STUDIJSKI PROGRAM ENERGETIKA, ELEKTRONIKA I TELEKOMUNIKACIJE

K TET

Katedra za teorijsku elektrotehniku

www.ktet.ftn.uns.ac.rs

Osnovi elektrotehnike 2 (I kolokvijum)

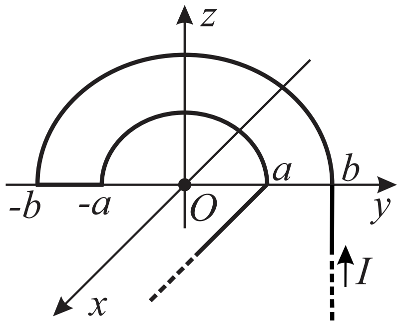
K1

21.10.2017.

ZADACI

Zadatak 1. Veoma dugačak žičani provodnik, sa vremenski konstantnom strujom jačine I, savijen je kao što je prikazano na slici 1. Provodnik se sastoji od tri pravolinijska i dva lučna segmenta, pri čemu jedan pravolinijski segment leži u y-z ravni Dekartovog pravouglog koordinatnog sistema, paralelno sa z osom, drugi leži na y osi, dok je treći u x-y ravni, paralelno sa x osom. Lučni segmenti, u obliku polovine kruga, poluprečnika a i b, leže u y-z ravni. Odrediti intenzitet vektora magnetske indukcije u koordinatnom početku, tačka O. Sredina je vazduh.

Brojne vrednosti: $I = 1\ A$, $a = 2\ cm$, $b = 3\ cm$.

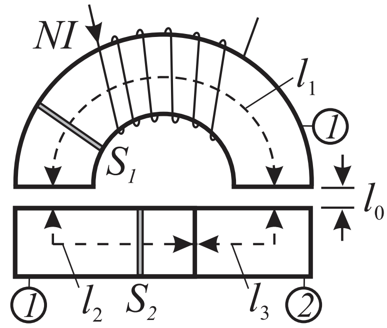


Slika 1.

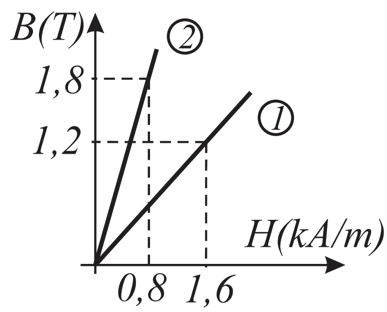
Zadatak 2. Na slici 2a je prikazano magnetsko kolo elektromagneta sa kotvom. Jezgro elektromagneta je načinjeno od materijala 1, dok je kotva načinjena od materijala 1 i 2. Idealizovane krive magnetisanja ovih materijala date su na slici 2b. Da bi elektromagnet privukao kotvu, u procepima treba ostvariti magnetsko polje indukcije $B_0 = 1{,}2\ T$.

a) Odrediti potrebnu vrednost pobude elektromagneta, NI.
b) Izračunati energiju koja se utroši na uspostavljanje magnetskog polja u jezgru elektromagneta i kotvi.

Površina poprečnog preseka jezgra elektromagneta je $S_1 = 2{,}5\ cm^2$, a kotve $S_2 = 2\ cm^2$. Dužine njihovih središnjih linija su $\ell_1 = 30\ cm$, $\ell_2 = 12\ cm$ i $\ell_3 = 8\ cm$. Efektivna površina preseka procepa je $S_0 = 3\ cm^2$. Širina procepa je $\ell_0 = 0{,}1\ mm$.



Slika 2a.



Slika 2b.

PRAVILA POLAGANJA

Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

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ZADACI

Zadatak 1. U mreži prostoperiodične struje prikazanoj na slici 1:

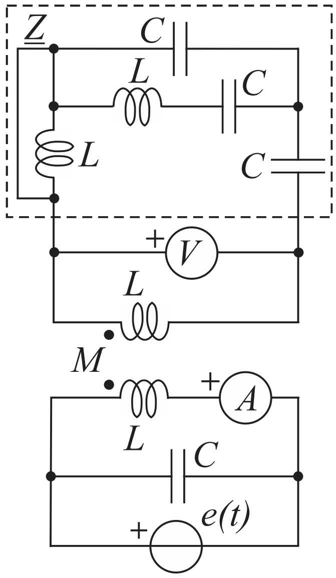
a) Odrediti sve rezonantne i antirezonantne kružne učestanosti prijemnika impedanse $\underline{Z}$.

b) Ako u kolu generator elektromotorne sile *e(t)* radi na većoj antirezonantnoj učestanosti, odrediti pokazivanje idealnih mernih instrumenata.

c) Napisati izraz za trenutnu vrednost jačine struje generatora elektromotorne sile *e(t)*, na većoj antirezonantnoj učestanosti.

Brojni podaci su: $e(t) = 10\sqrt{2}\cos\left(\omega t + 30^\circ\right) V$,

$L = 10\ mH$, $C = 200\ \mu F$, $M = 4\ mH$.



Slika 1.

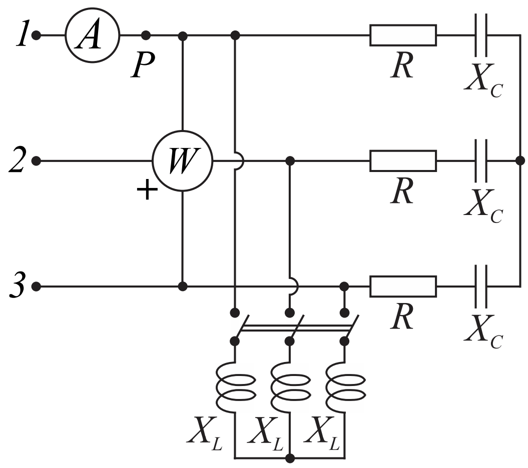
Zadatak 2. Na slici 2 je prikazan simetričan trofazni prijemnik, sastavljen od otpornika otpornosti $R = 60\ \Omega$ i kondenzatora reaktanse $X_C = 180\ \Omega$. Prijemnik je priključen na mrežu faznog napona $\underline{U}_1 = 230\ V$.

a) Odrediti reaktanse kalemova, koje je potrebno povezati, kako bi se faktor snage prijemnika popravio na jedinicu.

b) Odrediti pokazivanja idealnih mernih instrumenata posle priključivanja kalemova.

c) Odrediti pokazivanja idealnih mernih instrumenata, dok su kalemovi priključeni, ali kada se desi prekid provodnika prve faze, u tački *P*.

d) Na istom fazorskom dijagramu prikazati fazore svih veličina od kojih zavise pokazivanja instrumenata.



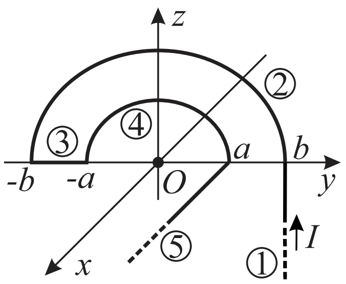
Slika 2.

PRAVILA POLAGANJA

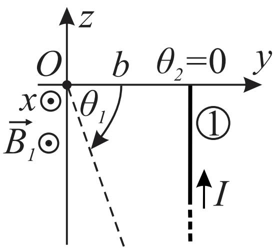
Za položen kolokvijum neophodno je tačno uraditi više od 50% svakog od zadataka. Svaki zadatak se boduje sa 25 poena. Kolokvijum traje jedan sat i trideset minuta.

Ispit iz OET2, 21.10.2017.

I-1



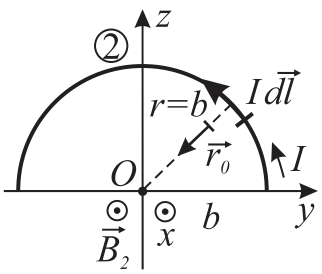
$$\boxed{\vec{B}_3 = 0} \qquad \sin\sphericalangle\left(\vec{dl},\vec{r}_0\right) = 0$$



$\boxed{1}$ $\qquad d = b, \qquad \theta_1 \to -\dfrac{\pi}{2}, \qquad \theta_2 = 0$

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi b}\left[0-(-1)\right] = \frac{\mu_0 I}{4\pi b}$$

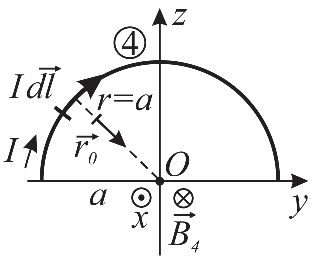
$$\boxed{\vec{B}_1 = \frac{\mu_0 I}{4\pi b}\cdot\vec{i}_x}$$



$$\vec{dB}_2 = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_2 = \frac{\mu_0}{4\pi}\frac{I\,dl}{b^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_2 = \int dB_2 = \frac{\mu_0 I}{4\pi b^2}\int_0^{\frac{1}{2}2b\pi} dl = \frac{\mu_0 I}{4\pi b^2}b\pi = \frac{\mu_0 I}{4b}$$

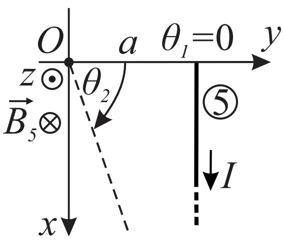
$$\boxed{\vec{B}_2 = \frac{\mu_0 I}{4b}\cdot\vec{i}_x}$$



$$\vec{dB}_4 = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}_0}{r^2} \qquad dB_4 = \frac{\mu_0}{4\pi}\frac{I\,dl}{a^2} \qquad \sphericalangle\left(\vec{dl},\vec{r}_0\right) = \frac{\pi}{2}$$

$$B_4 = \int dB_4 = \frac{\mu_0 I}{4\pi a^2}\int_0^{\frac{1}{2}2a\pi} dl = \frac{\mu_0 I}{4\pi a^2}a\pi = \frac{\mu_0 I}{4a}$$

$$\boxed{\vec{B}_4 = \frac{\mu_0 I}{4a}\cdot\left(-\vec{i}_x\right)}$$



$\boxed{5}$ $\qquad d = a, \qquad \theta_1 = 0, \qquad \theta_2 \to \dfrac{\pi}{2}$

$$B_5 = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) = \frac{\mu_0 I}{4\pi a}(1-0) = \frac{\mu_0 I}{4\pi a}$$

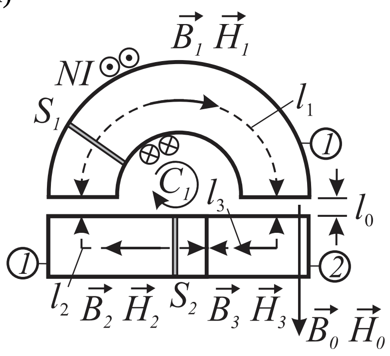
$$\boxed{\vec{B}_5 = \frac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right)}$$

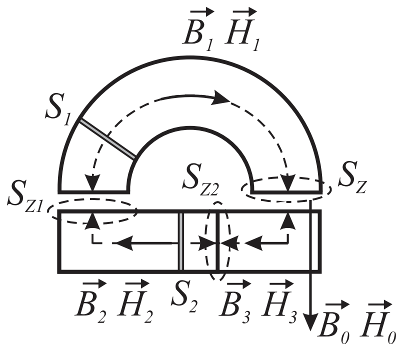
$$\boxed{\begin{array}{l}\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = \left(\dfrac{\mu_0 I}{4\pi b} + \dfrac{\mu_0 I}{4b} - \dfrac{\mu_0 I}{4a}\right)\cdot\vec{i}_x + \dfrac{\mu_0 I}{4\pi a}\cdot\left(-\vec{i}_z\right) \\ \vec{B}_O = 1{,}9\ \mu T\cdot\left(-\vec{i}_x\right) + 5\ \mu T\cdot\left(-\vec{i}_z\right)\end{array}}$$

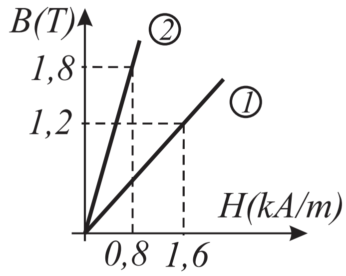
$$\left|\vec{B}_O\right| = \sqrt{\left(1{,}9\ \mu T\right)^2 + \left(5\ \mu T\right)^2} \qquad \boxed{\left|\vec{B}_O\right| = 5{,}35\ \mu T}$$

I-2

a)







$$\oint_{S_Z} \vec{B}\cdot\vec{ds}=0$$

$S_Z:\quad -B_1 S_1 + B_0 S_0 = 0 \qquad (1)$

$S_{Z1}:\quad -B_2 S_2 + B_0 S_0 = 0 \qquad (2)$

$S_{Z2}:\quad -B_3 S_2 + B_2 S_2 = 0$

$B_3 = B_2 \qquad (3)$

$$\oint_{C} \vec{H}\cdot\vec{dl}=NI$$

$C_1:\quad H_1 l_1 + 2H_0 l_0 + H_3 l_3 + H_2 l_2 = NI \qquad (4)$

$H_0 = \dfrac{B_0}{\mu_0} \qquad (5)$

Iz uslova zadatka sledi $B_0 = 1,2\,T$, $\Rightarrow$ $H_0 = \dfrac{B_0}{\mu_0} = \dfrac{1,2}{4\pi\cdot 10^{-7}} = 955,41\,kA/m$

Iz (1) sledi $B_1 = B_0\dfrac{S_0}{S_1} = 1,2\cdot\dfrac{3\cdot 10^{-4}}{2,5\cdot 10^{-4}} = 1,44\,T$

Sa krive magnetisanja sledi $H_1 = \dfrac{B_1}{\mu_1} = \dfrac{1,44}{\dfrac{1,2}{1600}} = 1920\,A/m$

Iz (2) sledi $B_2 = B_0\dfrac{S_0}{S_2} = 1,2\cdot\dfrac{3\cdot 10^{-4}}{2\cdot 10^{-4}} = 1,8\,T$

Sa krive magnetisanja sledi $H_2 = \dfrac{B_2}{\mu_1} = \dfrac{1,8}{\dfrac{1,2}{1600}} = 2400\,A/m$

Iz (3) sledi $B_3 = B_2 = 1,8\,T$

Sa krive magnetisanja sledi $H_3 = \dfrac{B_3}{\mu_2} = \dfrac{1,8}{\dfrac{1,8}{800}} = 800\,A/m$

Iz (4) sledi $NI = H_1 l_1 + 2H_0 l_0 + H_3 l_3 + H_2 l_2 = 1920\cdot 0,3 + 2\cdot 955,41\cdot 10^{3}\cdot 0,1\cdot 10^{-3} + 800\cdot 0,08 + 2400\cdot 0,12$

$\boxed{NI = 1119,1\,Azav}$

b)

$$W_{mJ} = \frac{1}{2}B_1 H_1 V_1 = \frac{1}{2}B_1 H_1 l_1 S_1 = \frac{1}{2}\cdot 1,44\cdot 1920\cdot 0,3\cdot 2,5\cdot 10^{-4} \qquad \boxed{W_{mJ} = 103,68\,mJ}$$

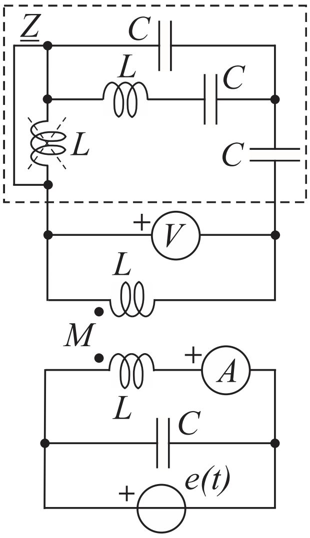
$$W_{mK} = \frac{1}{2}B_2 H_2 V_2 + \frac{1}{2}B_3 H_3 V_3 = \frac{1}{2}B_2 H_2 l_2 S_2 + \frac{1}{2}B_3 H_3 l_3 S_2$$

$$W_{mK} = \frac{1}{2}\cdot 1,8\cdot 2400\cdot 0,12\cdot 2\cdot 10^{-4} + \frac{1}{2}\cdot 1,8\cdot 800\cdot 0,08\cdot 2\cdot 10^{-4} \qquad \boxed{W_{mK} = 63,36\,mJ}$$

Ispit iz OET2, 21.10.2017.

II-1

a)



$$\underline{Z}_1 = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

$$\underline{Z}_2 = \underline{Z}_1 \parallel \frac{1}{j\omega C} = \frac{\frac{1-\omega^2 LC}{j\omega C}\,\frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C} + \frac{1}{j\omega C}} = \frac{\frac{1-\omega^2 LC}{j\omega C}\,\frac{1}{j\omega C}}{\frac{2-\omega^2 LC}{j\omega C}} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

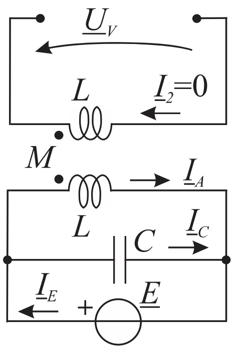
$$\underline{Z} = \underline{Z}_2 + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)} + \frac{2-\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}$$

$$\boxed{\underline{Z} = \frac{3-2\omega^2 LC}{j\omega C\left(2-\omega^2 LC\right)}}$$

$$\mathrm{Im}\{\underline{Z}\} = 0 \quad \Rightarrow \quad \boxed{\omega_{r1} = \sqrt{\frac{3}{2LC}}} \qquad \boxed{\omega_{r2} \to \infty}$$

$$\mathrm{Im}\{\underline{Y}\} = 0 \quad \Rightarrow \quad \boxed{\omega_{ar1} = 0} \qquad \boxed{\omega_{ar2} = \sqrt{\frac{2}{LC}}}$$

b)



$$e(t) = 10\sqrt{2}\cos\left(\omega t + 30^\circ\right) V$$

$$\underline{E} = \frac{10\sqrt{2}}{\sqrt{2}}\, e^{j30^\circ}\ V = 10\, e^{j30^\circ}\ V$$

$$\omega = \omega_{ar2} = \sqrt{\frac{2}{LC}} = 1000\,\frac{rad}{s}$$

$$j\omega L = j10, \qquad j\frac{1}{\omega C} = j5, \qquad j\omega M = j4$$

$$\underline{E} = j\omega L\,\underline{I}_A - j\omega M\,\cancel{\underline{I}_2}^{\,0} \quad \Rightarrow \quad \underline{I}_A = \frac{\underline{E}}{j\omega L} = \frac{10\, e^{j30^\circ}}{j10} = \frac{10\, e^{j30^\circ}}{10\, e^{j90^\circ}} = 1\, e^{-j60^\circ}\ A$$

$$\boxed{I_A = \left|\underline{I}_A\right| = 1\,A}$$

$$\underline{U}_V = -j\omega L\,\cancel{\underline{I}_2}^{\,0} + j\omega M\,\underline{I}_A = j4\cdot 1\, e^{-j60^\circ} = 4\, e^{j90^\circ}\cdot 1\, e^{-j60^\circ} = 4\, e^{j30^\circ}\, V$$

$$\boxed{U_V = \left|\underline{U}_V\right| = 4\,V}$$

c)

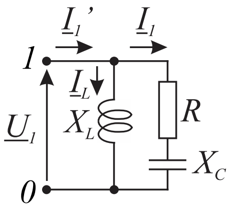
$$\underline{E} = -j\frac{1}{\omega C}\underline{I}_C \quad \Rightarrow \quad \underline{I}_C = \frac{\underline{E}}{-j\frac{1}{\omega C}} = \frac{10\, e^{j30^\circ}}{-j5} = \frac{10\, e^{j30^\circ}}{5\, e^{-j90^\circ}} = 2\, e^{j120^\circ}\ A$$

$$\underline{I}_E = \underline{I}_C + \underline{I}_A = 2\, e^{j120^\circ} + 1\, e^{-j60^\circ} = \left(-1 + j\sqrt{3}\right) + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 1\, e^{j120^\circ}\ A$$

$$\boxed{i_E(t) = \sqrt{2}\cos\left(1000t + 120^\circ\right) A}$$

II-2

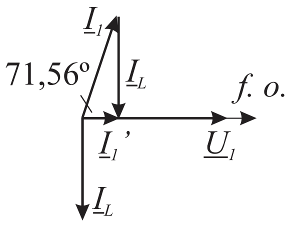
a)



$$\underline{Z} = R - jX_C = (60 - j180)\,\Omega = 189,7\,e^{-j71,56^\circ}\ \Omega$$

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \frac{U_1\,e^{j0^\circ}}{Z\,e^{j\varphi}} = \frac{230}{189,7\,e^{-j71,56^\circ}} = 1,21\,e^{j71,56^\circ}\ A$$

$$\underline{I}_L = \frac{U_1\,e^{j0}}{jX_L} = \frac{U_1\,e^{j0}}{X_L\,e^{j\frac{\pi}{2}}} = \frac{U_1}{X_L}\,e^{-j\frac{\pi}{2}}\ A$$



$$I_L = I_1\,\sin 71,56^\circ$$

$$\frac{U_1}{X_L} = \frac{U_1}{Z}\sin 71,56^\circ \quad\Rightarrow\quad \frac{1}{X_L} = \frac{\sin 71,56^\circ}{Z} \quad\Rightarrow\quad X_L = \frac{189,7}{\sin 71,56^\circ}$$

$$\boxed{X_L = 199,97\,\Omega}$$

I KZ: $\underline{I}_1' = \underline{I}_L + \underline{I}_1$

$$I_1' = I_1\,\cos 71,56^\circ = 1,21\cdot\cos 71,56^\circ = 0,38\ A \qquad \underline{I}_1' = 0,38\,e^{j0^\circ}\ A$$

$$\frac{U_1}{R_e} = \frac{U_1}{Z}\cos 71,56^\circ \quad\Rightarrow\quad \frac{1}{R_e} = \frac{\cos 71,56^\circ}{Z} \quad\Rightarrow\quad R_e = \frac{Z}{\cos 71,56^\circ} = \frac{189,7}{\cos 71,56^\circ}$$

$$R_e = 599,73\,\Omega$$

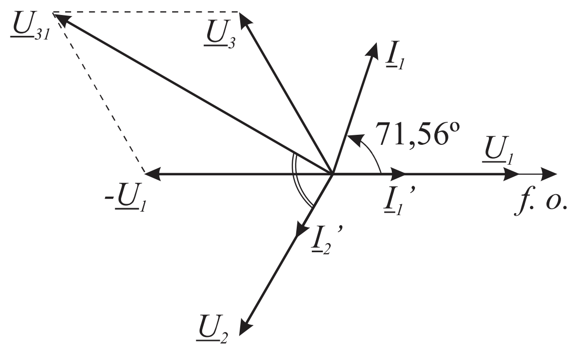
b)

$$\boxed{I_A^{(b)} = \left|\underline{I}_1'\right| = 0,38\ A}$$

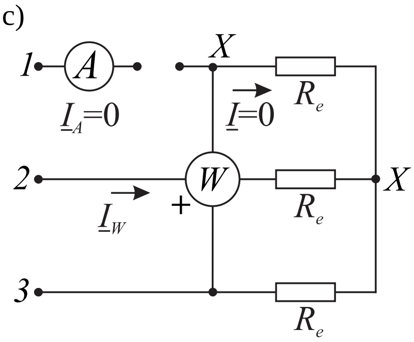
$$P_W^{(b)} = \mathrm{Re}\left\{\underline{U}_{31}\,\underline{I}_2'^{\,*}\right\} = U_{31}\,I_2'\cos\measuredangle\left(\underline{U}_{31},\underline{I}_2'\right)$$

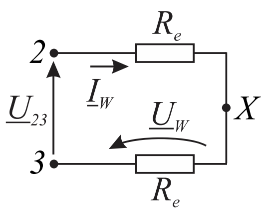
$$P_W^{(b)} = \sqrt{3}\,U_1\,I_1'\cos\left(30^\circ + 60^\circ\right) = \sqrt{3}\cdot 230\cdot 0,38\cdot\cos 90^\circ$$

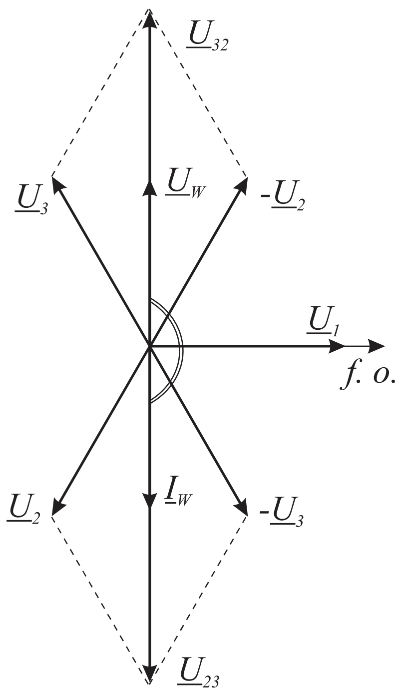
$$\boxed{P_W^{(b)} = 0\,W}$$



c)







$$\boxed{I_A^{(c)} = 0\ A}$$

$$\underline{U}_W = \underline{U}_{3X} = \frac{1}{2}\underline{U}_{32} = \frac{1}{2}\sqrt{3}\cdot 230\,e^{j90^\circ}\ V$$

$$\underline{I}_W = \frac{\underline{U}_{23}}{2R_e} = \frac{\sqrt{3}\cdot 230\,e^{-j90^\circ}}{2\cdot 599,73} = 0,33\,e^{-j90^\circ}\ A$$

$$P_W^{(c)} = \mathrm{Re}\left\{\underline{U}_W\,\underline{I}_W^{\,*}\right\} = U_W\,I_W\cos\measuredangle\left(\underline{U}_W,\underline{I}_W\right)$$

$$P_W^{(c)} = \frac{1}{2}\sqrt{3}\cdot 230\cdot 0,33\cdot\cos 180^\circ$$

$$\boxed{P_W^{(c)} = -65,73\,W}$$