STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET
Katedra za teorijsku elektrotehniku
www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2
(I kolokvijum)

K1

15.04.2018.

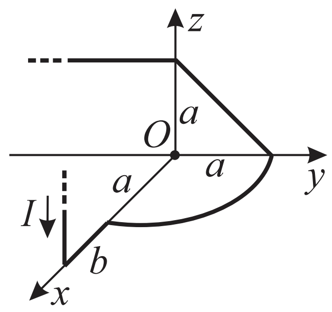
ZADACI

Zadatak 1. Veoma dugačak žičani provodnik sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od četiri pravolinijska segmenta, od kojih dva leže u x-z ravni zadatog koordinatnog sistema, a dva u y-z ravni, i jednog dela u obliku četvrtine kruga, poluprečnika a, koji leži u x-y ravni.

a) Odrediti, u opštim brojevima, izraz za vektor magnetske indukcije u tački O (centar koordinatnog sistema).

b) Izračunati intenzitet ovog vektora.

Brojni podaci: $I = 5\ A,\ a = 3\ cm,\ b = 1\ cm,\ \mu_0 = 4\pi \cdot 10^{-7}\ H/m$.



Slika 1.

Zadatak 2. Ne debelom torusnom feromagnetskom jezgru načinjenom od dva feromagnetska materijala prikazanom na slici 2a, nalazi se namotaj sa $N = 314$ zavojaka tanke bakarne žice, namotanih ravnomerno i gusto po celom torusu. Krive prvobitnog magnetisanja oba materijala su prikazane na slici 2b.

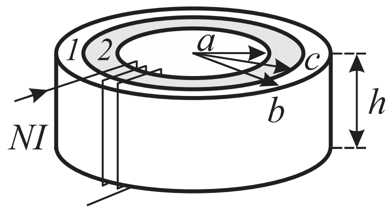
a) Izračunati jačinu vremenski konstantne struje I u namotajima tako, da ceo spoljašnji sloj torusa bude u linearnom režimu.

b) Odrediti režim rada unutrašnjeg sloja pri jačini struje izračunatoj pod a).

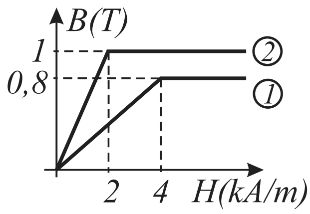
c) Izračunati magnetsku energiju utrošenu na uspostavljanje polja u unutrašnjem sloju.

d) Izračunati energiju utrošenu na zagrevanje spoljašnjeg sloja jezgra usled histerezisa, u intervalu vremena od 2 minuta, pod pretpostavkom da su namotaji priključeni na generator prostoperiodične struje, amplitude jednake vrednosti izračunatoj pod a), učestanosti $f = 50\ Hz$. Histerezisna petlja feromagnetskog materijala od kojeg je načinjen spoljašnji sloj torusnog jezgra prikazana je na slici 2c.

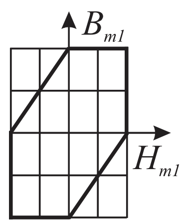
Dimenzije jezgra su $a = 4\ cm$, $c = 6\ cm$, $b = 8\ cm$, $h = 3\ cm$.



Slika 2a.



Slika 2b.



Slika 2c.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K	TET	Osnovi elektrotehnike 2 (II kolokvijum)	K2
Katedra za teorijsku elektrotehniku www.ktet.ftn.uns.ac.rs			15.04.2018.

ZADACI

Zadatak 1. Prijemnik sa slike 1a se priključuje na sekundar transformatora (tačke A i B) sa slike 1b.

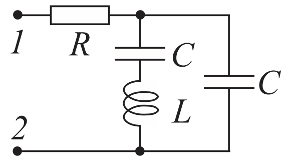
a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika prikazanog na slici 1a.

b) Nacrtati električnu šemu kola kada je priključen prijemnik, a frekvencija generatora prostoperiodičnog napona $\underline{E}_1$ jednaka manjoj rezonantnoj učestanosti određenoj pod a). Na električnoj šemi označiti krajeve namotaja tačkicama, koje određuju znak njihove međuinduktivnosti.

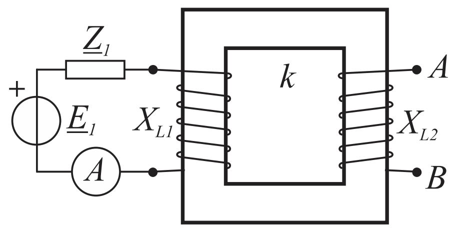
c) Napisati jednačine po drugom Kirhofovom zakonu za kolo prikazano pod b). Jednačine treba napisati za oba dela kola, primarni i sekundarni.

d) Ako je pokazivanje ampermetra jednako $I_{amp} = 1\ A$, odrediti amplitudu generatora elektromotorne sile i fazni pomeraj između tog napona i struje u grani sa ampermetrom.

Brojni podaci: $L = 10\ mH$, $C = 100\ \mu F$, $R = 20\ \Omega$, $\underline{Z}_1 = (20+j60)\ \Omega$, $X_{L1} = 4 \cdot X_{L2} = 80\ \Omega$, k = 0.9.



Slika 1a.



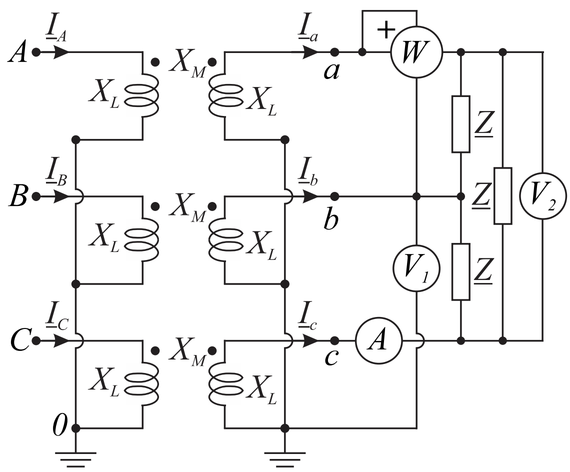
Slika 1b.

Zadatak 2. Trofazni prijemnik, impedansi $\underline{Z} = (30–j60)\ \Omega$ vezanih u trougao, priključen je na sekundar trofaznog transformatora, kao što je prikazano na slici 2. Primar transformatora je priključen na simetričnu mrežu faznog napona prve faze $U_A = 230\ V$. Brojne vrednosti reaktansi transformatora su: $X_L = 20\ \Omega$ i $X_M = 10\ \Omega$.

a) Izračunati kompleksne jačine struja primara i sekundara kao i kompleksne fazne napone sekundara transformatora.

b) Odrediti pokazivanja idealnih mernih instrumenata.

c) Na istom fazorskom dijagramu prikazati fazore faznih napona sekundara i fazore svih veličina od kojih zavise pokazivanja idealnih mernih instrumenata.



Slika 2.

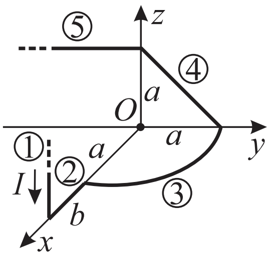
PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

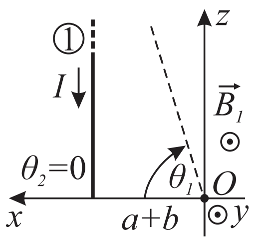
Ispit iz OET2, 15.04.2018.

I-1

a)



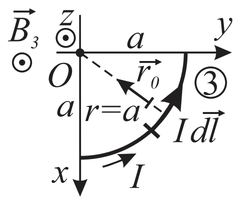
$$\boxed{\vec{B}_2 = 0} \qquad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$$



$$d = a+b, \qquad \theta_1 \to -\frac{\pi}{2}, \qquad \theta_2 = 0$$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi(a+b)}\left[0-(-1)\right] = \frac{\mu_0 I}{4\pi(a+b)}$$

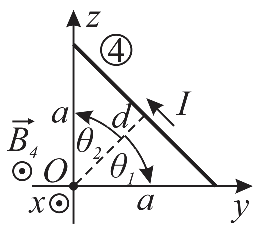
$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{4\pi(a+b)}\cdot\vec{i}_y}$$



$$\vec{dB}_3 = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_3 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_3 = \int dB_3 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{1}{4}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}\frac{a\pi}{2} = \frac{\mu_0 I}{8a}$$

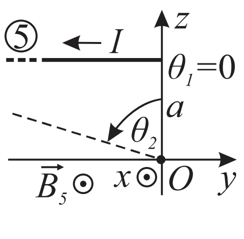
$$\boxed{\vec{B}_3 = \frac{\mu_0 I}{8a}\cdot\vec{i}_z}$$



$$d = \frac{a\sqrt{2}}{2}, \qquad \theta_1 = -\frac{\pi}{4}, \qquad \theta_2 = \frac{\pi}{4}$$

$$B_4 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi\frac{a\sqrt{2}}{2}}\left[\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right] = \frac{\mu_0 I}{2\pi a\sqrt{2}}\,2\frac{\sqrt{2}}{2} = \frac{\mu_0 I}{2\pi a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I}{2\pi a}\cdot\vec{i}_x}$$



$$d = a, \qquad \theta_1 = 0, \qquad \theta_2 \to \frac{\pi}{2}$$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}\left[1-0\right] = \frac{\mu_0 I}{4\pi a}$$

$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{4\pi a}\cdot\vec{i}_x}$$

$$\boxed{\vec{B}_{uk} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \left(\frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{2\pi a}\right)\cdot\vec{i}_x + \frac{\mu_0 I}{4\pi(a+b)}\cdot\vec{i}_y + \frac{\mu_0 I}{8a}\cdot\vec{i}_z = \frac{3\mu_0 I}{4\pi a}\cdot\vec{i}_x + \frac{\mu_0 I}{4\pi(a+b)}\cdot\vec{i}_y + \frac{\mu_0 I}{8a}\cdot\vec{i}_z}$$

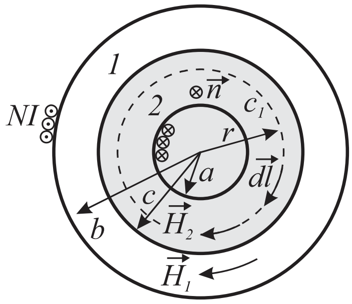
b)

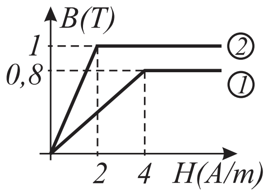
$$\vec{B}_{uk} = 50\,\mu T\cdot\vec{i}_x + 12{,}5\,\mu T\cdot\vec{i}_y + 26{,}2\,\mu T\cdot\vec{i}_z$$

$$\left|\vec{B}_{uk}\right| = \sqrt{\left(50\,\mu T\right)^2 + \left(12{,}5\,\mu T\right)^2 + \left(26{,}2\,\mu T\right)^2} \qquad \boxed{\left|\vec{B}_{uk}\right| = 57{,}82\,\mu T}$$

Ispit iz OET2, 15.04.2018.

I-2





Granični uslov: $H_{1t} = H_{2t} = H$

$$\oint_{c1} \vec{H} \cdot \vec{dl} = NI$$

$$H\,2r\pi = NI$$

$$H = \frac{NI}{2r\pi} \qquad a < r < b$$

a)

$$H_{1\min}(r = c) = \frac{NI}{2\pi c} \le H_{c1} = 4000\ A/m$$

$$I = \frac{H_{c1}\,2\pi c}{N} = \frac{4000 \cdot 2\pi \cdot 0,06}{314} \qquad \boxed{I = 4,8\ A}$$

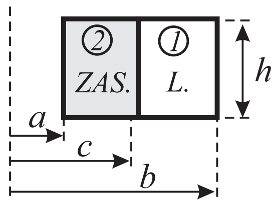
b)

$$H_{2\max}(r = a) = \frac{NI}{2\pi a} = 6000\ A/m$$

$$H_{2\min}(r = c) = \frac{NI}{2\pi c} = 4000\ A/m$$

Sledi da je materijal 2 u potpunom zasićenju.

c)

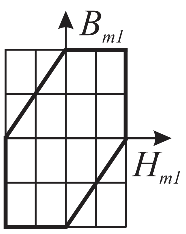


$$W_{m2} = \frac{1}{2} B_{c2} H_{c2} V_2 = \frac{1}{2} B_{c2} H_{c2} \left(c^2 - a^2\right) \pi\, h$$

$$W_{m2} = \frac{1}{2} \cdot 1 \cdot 2000 \cdot \left(0,06^2 - 0,04^2\right) \cdot 3,14 \cdot 0,03$$

$$\boxed{W_{m2} = 188,5\ mJ}$$

d)



$$w_{h1} = 12 \frac{B_{m1}}{2} \frac{H_{m1}}{2} = 3 B_{m1} H_{m1} = 3 \mu_1 \left(\frac{NI_m}{2r\pi} \right)^2$$

$$W_{h1} = f\ t\, W_{h1\,per} = f\ t \int_{V_1} w_{h1}\, dV = f\ t \int_c^b 3 \mu_1 \left(\frac{NI_m}{2r\pi} \right)^2 2r\pi\, dr\, h$$

$$W_{h1} = f\ t\, 3\, \mu_1 \frac{\left(NI_m\right)^2}{2\pi} h \int_c^b \frac{dr}{r} = f\ t\, 3\, \mu_1 \frac{\left(NI_m\right)^2}{2\pi} h \ln \frac{b}{c}$$

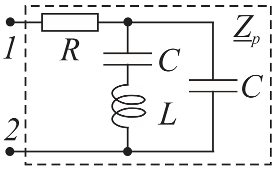
$$W_{h1} = 50 \cdot 120 \cdot 3 \cdot \frac{0,8}{4000} \cdot \frac{\left(314 \cdot 4,8\right)^2}{2 \cdot 3,14} \cdot 0,03 \cdot \ln \frac{0,08}{0,06}$$

$$\boxed{W_{h1} = 11238,8\ J}$$

Ispit iz OET2, 15.04.2018.

II-1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \,\|\, \frac{1}{j\omega C} = \frac{\frac{1-\omega^2 LC}{j\omega C}\frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C}+\frac{1}{j\omega C}} = \frac{\frac{1-\omega^2 LC}{(j\omega C)^2}}{\frac{2-\omega^2 LC}{j\omega C}} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} = -j\frac{1-\omega^2 LC}{\omega C\left(2-\omega^2 LC\right)}$$

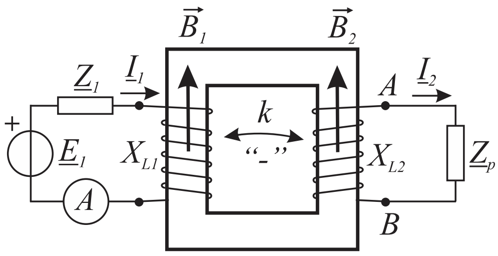
$\underline{Z}_p = R + \underline{Z}_2$ $\qquad$ $\boxed{\underline{Z}_p = R - j\frac{1-\omega^2 LC}{\omega C\left(2-\omega^2 LC\right)}}$

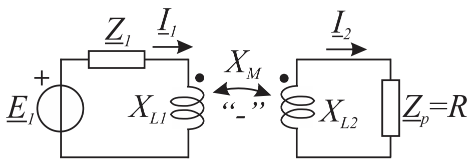
$\mathrm{Im}\{\underline{Z}_p\} = 0 \quad \Rightarrow \quad 1-\omega^2 LC = 0 \qquad \boxed{\omega_{r1} = \frac{1}{\sqrt{LC}}} \qquad \boxed{\omega_{r2} \to \infty}$

$\mathrm{Im}\{\underline{Y}_p\} = 0 \quad \Rightarrow \quad \omega C = 0 \qquad \boxed{\omega_{ar1} = 0}$

$2-\omega^2 LC = 0 \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$

b)





$$\omega = \omega_{r1} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\cdot 10^{-3}\cdot 100\cdot 10^{-6}}} = 1000\,\frac{rad}{s}$$

$$X_M = k\sqrt{X_{L1}\,X_{L2}} = 0{,}9\cdot\sqrt{80\cdot 20} = 36\,\Omega$$

c)

(1) $\quad -\underline{E}_1 + \underline{Z}_1\underline{I}_1 + jX_{L1}\underline{I}_1 - jX_M\,\underline{I}_2 = 0$

(2) $\quad jX_{L2}\underline{I}_2 - jX_M\,\underline{I}_1 + R\underline{I}_2 = 0$

d)

$I_{amp} = 1\,A = \left|\underline{I}_1\right|$

(2) $\quad \underline{I}_2 = \frac{jX_M}{R + jX_{L2}}\underline{I}_1$

(1) $\quad \underline{E}_1 = \underline{Z}_1\underline{I}_1 + jX_{L1}\underline{I}_1 - jX_M\frac{jX_M}{R+jX_{L2}}\underline{I}_1 = \left(\underline{Z}_1 + jX_{L1} + \frac{(X_M)^2}{R+jX_{L2}}\right)\underline{I}_1 = \left((20+j60)+j80+\frac{(36)^2}{20+j20}\right)\underline{I}_1$

$$\underline{E}_1 = \left(52{,}4 + j107{,}6\right)\underline{I}_1 = 119{,}68\,e^{j64{,}03^\circ}\,\underline{I}_1 = 119{,}68\,e^{j64{,}03^\circ}\cdot 1\,e^{j\psi} = 119{,}68\,e^{j(64{,}03^\circ+\psi)} = E_1\,e^{j\theta}\;V$$

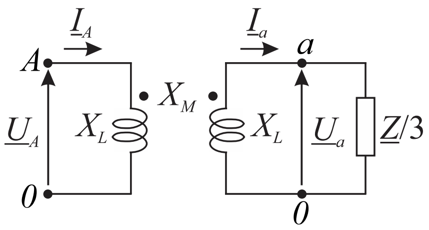
$E_{1m} = \sqrt{2}\,E_1 = \sqrt{2}\cdot 119{,}68\,V \qquad \boxed{E_{1m} = 169{,}25\,V}$

$\boxed{fazni\;\;pomeraj = \theta - \psi = 64{,}03^\circ}$

Ispit iz OET2, 15.04.2018.

II-2

a)



(1) $\underline{U}_A = jX_L \underline{I}_A - jX_M \underline{I}_a$

(2) $\underline{U}_a = -jX_L \underline{I}_a + jX_M \underline{I}_A$

(3) $\underline{U}_a = \dfrac{\underline{\underline{Z}}}{3} \underline{I}_a$

(3) $\Rightarrow$ (2) $\quad -jX_L \underline{I}_a + jX_M \underline{I}_A = \dfrac{\underline{\underline{Z}}}{3} \underline{I}_a \quad \Rightarrow \quad \underline{I}_a = \dfrac{jX_M}{\dfrac{\underline{\underline{Z}}}{3} + jX_L} \underline{I}_A = \dfrac{j10}{(10 - j20) + j20} \underline{I}_A = j1 \underline{I}_A$

(1) $\Rightarrow \quad \underline{U}_A = jX_L \underline{I}_A - jX_M \cdot j1 \underline{I}_A = jX_L \underline{I}_A + X_M \underline{I}_A$

$$\underline{I}_A = \frac{\underline{U}_A}{jX_L + X_M} = \frac{230\, e^{j0^\circ}}{10 + j20} = \frac{230\, e^{j0^\circ}}{22,36\, e^{j63,43^\circ}}$$

$$\boxed{\underline{I}_A = 10,28\, e^{-j63,43^\circ}\ A}$$

$$\boxed{\begin{aligned}\underline{I}_B &= 10,28\, e^{-j183,43^\circ}\ A\\ \underline{I}_C &= 10,28\, e^{-j303,43^\circ}\ A\end{aligned}}$$

$$\underline{I}_a = j1 \underline{I}_A = 1 e^{j90^\circ} \cdot 10,28\, e^{-j63,43^\circ}$$

$$\boxed{\underline{I}_a = 10,28\, e^{j26,57^\circ}\ A}$$

$$\boxed{\begin{aligned}\underline{I}_b &= 10,28\, e^{-j93,43^\circ}\ A\\ \underline{I}_c &= 10,28\, e^{-j213,43^\circ}\ A\end{aligned}}$$

(3) $\Rightarrow \quad \underline{U}_a = \dfrac{\underline{\underline{Z}}}{3} \underline{I}_a = (10 - j20) \cdot 10,28\, e^{j26,57^\circ} = 22,36\, e^{-j63,43^\circ} \cdot 10,28\, e^{j26,57^\circ}$

$$\boxed{\underline{U}_a = 229,86\, e^{-j36,86^\circ}\ V}$$

$$\boxed{\begin{aligned}\underline{U}_b &= 229,86\, e^{-j216,86^\circ}\ V\\ \underline{U}_c &= 229,86\, e^{-j336,86^\circ}\ V\end{aligned}}$$

b)

$$\boxed{I_{amp} = |\underline{I}_c| = 10,28\, A}$$

$$\boxed{U_{V1} = |\underline{U}_b| = 229,86\, V}$$

$$\boxed{U_{V2} = |\underline{U}_{ac}| = \sqrt{3}\, |\underline{U}_a| = \sqrt{3} \cdot 229,86 = 398,13\, V}$$

$$P_W = \mathrm{Re}\left\{\underline{U}_{ab}\, \underline{I}_a^{\,*}\right\} = U_{ab}\, I_a \cos\sphericalangle\left(\underline{U}_{ab}, \underline{I}_a\right) = \sqrt{3}\, U_a\, I_a \cos\left(30^\circ - 26,57^\circ\right) = \sqrt{3} \cdot 229,86 \cdot 10,28 \cdot \cos 3,43^\circ$$

$$\boxed{P_W = 4085,44\, W}$$

c)