

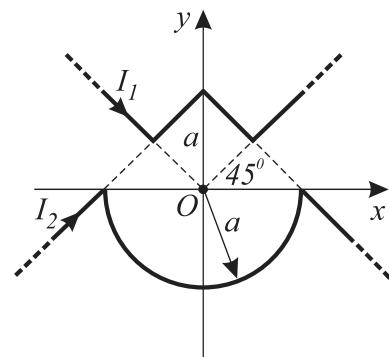
ZADACI

Zadatak 1. Dva veoma dugačka žičana provodnika sa vremenski konstantnim strujama jačine I_1 i I_2 , savijena su kao što je prikazano na slici 1. Provodnici se nalaze u x - y ravni Dekartovog pravouglog koordinatnog sistema.

- Odrediti u opštim brojevima izraz za vektor magnetske indukcije u koordinatnom početku koji stvara kontura sa strujom jačine I_1 .
- Odrediti jačinu struje u drugom provodniku, I_2 , tako, da ukupan vektor magnetske indukcije u koordinatnom početku bude jednak 0.

Sredina je vazduh.

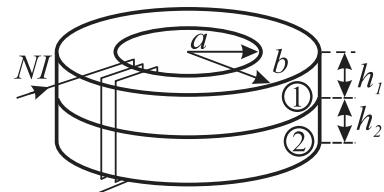
Brojne vrednosti su: $I_1 = 1 \text{ A}$, $a = 2 \text{ cm}$, $\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$.



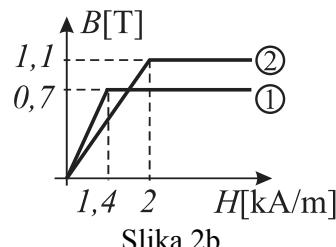
Slika 1.

Zadatak 2. Na slici 2 je prikazano debelo torusno jezgro, načinjeno od dva različita materijala i njihove krive prvobitnog magnetisanja. Na jezgro je ravnomerno namotano $N = 180$ zavojaka u kojima se uspostavila struja jačine I . Odrediti:

- jačinu struje, I , pri kojoj će jedna četvrtina dela jezgra načinjenog od materijala 2 biti u linearном režimu,
- režim rada u materijalu 1 pri struji određenoj pod a),
- fluks vektora magnetske indukcije kroz poprečni presek jezgra, pri struji određenoj pod a),
- energiju utrošenu na uspostavljanje magnetskog polja u jezgru, pri izračunatoj jačini struje.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

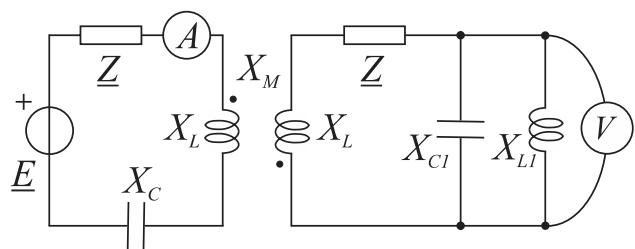
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

ZADACI

Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1, odrediti:

- Pokazivanje idealnog ampermetra i voltmatra.
- Aktivnu snagu kondenzatora X_{C1} i reaktivnu snagu kalema X_{L1} .
- Trenutnu vrednost jačine struje $i_{C1}(t)$ koja teče u grani sa kalemom X_{C1} .

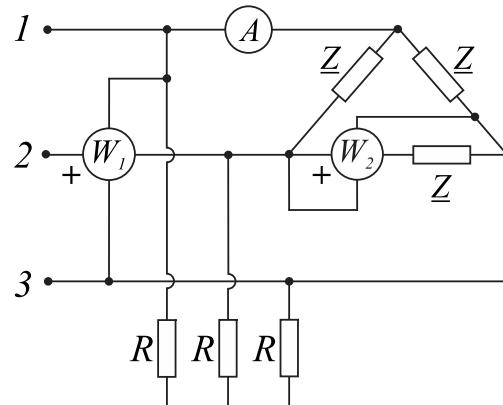
Brojni podaci su: $E = j10$ V, $\omega=1000\pi$ rad/s, $X_L = X_C = 4 \Omega$, $X_{L1} = X_{C1} = 1 \Omega$, $X_M = 2 \Omega$, $Z = (5+j5) \Omega$.



Slika 1.

Zadatak 2. Dva trofazna prijemnika, jedan otpornosti $R = 10 \Omega$, a drugi impedanse $Z = (30-j30) \Omega$, priključena su na simetričnu trofaznu mrežu faznog napona prve faze $U_1 = 230$ V, kao što je prikazano na slici 2. Odrediti:

- Pokazivanje idealnog vatmetra W_1 . Na fazorskom dijagramu nacrtati fazore veličina od kojih zavisi pokazivanje vatmeta.
- Pokazivanje idealnog vatmetra W_2 . Na fazorskom dijagramu nacrtati fazore veličina od kojih zavisi pokazivanje vatmeta.
- Pokazivanje idealnog ampermetra.
- Snagu Džulovih gubitaka trofaznog prijemnika koji se sastoji od otpornika.



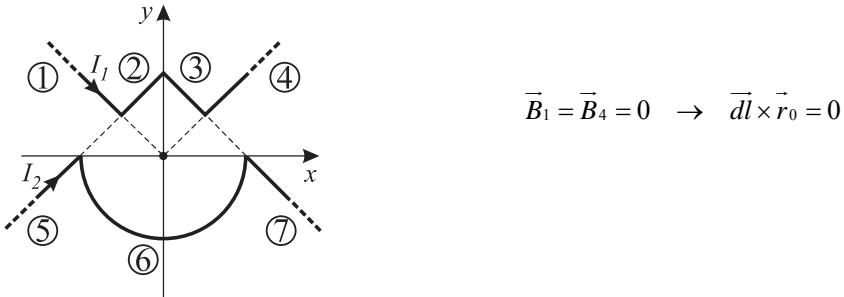
Slika 2.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

K1 Z1

a)



$$\theta_1 = 0, \quad \theta_2 = \frac{\pi}{4}, \quad d = \frac{a\sqrt{2}}{2}$$

$$B_2 = \frac{\mu_0 I_1}{4\pi d} (\sin \theta_2 - \sin \theta_1) = \frac{\mu_0 I_1}{4\pi \frac{a\sqrt{2}}{2}} \left(\frac{\sqrt{2}}{2} - 0 \right)$$

$$B_2 = \boxed{\frac{\mu_0 I_1}{4\pi a}}$$

$$\boxed{\vec{B}_2 = \vec{B}_3 = \frac{\mu_0 I_1}{4\pi a} \cdot (-\vec{i}_z)}$$

b)

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}_0}{r^2}$$

$$dB_6 = \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}}{a^2}$$

$$B_6 = \frac{\mu_0 I_2}{4\pi a^2} \int_0^{a\pi} dl = \frac{\mu_0 I_2}{4\pi a^2} \cdot \cancel{\cancel{a\pi}}$$

$$B_6 = \boxed{\frac{\mu_0 I_2}{4a}}$$

$$\boxed{\vec{B}_6 = \frac{\mu_0 I_2}{4a} \cdot (\vec{i}_z)}$$

$$\theta_1 = -\frac{\pi}{2}, \quad \theta_2 = -\frac{\pi}{4}, \quad d = \frac{a\sqrt{2}}{2}$$

$$B_5 = \frac{\mu_0 I_2}{4\pi d} \left(-\sin \frac{\pi}{4} - \left(-\sin \frac{\pi}{2} \right) \right) = \frac{\mu_0 I_2}{4\pi d} \left(1 - \frac{\sqrt{2}}{2} \right)$$

$$\boxed{B_5 = \frac{\mu_0 I_2 \sqrt{2}}{4\pi a} \left(1 - \frac{\sqrt{2}}{2} \right)}$$

$$\boxed{\vec{B}_5 = \vec{B}_7 = \frac{\mu_0 I_2 \sqrt{2}}{4\pi a} \left(1 - \frac{\sqrt{2}}{2} \right) \cdot (-\vec{i}_z)}$$

c)

$$\vec{B}_0 = \sum_{i=1}^7 \vec{B}_i = (0 - B_2 - B_3 + 0 - B_5 + B_6 - B_7) \cdot (\vec{i}_z)$$

Po uslovu zadatka:

$$-B_2 - B_3 - B_5 + B_6 - B_7 = 0$$

$$B_6 = B_2 + B_3 + B_5 + B_7 = 2B_2 + 2B_5$$

$$\frac{\mu_0 I_2}{\mathcal{A}\mu} = 2 \frac{\mu_0 I_1}{\mathcal{A}\pi\mu} + 2 \frac{\mu_0 I_2 \sqrt{2}}{\mathcal{A}\pi\mu} \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$I_2 = \frac{2I_1}{\pi} + \frac{2I_2 \sqrt{2}}{\pi} \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$I_2 \left(1 - \frac{2\sqrt{2}}{\pi} \left(1 - \frac{\sqrt{2}}{2}\right)\right) = \frac{2I_1}{\pi}$$

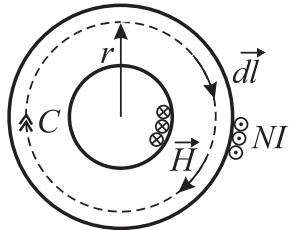
$$I_2 \left(1 - \frac{2\sqrt{2}}{\pi} + \frac{2}{\pi}\right) = \frac{2I_1}{\pi} / \cdot \pi$$

$$I_2 (\pi - 2\sqrt{2} + 2) = 2I_1$$

$$I_2 = \frac{2I_1}{\pi - 2\sqrt{2} + 2} = [0,865A]$$

K1 Z2

a)



$$\text{Granični uslov: } H_{1t} = H_{2t} = H$$

$$\oint_c \vec{H} \cdot d\vec{l} = NI$$

$$H \cdot 2r\pi = NI$$

$$H = \frac{NI}{2r\pi}, \quad a < r < b$$

$$H(r_c) = \frac{NI}{2r_c\pi} = H_{c2}$$

$$r_c = a + \frac{3}{4}(b - a) = 3cm + \frac{3}{4}(5cm - 3cm) = 4,5cm$$

$$I = \frac{2r_c \pi H_{c2}}{N} = \frac{2 \cdot 4,5 \cdot 10^{-2} \cdot \pi \cdot 2 \cdot 10^3}{180} = [3,14A]$$

b)

$$H_1(a) = \frac{NI}{2a\pi} = 3000 A/m > H_{c1}$$

$$H_1(b) = \frac{NI}{2b\pi} = 1800 A/m > H_{c1}$$

Materijal 1 je u zasićenju.

c)

$$\Phi = \Phi_1 + \Phi_2 = \Phi_{1Z} + \Phi_{2Z} + \Phi_{2L}$$

$$\Phi = B_{c1}(b-a)h_1 + B_{c2}(r_c-a)h_2 + \int_{r_c}^b \mu_2 \frac{NI}{2\pi r} dr \cdot h_2$$

$$\Phi = B_{c1}(b-a)h_1 + B_{c2}(r_c-a)h_2 + \frac{1,1}{2000} \frac{NI}{2\pi} h_2 \ln \frac{b}{r_c}$$

$$\Phi = 280 \mu Wb + 330 \mu Wb + 104,3 \mu Wb = \boxed{714,3 \mu Wb}$$

d)

$$W_m = W_{m1} + W_{m2} = W_{m1Z} + W_{m2Z} + W_{m2L}$$

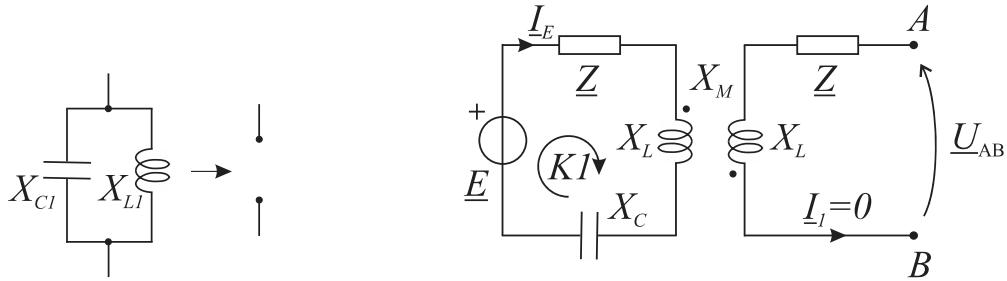
$$W_m = \frac{1}{2} B_{c1} H_{c1} (b^2 - a^2) \pi h_1 + \frac{1}{2} B_{c2} H_{c2} (r_c^2 - a^2) \pi h_2 + \int_{r_c}^b \frac{1}{2} \mu_2 \left(\frac{NI}{2\pi r} \right)^2 2r \pi \cdot dr \cdot h_2$$

$$W_m = \frac{1}{2} B_{c1} H_{c1} (b^2 - a^2) \pi h_1 + \frac{1}{2} B_{c2} H_{c2} (r_c^2 - a^2) \pi h_2 + \frac{1}{2} \cdot \frac{1,1}{2000} \frac{(NI)^2}{2\pi} h_2 \ln \frac{b}{r_c}$$

$$W_m = 49,2 mJ + 77,75 mJ + 29,49 mJ = \boxed{156,44 mJ}$$

K2 Z1

a)



$$K1: \quad -\underline{E} + \underline{Z} \underline{I}_E + \left(jX_L \underline{I}_E - \underbrace{jX_M \underline{I}_1}_0 \right) - jX_C \underline{I}_E = 0$$

$$-j10 + (5 + j5)\underline{I}_E + \cancel{j4\underline{I}_E} - \cancel{j4\underline{I}_E} = 0$$

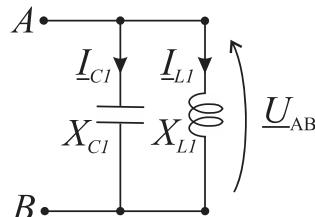
$$\underline{I}_E = \frac{j10}{5 + j5} = \frac{j2}{1+j} \cdot \frac{1-j}{1-j} = \frac{j\cancel{2}(1-j)}{\cancel{2}} = (1+j)A = \sqrt{2}e^{j45^\circ} A$$

$$\boxed{I_A = \sqrt{2}A}$$

$$\underline{U}_{AB} = \underbrace{\underline{Z} \underline{I}_1}_0 + \left(\underbrace{jX_L \underline{I}_1}_0 - jX_M \underline{I}_E \right) = -j2(1+j) = 2(1-j) = 2\sqrt{2}e^{-j45^\circ} V$$

$$\boxed{U_V = 2\sqrt{2}V}$$

b)



$$\boxed{P_{C1} = 0 W}$$

$$\underline{I}_{L1} = \frac{\underline{U}_{AB}}{jY_{L1}} = \frac{2\sqrt{2}e^{-j45^\circ}}{j\cancel{2}e^{j90^\circ}} = 2\sqrt{2}e^{-j135^\circ} A$$

$$\underline{S}_{L1} = \underline{U}_{AB} \cdot \underline{I}_{L1}^* = 2\sqrt{2}e^{-j45^\circ} \cdot 2\sqrt{2}e^{-j135^\circ} = 8e^{j90^\circ} = j8VA$$

$$\boxed{Q_{L1} = 8VAr}$$

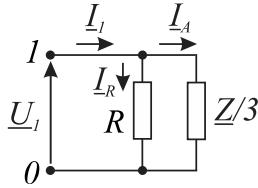
c)

$$\underline{I}_{C1} = -\underline{I}_{L1} = -2\sqrt{2}e^{-j135^\circ} = 2\sqrt{2}e^{-j135^\circ} e^{j180^\circ} = 2\sqrt{2}e^{j45^\circ} A$$

$$\boxed{i_{C1}(t) = 4\cos(1000\pi t + 45^\circ) A}$$

K2 Z2

a)

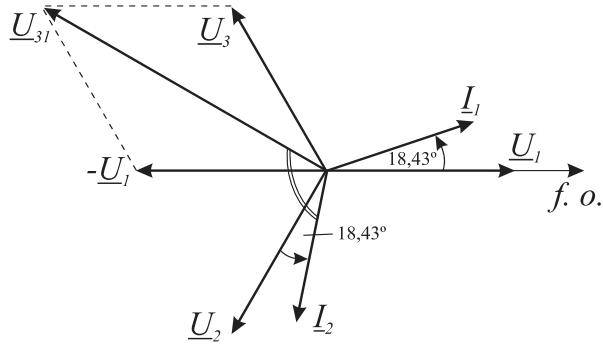


$$\underline{Z}_e = R \parallel \frac{\underline{Z}}{3} = \frac{10(10 - j10)}{20 - j10} \cdot \frac{2 + j}{2 + j} = \frac{30 - j10}{5} = 6 - 2j = 2\sqrt{10}e^{-j18,43^\circ} \Omega$$

$$I_1 = \frac{\underline{U}_1}{\underline{Z}_e} = \frac{230}{2\sqrt{10}e^{-j18,43^\circ}} = 36,37 e^{j18,43^\circ} A$$

$$P_{W1} = \operatorname{Re}\left\{\underline{U}_{31} \underline{I}_2^*\right\} = U_{31} I_2 \cos \angle(\underline{U}_{31}, \underline{I}_2) = \sqrt{3} U_1 I_1 \cos(90^\circ + 18,43^\circ) = \sqrt{3} \cdot 230 \cdot 36,37 \cdot \cos 108,43^\circ$$

$$\boxed{P_{W1} = -4580,6 W}$$



b)

$$I_{23} = \frac{\underline{U}_{23}}{\underline{Z}} = \frac{\sqrt{3} \cdot 230 e^{-j90^\circ}}{30\sqrt{2} e^{-j45^\circ}} = 9,4 e^{-j45^\circ} A$$

$$P_{W2} = \operatorname{Re}\left\{\underline{U}_{23} \underline{I}_{23}^*\right\} = U_{23} I_{23} \cos \angle(\underline{U}_{23}, \underline{I}_{23})$$

$$P_{W2} = \sqrt{3} \cdot 230 \cdot 9,4 \cdot \cos(90^\circ - 45^\circ)$$

$$\boxed{P_{W2} = 2647,9 W}$$

c)

$$I_A = \frac{U_1}{\frac{\underline{Z}}{3}} = 16,28 A = I_{23} \cdot \sqrt{3}$$

d)

$$P_R = R \cdot I_R^2 = R \cdot \left(\frac{U_1}{R}\right)^2 = 5290 W$$

$$P_{uk} = 3 \cdot P_R = \boxed{15870 W}$$

