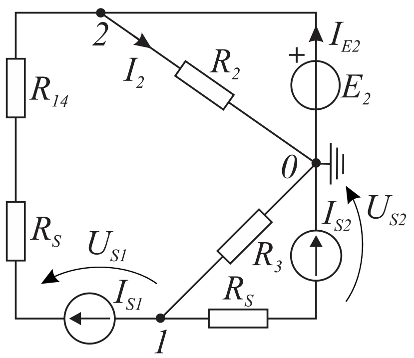
$$R_{14} = \frac{R_1 R_4}{R_1 + R_4} = \frac{10 \cdot 15}{10 + 15} = 6\,\Omega$$

$$U_{AB}' = \left(R_{14} + R_3\right) I_{S1}$$

$$I_{S1} = \frac{U_{AB}'}{R_{14} + R_3} = \frac{24}{6+6}$$

$$\boxed{I_{S1} = 2\,A}$$

b)



$$n_{č} = 3, \quad n_{i.n.g.} = 1$$

$$MPČ: \quad n_{č} - 1 - n_{i.n.g.} = 3 - 1 - 1 = 1$$

$$V_0 = 0\,V, \qquad V_2 = E_2 = 10\,V$$

$$V_1\left(\frac{1}{R_{14} + R_S + \infty} + \frac{1}{R_3} + \frac{1}{R_S + \infty}\right) - V_2\left(\frac{1}{R_{14} + R_S + \infty}\right) = -I_{S1} - I_{S2} \qquad / \cdot R_3$$

$$V_1 = \left(-I_{S1} - I_{S2}\right) R_3 = \left(-2 - 1\right) \cdot 6$$

$$V_1 = -18\,V$$

$$I_2 = \frac{V_2 - V_0}{R_2} = \frac{10}{20} = 0,5\,A$$

$$I_{E2} = I_2 - I_{S1} = 0,5 - 2 = -1,5\,A$$

$$P_{E2} = E_2 I_{E2} = 10 \cdot \left(-1,5\right) \qquad \boxed{P_{E2} = -15\,W}$$

$$U_{S1} = \left(R_S + R_{14}\right) I_{S1} + \left(V_2 - V_1\right) = \left(5 + 6\right) \cdot 2 + \left(10 - \left(-18\right)\right) = 50\,V$$

$$P_{S1} = U_{S1} I_{S1} = 50 \cdot 2 \qquad \boxed{P_{S1} = 100\,W}$$

$$U_{S2} = \left(V_0 - V_1\right) + R_S I_{S2} = \left(0 - \left(-18\right)\right) + 5 \cdot 1 = 23\,V$$

$$P_{S2} = U_{S2} I_{S2} = 23 \cdot 1 \qquad \boxed{P_{S2} = 23\,W}$$